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SHIFT-OPERATOR FINITE DIFFERENCE TIME DOMAIN ANALYSIS OF CHIRAL MEDIUM

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Abstract—Shift-Operator Finite difference Time Domain (SO-FDTD) method is introduced as a new efficient technique for simulating electromagnetic wave interaction with chiral medium. The dispersive properties of this medium are presented as polynomials of $j\omega$. These polynomials are converted to time domain by replacing $j\omega$ by the time derivative operator. Then this time derivative operator is converted to the corresponding time shift operator which is used directly to obtain the corresponding update equations of electric and magnetic field components. The resulting update equations do not require time convolution or additional vector components. The present analysis does not require also any transformation. Significant improvement is obtained in memory requirements by using this method while the computational time is nearly the same compared with other similar techniques like Z-transformation FDTD.

1. INTRODUCTION

Chiral materials have received a great interest due to their unique properties that include magnetoelectric coupling and polarization rotation. From theoretical point of view, chiral medium represents a general form for different materials including lossy dispersive material and meta-materials. On the other hand, from practical point of view, chiral medium is a good candidate for different applications like wave depolarizers and anti-reflection surfaces.

Different approaches have been developed for studying electromagnetic wave interaction with structures composed of or include chiral media [1–7]. These approaches include analytical solutions for simple

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problems like plane wave propagation through a chiral slab and chirowaveguide [1]. Integral equation formulation combined with MoM solution was introduced for solving electromagnetic wave interaction with more complicated configuration [2]. FDTD was also introduced as an efficient tool for studying similar problems over a wide frequency range in a single simulation. FDTD is also more suitable for studying the short pulse response for these problems [3–6, 8–10]. This short pulse response represents an important characterization parameter for newly developing applications related to UWB.

One of the earliest rigorous formulations for solving chiral media problems by using FDTD was introduced by Akyurtlu and Werner [3]. Their analysis is based on calculating the dispersive properties of chiral medium as temporal convolution integrations. A similar approach was introduced by Barba et al. [4] where they converted the problem to discrete time convolution. Demir et al. [5] introduced another approach based on using Z transformation to treat these dispersive properties. The advantage of this approach is that it does not require evaluating convolution integration. However, it requires introducing additional intermediate vector components. It is also based on calculating electric and magnetic flux densities separately, then obtaining electric and magnetic field components. Thus the total problem requires much more storage requirements. A similar approach was introduced by Pereda et al. [6] where they used Mobius transformation which is mainly based on Laplace transformation. Then the Laplace s operator is replaced with the corresponding Z transform operator to formulate the problem in a discrete time form. Grande et al. [8] showed the equivalence of these different formulations for similar problems related to metamaterials.

Recently, a new form of FDTD has been developed to simulate wave propagation in anisotropic lossy dispersive plasma. This method is based on introducing a time-shift operator that can replace the time derivative operator. The basic theory of this shift-operator FDTD is discussed in [11–14]. The main idea of this method is to present the dispersive relations between electric and magnetic fields in frequency domain as polynomials of $j\omega$. These polynomials are converted to time domain by replacing $j\omega$ by a time derivative operator. Then this time derivative operator is replaced by the corresponding shift operator. Thus, the problem is converted into polynomials of this shift operator. The order of each term of these polynomials corresponds to the number of delayed time steps of this term. By arranging these terms, one can obtain the required update equations for the dispersive medium. Recently, Ramadan [16] has shown that this shift-operator FDTD is simply equivalent to the bilinear frequency approximation technique which is used in the field of digital signal processing [17].

In this paper, we apply this SO-FDTD to simulate wave propagation in chiral medium. The first part in the following section presents a brief discussion of SO-FDTD. Then the formulation of the SO-FDTD formulation for chiral medium is discussed in detail. In Section 3, a comparison between the result of this SO-FDTD and analytical technique is presented for transmission and reflection of normally incident pulsed plane wave on a chiral slab as an example for validating the accuracy of this technique.

2. THEORY

2.1. Basic Theory of Shift-operator FDTD

To show the basic idea of shift-operator FDTD we start with a simple differential equation

$$y(t) = \frac{df(t)}{dt}. \tag{1}$$

This equation can be presented in a central difference form as follows:

$$y^{n+\frac{1}{2}} = \frac{f^{n+1} - f^n}{\Delta t} \tag{2}$$

Assuming that the time-updated variables are related to previous values by a simple linear relation as follows:

$$f^{n+1} = z_t f^n \tag{3}$$

where z_t represents the time shift operator. By applying (3) in (2), it can be shown that

$$y^{n+\frac{1}{2}} = h\,(z_t - 1)\,f^n \tag{4}$$

where $h = 1/\Delta t$. Then by comparing (4) with (1) it is shown that the time differential operator can be represented in terms of the time shift operator as:

$$d/dt \equiv h\,(z_t - 1) \tag{5}$$

It should be noted here that this shift operator has quite similar form to Z-transformation discussed in [5]. However, the usage of the present shift operator does not require performing any transformation to the Z-domain as it is shown in the following parts of this paper.

For problems with dispersive characteristics that can be presented as polynomials of $j\omega$, each $j\omega$ can be replaced by d/dt and subsequently it can be replaced by $h(z_t - 1)$. Thus, by a simple algebraic treatment, this dispersive relation can be presented as a polynomial of z_t. This polynomial is normalized such that its maximum order is zero. Each term of this normalized polynomial represents v-steps time delayed value of the corresponding variable where v is the order of the time shift operator of this term. This means that

$$z_t^{-v} f^n = f^{n-v} \tag{6}$$

2.2. Shift-operator FDTD Analysis of Chiral Medium

There are three equivalent constitutive relations which are used to describe chiral medium [15]. The first one represents relations between electric and magnetic flux densities (**D** and **B**) and the corresponding field intensities (**E** and **H**) which is known as Tellegen relation. The second representation introduces electric flux density **D** as combination between electric field intensity **E** and magnetic flux density **B** and vice versa for the magnetic flux density. This representation is known as Post relation. The third representation, known as Drude-Born-Fedorov relation, introduces a relation between the electric flux density and both electric field intensity and its curl. It introduces another similar relation between the magnetic flux density and both magnetic field intensity and its curl. Based on the first representation, Maxwell's curl equations in time-harmonic forms are given by [5]:

$$\nabla \times \mathbf{E} = -j\omega\mu\left(\omega\right)\mathbf{H} + \omega\kappa\left(\omega\right)\sqrt{\mu_o\varepsilon_o}\,\mathbf{E} \tag{7a}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\left(\omega\right)\mathbf{E} + \omega\kappa\left(\omega\right)\sqrt{\mu_o\varepsilon_o}\,\mathbf{H} \tag{7b}$$

where the above constitutive coefficients are:

$$\varepsilon\left(\omega\right) = \varepsilon_0\left(\varepsilon_\infty + \frac{\left(\varepsilon_s - \varepsilon_\infty\right)\omega_\varepsilon^2}{\omega_\varepsilon^2 + 2j\omega_\varepsilon\xi_\varepsilon\omega - \omega^2}\right) \tag{8a}$$

$$\mu\left(\omega\right) = \mu_0\left(\mu_\infty + \frac{\left(\mu_s - \mu_\infty\right)\omega_\mu^2}{\omega_\mu^2 + 2j\omega_\mu\xi_\mu\omega - \omega^2}\right) \tag{8b}$$

$$\kappa\left(\omega\right) = \frac{\tau_\kappa\omega_\kappa^2\omega}{\omega_\kappa^2 + 2j\omega_\kappa\xi_\kappa\omega - \omega^2} \tag{8c}$$

where κ is the chirality parameter, ε_∞ and μ_∞ are the relative permittivity and relative permeability at infinite frequency, ε_s and μ_s are the relative permittivity and relative permeability at zero frequency, ω_ε, ω_μ and ω_κ are the resonant angular frequencies of the permittivity, permeability and chirality respectively, and ξ_ε, ξ_μ and ξ_κ are the corresponding damping coefficients.

Based on (7b), (8a) and (8b), Ampere's curl equation can be presented as function of ω as follows:

$$\nabla \times \mathbf{H} = j\omega\varepsilon_o\left(\varepsilon_\infty + \frac{\left(\varepsilon_s - \varepsilon_\infty\right)\omega_\varepsilon^2}{\omega_\varepsilon^2 + 2j\omega_\varepsilon\xi_\varepsilon\omega - \omega^2}\right)\mathbf{E} + \omega\frac{\tau_\kappa\omega_\kappa^2\omega}{\omega_\kappa^2 + 2j\omega_\kappa\xi_\kappa\omega - \omega^2}\sqrt{\mu_o\varepsilon_o}\,\mathbf{H} \tag{9}$$

This equation can be arranged as follows:

$$\begin{aligned}&\left((j\omega)^2+2\omega_\varepsilon\xi_\varepsilon(j\omega)+\omega_\varepsilon^2\right)\left((j\omega)^2+2\omega_\kappa\xi_\kappa(j\omega)+\omega_\kappa^2\right)\nabla\times\mathbf{H}\\&=\left((j\omega)^2+2\omega_\kappa\xi_\kappa(j\omega)+\omega_\kappa^2\right)\left(\varepsilon_o\varepsilon_\infty(j\omega)^3+2\omega_\varepsilon\xi_\varepsilon\varepsilon_o\varepsilon_\infty(j\omega)^2\right.\\&\left.+\omega_\varepsilon^2\varepsilon_o\varepsilon_s(j\omega)\right)\mathbf{E}-\left((j\omega)^2+2\omega_\varepsilon\xi_\varepsilon(j\omega)+\omega_\varepsilon^2\right)\left(\tau_\kappa\omega_\kappa^2\sqrt{\mu_o\varepsilon_o}(j\omega)^2\right)\mathbf{H}\end{aligned}\quad(10)$$

which can be directly presented as a polynomial form of $j\omega$ as follows:

$$\sum_{v=0}^{4}b_{v\varepsilon}(j\omega)^v(\nabla\times\mathbf{H})=\sum_{v=0}^{5}a_{v\varepsilon}(j\omega)^v\mathbf{E}-\sum_{v=0}^{4}c_{v\varepsilon}(j\omega)^v\mathbf{H}\quad(11a)$$

Similarly, Faraday's curl equation presented by Eq. (7a) can also be presented as a polynomial form of $j\omega$ as follows:

$$\sum_{v=0}^{4}b_{v\mu}(j\omega)^v(\nabla\times\mathbf{E})=-\sum_{v=0}^{5}a_{v\mu}(j\omega)^v\mathbf{H}-\sum_{v=0}^{4}c_{v\mu}(j\omega)^v\mathbf{E}\quad(11b)$$

where the coefficients of these polynomials are given by:

$$b_{0\vartheta}=\omega_\vartheta^2\omega_\kappa^2\quad(12a)$$
$$b_{1\vartheta}=2\omega_\vartheta\omega_\kappa(\xi_\vartheta\omega_\kappa+\xi_\kappa\omega_\vartheta)\quad(12b)$$
$$b_{2\vartheta}=\left(\omega_\vartheta^2+\omega_\kappa^2+4\omega_\vartheta\xi_\vartheta\omega_\kappa\xi_\kappa\right)\quad(12c)$$
$$b_{3\vartheta}=2(\omega_\vartheta\xi_\vartheta+\omega_\kappa\xi_\kappa)\quad(12d)$$
$$b_{4\vartheta}=1\quad(12e)$$

$$a_{0\vartheta}=0\quad(13a)$$
$$a_{1\vartheta}=\left(\omega_\vartheta^2\vartheta_0\vartheta_s\omega_\kappa^2\right)\quad(13b)$$
$$a_{2\vartheta}=2\omega_\vartheta\vartheta_0\left(\xi_\vartheta\vartheta_\infty\omega_\kappa^2+\xi_\kappa\vartheta_s\omega_\kappa\omega_\vartheta\right)\quad(13c)$$
$$a_{3\vartheta}=\left(\vartheta_0\vartheta_\infty\omega_\kappa^2+4\omega_\varepsilon\xi_\vartheta\vartheta_0\vartheta_\infty\omega_\kappa\xi_\kappa+\omega_\vartheta^2\vartheta_0\vartheta_s\right)\quad(13d)$$
$$a_{4\vartheta}=2\vartheta_0\vartheta_\infty(\omega_\kappa\xi_\kappa+\omega_\vartheta\xi_\vartheta)\quad(13e)$$
$$a_{5\vartheta}=\vartheta_0\vartheta_\infty\quad(13f)$$

$$c_{0\vartheta}=c_{1\vartheta}=0\quad(14a)$$
$$c_{2\vartheta}=\omega_\vartheta^2\tau_\kappa\omega_\kappa^2\sqrt{\mu_0\varepsilon_0}\quad(14b)$$
$$c_{3\vartheta}=2\omega_\vartheta\xi_\vartheta\tau_\kappa\omega_\kappa^2\sqrt{\mu_0\varepsilon_0}\quad(14c)$$
$$c_{4\vartheta}=\tau_\kappa\omega_\kappa^2\sqrt{\mu_0\varepsilon_0}\quad(14d)$$

where ϑ is either ε or μ.

By replacing $j\omega$ by the corresponding time shift operator $(z_t - 1)$, (11) can be rewritten as:

$$\sum_{v=0}^{4} b_{v\varepsilon}(h\,(z_t-1))^v\,(\nabla\times\mathbf{H})=\sum_{v=0}^{5} a_{v\varepsilon}(h\,(z_t-1))^v\mathbf{E}-\sum_{v=0}^{4} c_{v\varepsilon}(h\,(z_t-1))^v\mathbf{H} \quad (15a)$$

$$\sum_{v=0}^{4} b_{v\mu}(h\,(z_t-1))^v\,(\nabla\times\mathbf{E})=-\sum_{v=0}^{5} a_{v\mu}(h\,(z_t-1))^v\mathbf{H}-\sum_{v=0}^{4} c_{v\mu}(h\,(z_t-1))^v\mathbf{E} \quad (15b)$$

Equation (15) can be rearranged to be in the form of polynomials of z_t as follows:

$$\sum_{v=0}^{4} \beta_{v\varepsilon} z_t^v\,(\nabla\times\mathbf{H}) = \sum_{v=0}^{5} \alpha_{v\varepsilon} z_t^v\mathbf{E} - \sum_{v=0}^{4} \gamma_{v\varepsilon} z_t^v\mathbf{H} \quad (16a)$$

$$\sum_{v=0}^{4} \beta_{v\mu} z_t^v\,(\nabla\times\mathbf{E}) = -\sum_{v=0}^{5} \alpha_{v\mu} z_t^v\mathbf{H} - \sum_{v=0}^{4} \gamma_{v\mu} z_t^v\mathbf{E} \quad (16b)$$

where the coefficients of these polynomials are given by:

$$\begin{bmatrix} \alpha_{0\vartheta} \\ \alpha_{1\vartheta} \\ \alpha_{2\vartheta} \\ \alpha_{3\vartheta} \\ \alpha_{4\vartheta} \\ \alpha_{5\vartheta} \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 & 1 & -1 \\ 5 & -4 & 3 & -2 & 1 \\ -10 & 6 & -3 & 1 & 0 \\ 10 & -4 & 1 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{5\vartheta}h^5 \\ a_{4\vartheta}h^4 \\ a_{3\vartheta}h^3 \\ a_{2\vartheta}h^2 \\ a_{1\vartheta}h \end{bmatrix} \quad (17a)$$

$$\begin{bmatrix} \beta_{0\vartheta} \\ \beta_{1\vartheta} \\ \beta_{2\vartheta} \\ \beta_{3\vartheta} \\ \beta_{4\vartheta} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -4 & 3 & -2 & 1 & 0 \\ 6 & -3 & 1 & 0 & 0 \\ -4 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_{4\vartheta}h^4 \\ b_{3\vartheta}h^3 \\ b_{2\vartheta}h^2 \\ b_{1\vartheta}h \\ b_{0\vartheta} \end{bmatrix} \quad (17b)$$

$$\begin{bmatrix} \gamma_{0\vartheta} \\ \gamma_{1\vartheta} \\ \gamma_{2\vartheta} \\ \gamma_{3\vartheta} \\ \gamma_{4\vartheta} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -4 & 3 & -2 \\ 6 & -3 & 1 \\ -4 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{4\vartheta}h^4 \\ c_{3\vartheta}h^3 \\ c_{2\vartheta}h^2 \end{bmatrix} \quad (17c)$$

where ϑ is either ε or μ. By using (16) and applying the time shift operation of (6) one can obtain that

$$\mathbf{E}^{n+1} = -\sum_{v=0}^{4} \frac{\alpha_{v\varepsilon}}{\alpha_{5\varepsilon}}\mathbf{E}^{n+v-4} + \sum_{v=0}^{4} \frac{\beta_{v\varepsilon}}{\alpha_{5\varepsilon}}\left(\nabla\times\mathbf{H}^{n+v-3\frac{1}{2}}\right) + \sum_{v=0}^{4} \frac{\gamma_{v\varepsilon}}{\alpha_{5\varepsilon}}\mathbf{H}^{n+v-3\frac{1}{2}} \quad (18a)$$

$$\mathbf{H}^{n+\frac{1}{2}} = -\sum_{v=0}^{4} \frac{\alpha_{v\mu}}{\alpha_{5\mu}}\mathbf{H}^{n+v-4\frac{1}{2}} - \sum_{v=0}^{4} \frac{\beta_{v\mu}}{\alpha_{5\mu}}\left(\nabla\times\mathbf{E}^{n+v-4}\right) - \sum_{v=0}^{4} \frac{\gamma_{v\mu}}{\alpha_{5\mu}}\mathbf{E}^{n+v-4} \quad (18b)$$

Then, by performing the spatial curl operator in a finite difference form one can obtain directly the update equations for both electric and magnetic field components. As examples for these update equations,

$$\begin{aligned}
&E^{n+1}_{x,i,j+\frac{1}{2},k+\frac{1}{2}} \\
&= \sum_{v=0}^{4}\frac{\beta_{v\varepsilon}}{\alpha_{5\varepsilon}}\left(\frac{H^{n+v-3\frac{1}{2}}_{z,i,j+1,k+\frac{1}{2}}-H^{n+v-3\frac{1}{2}}_{z,i,j,k+\frac{1}{2}}}{\Delta y}-\frac{H^{n+v-3\frac{1}{2}}_{y,i,j+\frac{1}{2},k+1}-H^{n+v-3\frac{1}{2}}_{y,i,j+\frac{1}{2},k}}{\Delta z}\right) \\
&\quad -\sum_{v=0}^{4}\frac{\alpha_{v\varepsilon}}{\alpha_{5\varepsilon}}E^{n+v-4}_{x,i,j+\frac{1}{2},k+\frac{1}{2}}+\sum_{v=0}^{4}\frac{\gamma_{v\mu}}{8\alpha_{5\mu}}\Big(H^{n+v-3\frac{1}{2}}_{x,i+\frac{1}{2},j,k}+H^{n+v-3\frac{1}{2}}_{x,i+\frac{1}{2},j+1,k} \\
&\quad +H^{n+v-3\frac{1}{2}}_{x,i+\frac{1}{2},j+1,k+1}+H^{n+v-3\frac{1}{2}}_{x,i+\frac{1}{2},j,k+1}+H^{n+v-3\frac{1}{2}}_{x,i-\frac{1}{2},j,k}+H^{n+v-3\frac{1}{2}}_{x,i-\frac{1}{2},j+1,k} \\
&\quad +H^{n+v-3\frac{1}{2}}_{x,i-\frac{1}{2},j+1,k+1}+H^{n+v-3\frac{1}{2}}_{x,i-\frac{1}{2},j,k+1}\Big)
\end{aligned} \tag{19a}$$

$$\begin{aligned}
&H^{n+\frac{1}{2}}_{y,i,j+\frac{1}{2},k} \\
&= -\sum_{v=0}^{4}\frac{\beta_{v\mu}}{\alpha_{5\mu}}\left(-\frac{E^{n+v-4}_{z,i+\frac{1}{2},j+\frac{1}{2},k}-E^{n+v-4}_{z,i-\frac{1}{2},j+\frac{1}{2},k}}{\Delta x}+\frac{E^{n+v-4}_{x,i,j+\frac{1}{2},k+\frac{1}{2}}-E^{n+v-4}_{x,i,j+\frac{1}{2},k-\frac{1}{2}}}{\Delta z}\right) \\
&\quad -\sum_{v=0}^{4}\frac{\alpha_{v\mu}}{\alpha_{5\mu}}H^{n+v-4\frac{1}{2}}_{y,i,j+\frac{1}{2},k}-\sum_{v=0}^{4}\frac{\gamma_{v\mu}}{8\alpha_{5\mu}}\Big(E^{n+v-4}_{y,i+\frac{1}{2},j+1,k+\frac{1}{2}}+E^{n+v-4}_{y,i-\frac{1}{2},j+1,k+\frac{1}{2}} \\
&\quad +E^{n+v-4}_{y,i-\frac{1}{2},j+1,k-\frac{1}{2}}+E^{n+v-4}_{y,i+\frac{1}{2},j+1,k-\frac{1}{2}}+E^{n+v-4}_{y,i+\frac{1}{2},j,k+\frac{1}{2}}+E^{n+v-4}_{y,i-\frac{1}{2},j,k+\frac{1}{2}} \\
&\quad +E^{n+v-4}_{y,i-\frac{1}{2},j,k-\frac{1}{2}}+E^{n+v-4}_{y,i+\frac{1}{2},j,k-\frac{1}{2}}\Big)
\end{aligned} \tag{19b}$$

The remaining four field components can be obtained in a similar way.

By comparing these updating equations with the corresponding ones in [5] it can be noted that the present forms depends only on electric and magnetic field components only without the need to use other update equations for electric and magnetic flux densities or other additional intermediate vector components. Another important point is that both forms require storing previous field components. However, the present technique uses previous field components in direct way without the need to combine them in other intermediate components. This property makes the present forms more suitable to be extended for parallel computation [10]. It is also found that the present approach requires storing five temporal states for each field component while the approach of [5] requires storing seven parameters for each field component. On the other hand, the present approach requires storing thirty two coefficients which are represented by (17) while the Z-transform FDTD in [5] requires storing only nine coefficients which include chiral medium parameters. However, these coefficients are calculated and stored only one time for a homogenous chiral medium. Thus, this increase in the coefficients in the present approach does not represent a real overload on its memory requirements.

3. RESULTS AND DISCUSSIONS

In this section, we discuss a sample result for the present technique to show its validity and to compare it with the previous other techniques. It should be noted that the present analysis is based on 3-D FDTD formulation. However, to show a comparison with analytical solution, the present example is limited to 1-D problem of wave propagation through an infinite slab of chiral medium.

The parameters of the chiral medium are the same as in [5] as follows:

$$\begin{aligned} &\varepsilon_{r\infty} = 2, \quad \varepsilon_{rs} = 5, \quad \omega_{\varepsilon} = 2\pi \times 2 \times 10^9\,\text{rad/s}, \quad \xi_{\varepsilon} = 0.5 \\ &\mu_{r\infty} = 1.1, \quad \mu_{rs} = 1.8, \quad \omega_{\mu} = 2\pi \times 2 \times 10^9\,\text{rad/s}, \quad \xi_{\mu} = 0.5 \\ &\omega_{\kappa} = 2\pi \times 2 \times 10^9\,\text{rad/s}, \quad \xi_{\kappa} = 0.3, \quad \tau_{\kappa} = \frac{0.5}{\omega_{\kappa}} \end{aligned} \tag{20}$$

A normally incident pulse is assumed to be a first time derivative Gaussian pulse of x polarized field as follows:

$$E_{x\_\text{inc}}(t) = \left(\frac{t - 4T_0}{T_0}\right) \exp\left[\left(\frac{t - 4T_0}{T_0}\right)^2\right] \text{ V/m} \tag{21}$$

where $T_0 = 0.25\,\text{ns}$. The chiral slab has a thickness of 10 mm. The problem is simulated as a 1-D problem along the z-axis. The spatial discretization is chosen to be $dz = 1\,\text{mm}$ and the time step is $dt = 1.6666\,\text{ps}$. The observation point is 3 mm away from the interface of the chiral slab. Figs. 1 and 2 show comparisons between SO-FDTD and analytical solution for both transmitted and reflected pulsed plane waves. The same results are exactly obtained by using Z-transform FDTD but they are not presented here to obtain clear figures. We obtained an excellent agreement between the three results. The computational time of SO-FDTD and Z-transform FDTD was found to be nearly identical. Thus, SO-FDTD does not introduce a significant improvement in this point. However, the storage requirements for the chiral part of SO-FDTD are decreased by nearly 75% compared with Z-transform FDTD. It should be noted that the reflected wave from a chiral slab does not include cross polarized component [1]. This is also verified numerically by using SO-FDTD in Fig. 2. It can also be noted that the early response of the reflected wave has narrow temporal width while the late response and the transmitted wave has wider temporal responses. This can be explained due to dispersive properties

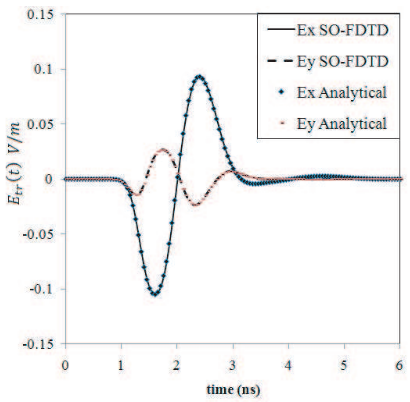

Figure 1. Transmitted pulsed plane wave through a chiral slab of thickness 10 mm. The observation point is located in free space on the other side of the slab at 3 mm from the interface of the slab. Parameter of the chiral slab are given by (20).

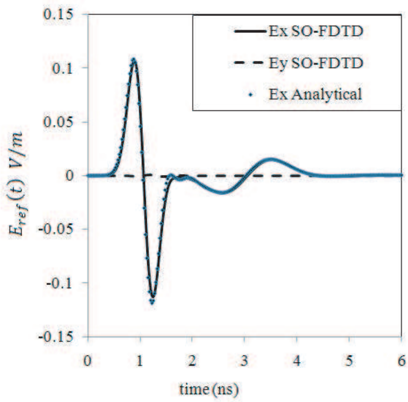

Figure 2. Reflected pulsed plane wave due to the chiral slab of Fig. 1 at 3 mm from the interface of the slab.

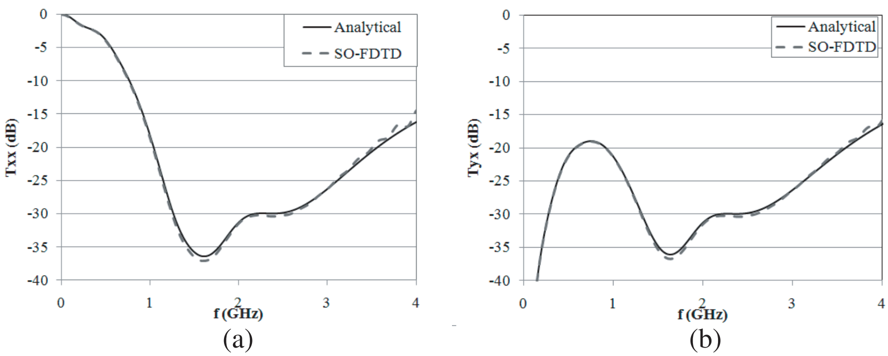

Figure 3. Co-polarized and cross-polarized transmission coefficients of the chiral slab of Fig. 1 in (dB). (a) Co-polarized transmission coefficient. (b) Cross-polarized transmission coefficient.

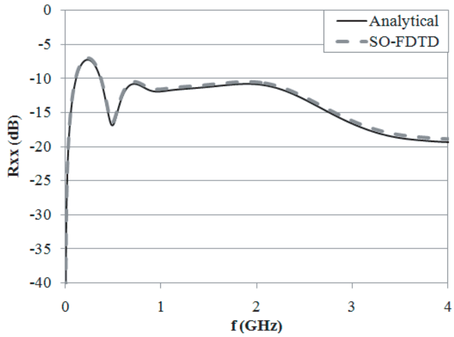

Figure 4. Reflection coefficient of the chiral slab of Fig. 1 in (dB).

of this chiral medium that attenuate the high frequency components of incident pulse. This property is quite clear in the multiple reflections that correspond to the late response of the reflection coefficient and also in the transmitted pulse. On the other hand, the early response of the reflected wave is not affected by these dispersive attenuating properties since it starts directly at the interface of the chiral slab. Finally, Figs. 3 and 4 show the same comparisons for the same chiral slab in frequency domain. It can be noticed the good agreement between the analytical solution and numerical solution obtained by using SO-FDTD.

4. CONCLUSION

A new efficient 3D finite difference time formulation based on time shift operator is used to simulate electromagnetic wave interaction with dispersive chiral medium. The resulting update equations depend only on electric and magnetic field components. There are no additional intermediate vector components, temporal convolution or transformations. The present technique shows a good improvement in memory requirements while keeping on the accuracy and the computational time compared with other similar methods.

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