Abstract—The coupling between an irradiated aperture and a monopole antenna into a complex enclosure is investigated. The aperture is realized at the one side of the enclosure and the monopole antenna at the other side. The proposed study uses Babinet's principle to extend the Random Coupling Model to determine the radiation impedance of apertures. An experimental study is carried out using a computer box as an enclosure. A high intensity external electromagnetic radiation is applied to the aperture. The induced voltages is measured along the monopole antenna. The simulated probability levels of the induced voltages agree well with the experimental ones.
1. INTRODUCTION

The significant progress in the development of civil and military modern electronic applications has led to the increase of electromagnetic interference sources. On the other hand, due to their high electronic characteristic performances (high architecture complexity, high sensitivity, large bandwidth, high bit rate, etc., . . . ), the modern electronic systems become more and more vulnerable to these electromagnetic sources which affect their performance and functionality in particular via Front-Door Coupling and Back-Door Coupling. Indeed, the performances of electronic circuit blocks and devices housed within the enclosure can be affected by these electromagnetic threats through various coupling ports commonly used in electronic systems. To predict these electromagnetic effects, Zheng et al. [4] have proposed a stochastic model based on Random Matrix Theory (RMT) of chaotic systems. The model allows to quantify the coupling process by the non statistical radiation impedance of the coupling ports. The scattering parameter of one port is characterized knowing the radiation impedance of this port (Hemmady [8]). This method has been generalized to treat multiport scattering problems (Zheng [5]) and addressed to the coupling between monopole antennas. It has been used to numerically determine the probability levels of induced voltages along the monopole antennas. It has been also experimentally validated for two monopole antennas (Hemmady [1, 2]). Method using the Babinet’s principle has been also proposed by Konefal [11] to study the prediction of shielding effectiveness of a box containing a rectangular aperture.

In this work, we use the Babinet’s principle to extend the Zheng’s approach [5] to predict the aperture coupling effects by determining the radiation impedance of apertures. The first paragraph presents the proposed method, particularly the determination of the expression of the radiation impedance. The second paragraph compares experimental results to those obtained from this method.

2. DETERMINATION OF THE IMPEDANCE OF THE APERTURE

Babinet’s principle allows us to express the impedance of an aperture in terms of a simpler complementary structure; a dipole antenna.

The impedance of the two complementary structures $Z_{aperture}$ and $Z_{dipole}$ are related by (1) [9–11].

$$Z_{aperture}Z_{dipole} = \frac{\eta^2}{4},$$

(1)
where \( Z_{\text{aperture}} \) is the impedance of the aperture (defined in the center of the aperture), \( Z_{\text{dipole}} \) is the impedance of the complementary dipole and \( \eta \) is the impedance of free space \( \eta = 120\pi \Omega \).

Moreover, if we have a planar dipole antenna of width \( a \) and length \( b \), in [7] Newman suggests that this structure is equivalent to a wire dipole of radius \( r \) given by \( 4r = a \). To find \( Z_{\text{dipole}} \), we solve the Pocklington’s integral equation for cylindrical dipoles of length \( b \) and radius \( r \) [9]:

\[
\int_{\text{dipole}} I_z (z') \left[ \left( \frac{d^2}{dz'^2} + k^2 \right) G(z, z') \right] dz' = -j\omega \epsilon E_z (\rho = r), \tag{2}
\]

where \( E_z (\rho = r) \) represents the tangential component of the electric field along the dipole (the wire antenna axis coincides with the \( z \)-axis). The Green’s function \( G(z, z') \) is given by:

\[
G(z, z') = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{e^{-jkR}}{4\pi R} d\phi', \tag{3}
\]

with

\[
R(\rho = r) = \sqrt{4r^2 \sin \frac{\phi'}{2} + (z - z')^2} \tag{4}
\]

In order to use (2)–(4), we need to know \( E_z (\rho = r) \). By using the method of “magnetic-frill generator”, \( E_z (\rho = r) \) is written as [9]:

\[
E_z (\rho = r) \simeq -V_i \left( \frac{kAe^{-jkR_0}}{8 \ln \left( \frac{b}{r} \right) R_0^2} \left\{ 2 \left[ \frac{1}{kR_0} + j \left( 1 - \frac{A}{2R_0} \right) \right] + \frac{r^2}{R_0} \left[ \left( \frac{1}{kR_0} + j \left( 1 - \frac{B}{2R_0} \right) \right) \left( -jk - \frac{2}{R_0} \right) + \left( -\frac{1}{kR_0^2} + j \frac{B}{R_0^3} \right) \right] \right\} \right), \tag{5}
\]

where \( V_i \) is the voltage supplied by the source, \( b \) is the outer radius of the equivalent annular aperture of the magnetic-frill generator, \( R_0 = \sqrt{z^2 + r^2} \), \( A = (b^2 - r^2) \) and \( B = (b^2 + r^2) \). Finally, we solve from (2) to (5) using the moment method (MoM). The radiation impedance of a dipole is determined by the ratio of the voltage to current [9].

\[
Z_{\text{dipole}} = \frac{V_i}{I_{in}} \tag{6}
\]

For our application, (1) can be rewritten as:

\[
Z_{\text{aperture}} = \frac{\eta^2}{4} \frac{1}{Z_{\text{dipole}}} = \frac{\eta^2}{4} \frac{I_{in}}{V_i} \tag{7}
\]

We model the cavity-monopole system as a 2-port network. We designate the aperture as Port 1 and the monopole as Port 2 (Figure 1).
A first experimental study using the computer box as a complex enclosure was carried out with two monopole antennas. A monopole designated as Port 1 and the second as Port 2. The monopole antennas are inside the cavity (Figure 1 right). This study enabled us to validate the Random Coupling Model with this type of antenna and to determine the loaded quality factor of the enclosure. Figure 2 depicts the scattering parameters measured:

We now transform these scattering parameters ($S$-parameters) to the impedance parameters ($Z$-parameters) using a simple transformation $Z = Z_0^{1/2}(I + S)(I - S)^{-1}Z_0^{1/2}$, where $Z$ is the impedance matrix, $S$ is the scattering matrix, $I$ is the identity matrix and $Z_0$ is the characteristic impedance matrix. The $Z_0$ matrix is a real diagonal matrix. Its elements are the characteristic impedances of the driving ports. Figure 3 presents the impedance parameters.
We can notice that $|Z_{11}| \gg |Z_{21}|$ ($\frac{|Z_{11}|}{|Z_{21}|} > 86$) and the quantity $|Z_{21}|$ is close to zero. Consequently, to simplify our study, we consider the radiation impedance matrix $Z_r$ as a diagonal matrix. In this condition, we can determine $Z_r$ of our system as:

$$Z_r = \begin{pmatrix} Z_{\text{aperture}} & 0 \\ 0 & Z_{\text{monopole}} \end{pmatrix}$$  \hspace{1cm} (8)

where the radiation impedance of the monopole antenna $Z_{\text{monopole}}$ could be easily determined either numerically or experimentally. Now, we introduce $Z_r$ in the model proposed by Zheng. The $Z$ matrix of our system is then given by:

$$Z = j \text{Im} (Z_r) + (\text{Re} (Z_r))^\frac{1}{2} \tilde{z} (\text{Re} (Z_r))^\frac{1}{2},$$  \hspace{1cm} (9)

with, $\tilde{z}$ being an universal fluctuating quantity called “normalized impedance” obtained considering that the cavity is overmoded and the eigenfunctions satisfy the “random plane wave” hypothesis. It describes the scalar cavity impedance of a cavity which is perfectly coupled to its driving ports, i.e., $Z_r = Z_0$. In this case, the imaginary part of $Z_r$ is equal to zero and the real part of $Z_r$ is equal to $Z_0$ (Zheng [4, 5], Hemmady [6]).

$$\tilde{z} = -\frac{j}{\pi} W \frac{1}{\lambda - j\alpha I} W^T$$  \hspace{1cm} (10)

The matrix $W$ shown in (10) is a random coupling-matrix with each element $W_{ij}$ representing the coupling between the $i$th driving port and the $j$th eigenmode of the cavity. Each $W_{ij}$ is an independent Gaussian distributed number with the mean equals to zero and the

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**Figure 3.** Magnitudes of the $Z$-parameters; $|Z_{11}|$ and $|Z_{21}|$.

**Figure 4.** The conical antenna and the computer box in the anechoic chamber (CE-SAME equipment).
variance equals to 1. $\lambda$ is a random diagonal matrix which represents the distribution of eigenfrequency of the cavity. These eigenvalues use the GOE (Gaussian Orthogonal Ensemble) of the Random Matrix Theory. $\alpha = (k^3V)(2\pi^2Q)^{-1}$ is called the cavity loss-parameter (Hemmady [1, 3]), $k = 2\pi fc^{-1}$ is the wave-number, $f$ and $c$ represent respectively the frequency of incident wave on driving ports and the speed of light in vacuum equal to $3 \times 10^8 \text{ms}^{-1}$, $V$ and $Q$ represent respectively the volume and the loaded-quality factor of the cavity. The Equations (9) and (10) have been established on the hypothesis that the wavelength is smaller than the characteristic dimensions of the box.

To calculate the induced voltages along the monopole antenna, we have determined the incident power on the aperture [11]:

$$P_{\text{aperture}}^i = P_1 = \frac{E^2}{\eta}S_{\text{aperture}}$$

where $S_{\text{aperture}}$ is the surface of the aperture. From (8)–(11), we are able to determine probability levels of the induced voltages along the target monopole antenna.

3. THEORETICAL AND EXPERIMENTAL RESULTS

The experimental set up has been developed to verify the theoretical part. The complex enclosure used is a computer box of dimensions $44 \times 43 \times 20 \text{cm}^3$. It is excited by an irradiated $14 \times 4 \text{cm}^2$ rectangular aperture (Port $n^\circ 1$). The target-circuit is modeled by a monopole antenna of $18 \text{mm}$ length (Port $n^\circ 2$). To modify the repartition of eigenfrequency, we have placed a mode stirrer in the computer box. Measurements were carried out for 18 positions of stirrer and for two polarizations ‘A’ and ‘B’, respectively perpendicular and parallel to the aperture. The aperture is excited using a conical antenna. Measurements have been taken in an anechoic chamber as show in Figure 4.

The experiments were carried out with a network analyzer from 1 GHz to 6 GHz. The experimental setup provides the transmission coefficients $S_{21}$. Equation (12) gives the relation between the induced voltage $V_2$ along the monopole antenna and various system parameters:

$$V_2 = \ln \left[ \cot \left( \frac{\theta_0}{2} \right) \right] d S_{21} E$$

$\theta_0$ is the angle of the conical antenna, $d$ is the distance between the computer box and the cone apex. The incident electric-field $E$ on the aperture is generated by the conical antenna. The loaded-quality factor
$Q$ of the cavity is determined from the $|S_{21}|^2$ parameters, which has a maximum value at the resonant frequency. The loaded cavity quality factor $Q$ is then the ratio of the resonant frequency to the $-3$ dB bandwidth. We assume that the losses do not change significantly in a narrow pool of frequency. We estimate the average value of the quantity $Q$ to 56 from 4 GHz to 5 GHz. Figure 5 depicts the probability density functions of the induced voltages at the monopole antenna with an electric field interpolated equal to $1 \text{ kV m}^{-1}$ from 4 GHz to 5 GHz. We find a good agreement between the experimental probability density functions and those determined from numerical model.

Table 1 summarizes the various numerical and experimental statistical data. The mean values were computed versus the frequency from 4 GHz to 5 GHz and for the 18 stirrer positions.

Using the Table 1, we notice that the perpendicular polarization is more effective in term of transmission than the parallel polarization ($\langle V_2 \rangle_{\text{polar.A}} > \langle V_2 \rangle_{\text{polar.B}}$). These agree well with the theory of

| Polarization ‘A’ | | Polarization ‘B’ |
|---|---|---|---|
| Numerical | Experimental | Numerical | Experimental |
| $\langle V_2 \rangle$ (V) | 2.31 | 2.25 | 1.41 | 1.42 |
| $\sigma_{V_2}$ (V) | 1.30 | 1.26 | 0.80 | 0.77 |

**Figure 5.** Probability density functions of the induced voltage along the monopole antenna for the two polarizations: (a) Polarization ‘A’ and (b) polarization ‘B’. The red stars represent the experimental data and the blue line represents numerical data.

**Table 1.** Statistical data of the induced voltage $V_2$ along the monopole antenna. $\langle V_2 \rangle$ corresponds to the average of $V_2$ and $\sigma_{V_2}$ is equal to the standard deviation of $V_2$. 

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Figure 6. The cumulative distribution functions of the quantity $V_2/\langle V_2 \rangle$ (dB) for the polarizations ‘A’ and ‘B’.

Table 2. Probabilities of $V_2$ for the two polarizations: (a) Polarization ‘A’ and (b) polarization ‘B’.

<table>
<thead>
<tr>
<th>Levels of $V_2$ (V)</th>
<th>Probabilities (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.5 \leq V_2 \leq 2.0$</td>
<td>17</td>
</tr>
<tr>
<td>$1.0 \leq V_2 \leq 2.5$</td>
<td>39</td>
</tr>
<tr>
<td>$0.5 \leq V_2 \leq 3.5$</td>
<td>46</td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>Levels of $V_2$ (V)</th>
<th>Probabilities (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.8 \leq V_2 \leq 1.2$</td>
<td>20</td>
</tr>
<tr>
<td>$0.5 \leq V_2 \leq 1.5$</td>
<td>44</td>
</tr>
<tr>
<td>$0.2 \leq V_2 \leq 1.8$</td>
<td>56</td>
</tr>
</tbody>
</table>

(b)

apertures. Nevertheless, the statistical prediction of the induced voltage levels is more precise in the case of the parallel polarization than in the case of the perpendicular polarization ($\sigma_{V_2 \text{ polar.A}} > \sigma_{V_2 \text{ polar.B}}$). Figure 6 depicts the cumulative distribution functions of the quantity $V_2/\langle V_2 \rangle$ in dB for the perpendicular polarization and for the parallel polarization.

As examples, the Table 2 provides the induced voltage probabilities for the two polarizations.
4. CONCLUSION

In this work, we have proposed an approach to study the coupling between an irradiated rectangular aperture and a monopole antenna in a complex cavity. We have shown that the radiation impedance of the aperture can be expressed with respect to the radiation impedance of a complementary dipole antenna. Our approach allows to predict quickly and accurately the statistical nature of the induced voltages along the monopole antenna, for an aperture type excitation. A good agreement is obtained between measurements and numerical model results and it confirms the validity of our approach. On the other hand, this approach is not restricted to rectangular apertures as it could be easily expanded to other aperture geometries such as, circular.

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