ENHANCEMENT OF ELECTROMAGNETIC FORCE BY LOCALIZED FIELDS IN ONE-DIMENSIONAL PHOTONIC CRYSTAL

J. M. Li, T. L. Dong, and G. J. Shan

Department of Electronics and Information Engineering
Huazhong University of Science and Technology
Wuhan 430074, China

Abstract—In 1-D photonic crystal with structural defects, localized mode results in strong electromagnetic fields around the position of the defect. Thus, the strong fields enhance the tangential force on a lossy dielectric layer, as well as normal force on the perfect dielectric slab. The results of this study suggest a class of micro-machines driven by electromagnetic wave, such as sunlight or microwave.

1. INTRODUCTION

Generally, radiation pressure is big enough for manipulating the micro-objects [1]. For instance, in optical tweezers [2] it is used to levitate the little matter including viruses, bacteria, cells, and subcellular organisms, etc. Motivated by rapid development of micro electromechanical system, people are searching for the novel working principles of electromagnetic-wave-driven micro-motors [3], but the main problem is that the radiation pressure is too small for this application. Resonance effects can be used to increase the force. A model is lossless dielectric blocks in waveguide [4, 5], and the direction of the force exerted on the dielectric is parallel to the waveguide axis. The other model for enhancing the electromagnetic force is Bragg waveguide [6], in which the force is normal to the wall of waveguide. Essentially, this structure is a Fabry-Perot cavity. The peak of that force only appears at the structure's resonant frequency. Specifically, the force tends to separate the omnidirectional mirrors (presented in [6]). At the same time that would reduce the force dramatically. Practically, a micro-machine driven by normal force

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Corresponding author: J. M. Li (lijianming001@163.com).

can be realized in piston mechanism, but the incident wave should have continuous frequency spectrum in order to ensure that a right frequency component can always work on driving the moving part. On the other hand, if tangential force could be used, it can drive a rotator in rotating motor.

To establish the principles of enhancement of electromagnetic force, we analyzed the radiation pressure on each layer in 1-D photonic crystal, which consisted of air and lossless media with defects. It is shown by the calculation of force that radiation pressure on the layers near the defects increases at certain frequency. The layers on both sides of the defects experienced the largest force. No matter what the incidence angle is, there is a normal force on each layer, and the maximal normal force is very susceptible to the change of the wave frequency and structural parameters. On the contrary, tangential force only exerted on the lossy layer. If a layer next to the defects is moveable and consisted of lossy medium, then this layer could serve as moving part of a micro-motor. The force exerted on this moveable layer depends upon the structural parameters and dielectric loss factor.

In this study, we demonstrate that the maximum normal or tangential force can be induced while the defect mode was excited in the structure. In comparison with the single layer in the air, the force's value is increased. Physically, the effect mentioned above is due to localized field in photonic crystal with defect, which enhances the fields [7]. Thus, the radiation pressure on some layers in photonic crystal is greatly enhanced. This principle is also applied to two or three dimensional photonic crystals, and the defects' configuration can be designed according to actual need. Localized field enhances the electromagnetic force, just like it enhances the nonlinear effects in photonic crystals [8], and the effect results from interference strengthening of wave in photonic crystal with defects.

This paper is organized as follows. In Section 2, the model is described, and the way how to calculate the normal force or tangential force is derived. In Section 3, the numerical analysis of the force at different frequencies or at different structural parameters is presented. The paper is concluded in Section 4 with the obtained results and potential applications.

2. FORMALISM

Assuming that 1-D photonic crystal consists of four kinds of materials as shown in Fig. 1. The defect parts have three layers; one is a lossy dielectric layer (blue) with permittivity $\varepsilon_{3r} - j\varepsilon_r''$; one is a lossless dielectric layer (yellow) with permittivity ε_{3r}; one is an air layer
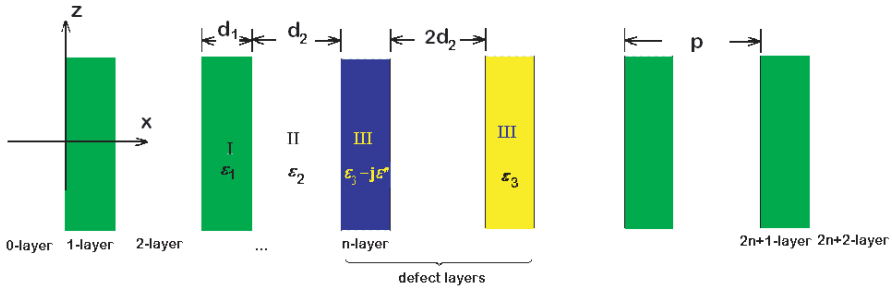
Figure 1. Configuration of the structure. The photonic crystal with defects is quarter-stack except the $(n+1)$th layer, and the $(n+1)$th layer's optical thickness is a half wavelength.

(white). Periodic layers consist of alternating high- and low-dielectric I (green) and II (white) with period $p = d_1 + d_2$, where d_1 and d_2 are I and II-layer's width respectively. The II-layer is air. All layers are nonmagnetic material.

Through characteristic matrix method [9], the fields in layer i is associated with a 2×2 matrix that is the function of the ith layer parameters

$$M_i(\omega, k_{xi}, d_i) = \begin{bmatrix} \cos k_{xi} d_i & j\frac{1}{p_i} \sin k_{xi} d_i \\ jp_i \sin k_{xi} d_i & \cos k_{xi} d_i \end{bmatrix} \tag{1}$$

where, $0 \leq i \leq 2n + 2$, and,

$$p_i = \begin{cases} \frac{k_{xi}}{\omega\mu} & TE \quad \text{mode} \\ \frac{\omega\varepsilon_i}{k_{xi}} & TM \quad \text{mode} \end{cases} \tag{2}$$

In above equations, 'k_{xi}' is the propagation constant along the x axis in the layer i; d_i is the thickness of the layer i; ω is the angular frequency of incident wave.

The characteristic matrix relates the total fields from one side to the other in layer i. By using the boundary conditions, the transmitted and reflected fields can be deduced from the incidence fields. Then, the distribution of fields in every layer can be determined layer by layer. Each layer may experience both normal and tangential forces. If one layer is made of lossless media, the tangential force is zero [10], regardless of incidence angle. Otherwise, if it is lossy, the tangential force is not zero [5]. So, we put a lossy layer in the position of localized field in order to receive the maximum tangential force. The direct application of Lorentz force method [11] is chosen to calculate the force. The interfacial bound charges on the interface between $(i-1)$th

and ith layers receive the surface force per unit area as,

$$f_{sx} = \frac{1}{2}\text{Re}\left\{\rho_P \frac{1}{2}\left[E_{i-1,x}\left(x_{i-1}^-\right) + E_{i,x}\left(x_{i-1}^+\right)\right]^*\right\} \tag{3}$$

$$f_{sz} = \frac{1}{2}\text{Re}\left\{\rho_P E_{i-1,z}(x_{i-1}^+)^*\right\} \tag{4}$$

where, ρ_P stands for interfacial bound charges density; $E_{i,x}(x_{i-1}^+)$ represents the electric field along x-direction in layer i at x_{i-1}^+. The volume force density is written

$$f_v = \frac{1}{2}\text{Re}\left\{\vec{J}_P \times \vec{B}^*\right\} \tag{5}$$

here, $\vec{J}_P = \partial\vec{P}/\partial t = -j\omega\varepsilon_0(\varepsilon_{ir} - 1)\vec{E}_i$ represents bound current density, and ε_{ir} is the i-layer's permittivity. The force exerted on layer i is the sum of surface force and volume force: $\vec{f} = \vec{f}_s + \vec{f}_v$, and surface forces must consider the contributions of two sides of layer i.

If we want to use normal force, the incident frequency should be in the photonic band gap. The easiest model is that all layers are consisted of lossless media with a single defect. The $(n + 1)$th layer that consists of air is defect layer, and the nth or $(n+2)$th layer has the same permittivity as the first layer if n is an odd number. Otherwise their permittivities are the same as the second layer. Assuming that the refractive index coefficients of those alternating media are n_1 and n_2, and incidence angle is zero. The band gap of structure is [12]

$$\begin{aligned}&[f_{\min}, f_{\max}]\\&= \left[c\left(2\frac{1}{\pi}\arcsin\frac{2\sqrt{n_1 n_2}}{n_1 + n_2}\right)/\lambda_0, 2c\left(1 - \frac{1}{\pi}\arcsin\frac{2\sqrt{n_1 n_2}}{n_1 + n_2}\right)/\lambda_0\right]\end{aligned} \tag{6}$$

where, λ_0 is the center wavelength of this quarter-stack. 'c' is the speed of light in vacuum. When the layers from $(n+2)$th to $(2n+1)$th were bonded in one and moved along x axis, the defect mode frequency would vary with x values of this movable part. Assuming that the air layer thickness is $X\lambda_0/4$, where 'X' means that the air layer thickness can vary continuously with the movement of the movable part, we obtain the defect mode frequency as,

$$f_d = 2cm/(X\lambda_0), \ (m = 1, 2, 3, \ldots) \tag{7}$$

where 'm' indicates that the times go forth and back. The distance covered by the light with defect mode frequency is $m\lambda_0$. The defect mode frequency is a continuous function of the $(n+1)$th layer thickness. Different $(n+1)$-layer's thicknesses mean that the movable part moved to different locations. So, wherever the movable part is, there is

always a light with defect mode frequency working on the moving part. Deducing from Equations (6) and (7), we got the minimum thickness of the air layer as $cm/(2f_{\max})$, and the maximum thickness is $cm/(2f_{\min})$. Then the maximum displacement of the movable part is $cm[1/f_{\min} - 1/f_{\max}]/2$. When the parameters were properly selected, a large range of movement can be achieved.

3. NUMERICAL EXAMPLES

Normal force and tangential force are numerically analyzed by the above method, and particularly, the permittivities are $\varepsilon_{1r} = 2.35^2$ and $\varepsilon_{3r} - j\varepsilon_r'' = 2.28^2 - j0.006$. $n = 15$, the layers' total number in this structure is 31, and the whole structure is in air. We chose 600 nm as the center wavelength for this quarter-stack. The incident wave is TM mode with incidence angle 45 degree, and the incident magnetic field's magnitude is H_0.

Firstly, we considered the relationship between force and frequency. The transmission coefficient and normal or tangential forces per unit area on the n-layer were calculated and shown in Fig. 2, The force varies with the frequency. Then the most representative frequencies, namely, band edge frequencies a and b, band gap frequency
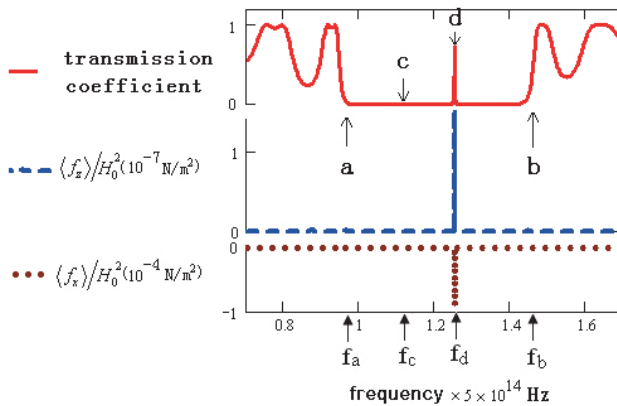

Figure 2. Time-average forces on the n-layer and transmission coefficient is shown. We focus on the character of force at four kinds of frequencies, which are marked with an arrow (a, b, c and d). Frequency a and b is the band edge, frequency c is in the band gap, and defect mode is excited at frequency d. Additionally, the peaks of the time-average normal (doted curve) or tangential (dashed curve) force on the n-layer appear at the frequency d.

c, defect mode frequency d, are marked in Fig. 2. It is worth noting that the maximum force appears at frequency d when defect mode is excited in the structure. Next, the fields and the magnitude of the normal or tangential force on each layer would be calculated at these four frequencies in this section.

Obviously, the normal force is larger than the tangential force at frequency d. The normal force on the nth layer is pulling effect, but if the nth layer changed its position even a little bit along the normal force's direction under irradiation of monochromatic wave, the normal force will almost vanish immediately. For tangential force, even though the n-layer moved along tangential force's direction, since the boundary condition did not change, the fields in the structure did not change too. This characteristic would make a machine conveniently work if the n-layer is used as rotator.

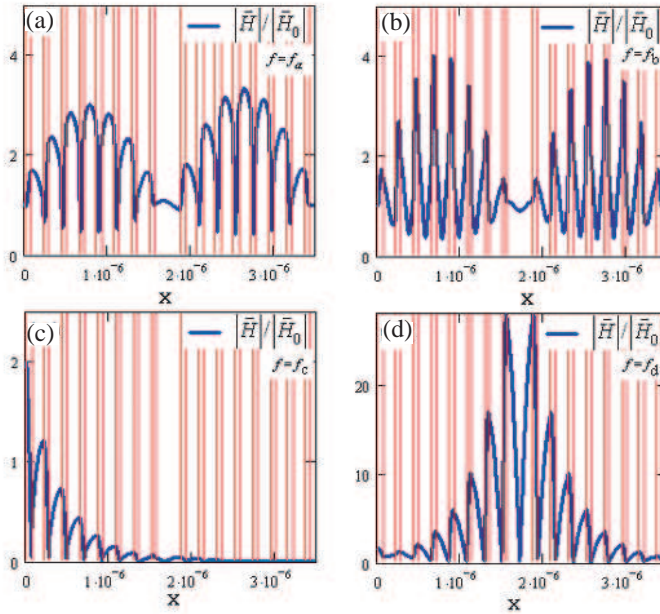Four kinds of distribution of magnetic fields at different frequencies

Figure 3. The localized fields appeared at three kinds of frequency a, b and d. Frequency a and b are called band edge, frequency d means that defect mode can be excited in that structure, especially, the value of magnetic fields' peak gain its maximum at frequency d. (d) shown that the field get its peak in the nth and $(n+2)$th layer. On the contrary, (c) draws an evanescent wave at frequency c, where light transmission is prohibited.

in photonic crystal are calculated and illustrated by Fig. 3.

Now, we calculate the normal force exerted on each layer as shown in Fig. 4. Out of the knowledge available, the normal force's effect does not always separate the layers of the structure, namely, at frequencies a and b, the normal force is to squeeze them. Furthermore, the largest normal force at frequencies a and b does not appear on the nth layer. In comparison, the normal force has more concentrated form at frequencies c and d. Remarkably, that normal force on the nth layer at frequency d was enhanced more than 100 times compared with the force when it is in the air. Besides, if the incidence angle is zero, enhancement will get a better effect. For example, all structural parameters does not change, but the wave frequency is 5×10^{14} Hz which is less than frequency c but more than frequency a. Anyhow, the
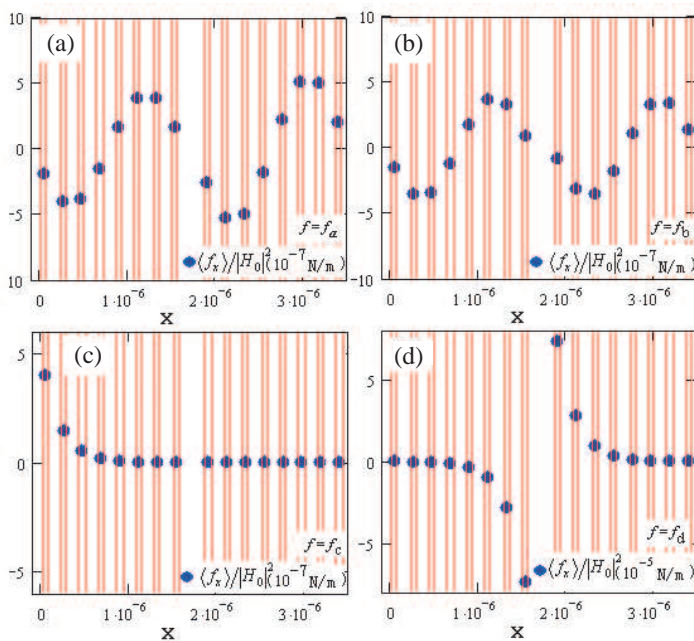

Figure 4. The normal force on each layer at frequency a and b are not only used to separate the layers of the structure but also to squeeze them. The maximum normal force acts on the nth layer at frequency d where the defect mode can be excited, and it is 862 times than the force when that single layer was under the same irradiation in the air. (c) shows that evanescent wave produce an evanescent force at frequency c.

defect mode can be excited under vertical incidence too. Additionally, $n = 25$, the normal force on the nth layer is $-1.061 \times 10^3 \, \text{N/m}^2$ when the layer is made of lossless media. Furthermore, the normal force can become bigger if the number of layers and parameters are optimized. For instance, if $n = 41$, the normal force on 41st layer gets $-1.032 \times 10^8 \, \text{N/m}^2$. Next, we calculate the force on the n-layer at defect mode frequency with different loss factors. It demonstrates that the normal force decrease as the loss factor rises. It is similar to [6], all photonic crystals with defects can induce considerable internal force, if it is under right conditions. This suggests that lossless photonic crystal with defects can be used as a spring driven by direct sunlight. Consider the $(n + 1)$-layer as a channel, and the whole structure is divided into two parts by this channel, although the channel width changes, there is always a right frequency component of sunlight acting its maximum normal force on two parts of the structure. Movement of one part does not change the effect that the $(n + 2)$th to $(2n + 1)$th layers receive a relative larger force, provided the incidence wave, such as the sunlight, has continuous frequency spectrum. A possible configuration of spring is illustrated in Fig. 5. All layers are made of lossless media, and the permittivities are $\varepsilon_{1r} = 2.35^2$, $\varepsilon_{3r} = 1.2$, $\varepsilon_{2r} = 1.2$. $n = 50$. The 51-layer is air, which is defect layer, and the center wavelength is 500 nm. According to Equation (6), the band gap is $3.33 \times 10^{14} - 8.67 \times 10^{14} \, \text{Hz}$, which almost covers the whole visible spectrum. The air layer's width can vary from 173 nm to 450 nm if $m = 1$, from 346 nm to 900 nm if $m = 2$, and from 519 nm to 1350 nm, if $m = 3$, etc. The movable part can receive the largest normal force in motion. If all frequency components' power is $1 \, \text{W/m}^2$, the total force is $1655 \, \text{N/m}^2$. Furthermore, there are more frequency components doing work when the movable part moved from 346 nm to 450 nm,
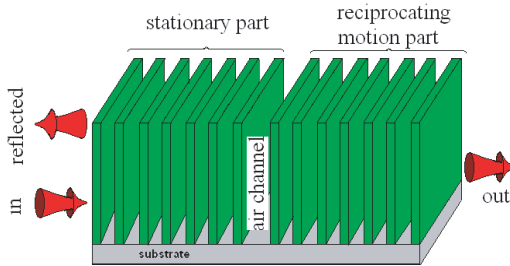

Figure 5. Incidence angle is zero, and irradiated by sunlight, the $(n + 2)$th to $(2n + 1)$th layers will serve as a piston. From qualitative analysis, this device can operate theoretically.
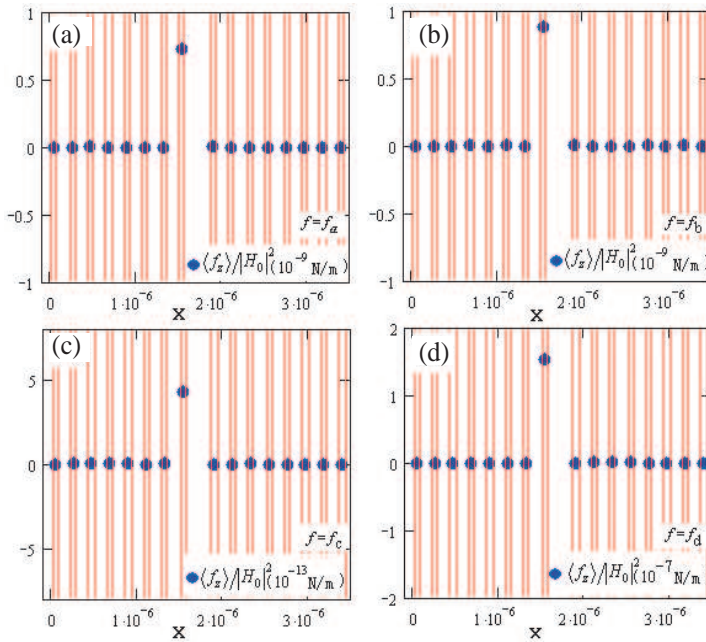
Figure 6. The tangential force on every layer at frequency a, b, c and d.

but it is a single frequency component doing work in the journey from 173 nm to 346 nm.

For tangential force, it does not act upon all layers. Only that lossy layer will experience the tangential force (shown in Fig. 6). Tangential force on the nth layer at frequencies a and b was of the same order of magnitude, and all of them were smaller than tangential force at frequency d. Due to the defect mode, the magnetic field was increased 27 times compared with the input ones (shown in Fig. 3(d)), which results in the enhancement of tangential force. Fig. 6(d) shows that the tangential force can get its peak in this structure on the n-layer. If incident magnetic field is a unit, the time average tangential force is 1.525×10^{-7} N/m^2. At the same time, supposing that the layer is not in photonic crystal, but in the air, the tangential force is only 6.301×10^{-10} N/m^2 under the same irradiation. As we expected, the tangential force on the n-layer in photonic crystal with defect was increased 242 times than the force when it was not in them. Although the tangential force is smaller than the normal force, the great advantage of this force is that tangential force did not change even if the nth layer moved, and this characteristic is suitable for making
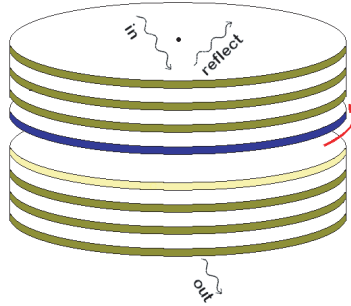
Figure 7. A possible model are designed for using tangential force to do work, the red arrow (online) shows the nth layer rotation's direction.

a rotating machine. When the nth layer moved at low velocity along the interface, the boundary condition did not change. Moreover, under low velocity condition, the relativistic effect can be ignored; the fields will remain the same as the rest stage. Fields do not change, and then naturally the tangential force keeps its original magnitude in motion. A possible model to make micro-motor using this structure is shown in Fig. 7.

In order to evaluate the influence of incidence angle, we calculated the tangential force at different incidence angles. We found that the tangential force existed only at oblique incidence. Subsequently, oblique incidence is required in rotator model, as illustrated in Fig. 7.

4. CONCLUSION

We analyzed the electromagnetic force in finite 1-D photonic crystal with defects. Fields localized around the defects resulted in the enhancement of force, especially the normal force at defect mode frequency. The normal force in photonic crystal with defects can not only to separate the structure, but also to squeeze the structure. These effects depend on which frequencies of wave are used. During the band gap, electromagnetic wave pushes the whole structure, but at the defect mode frequency, this force is to divide the structure into two parts, and at band edges, the normal force is not only to separate some layers of the structure but also to squeeze them. To generate a tangential force, two conditions are required here. First, there should be lossy layer; second, the wave should strike the structure obliquely.

Based on above principle, a novel micro-motor model is introduced, including two kinds of devices. We need to use the normal

force, because its maximum value is at defect mode, and at other frequencies, normal force is too small. So, the sunlight can be utilized because of its continuous spectrum. At the same time, all layers of structure are made of lossless media. Thus, different frequency components of sunlight make their own contributions to the movable layer in turn. Although the part is moved, there is always a light with certain frequency executing the largest force on the movable part. If we need to use the tangential force, an oblique incident wave micromotor is considered, because the tangential force appears when the layer is made of lossy media, and the incidence angle is not zero.

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