FREQUENCY TUNEABLE SINGLE-NEGATIVE BISTABLE HETEROSTRUCTURE

S. Roshan Entezar

Physics Department
University of Tabriz
Tabriz, Iran

Abstract—The nonlinear responses of a one-dimensional heterostructure containing two kinds of single-negative materials with an air gap are investigated. It is shown that the frequency of zero-phase gap bistable heterostructure can be tuned simply by adjusting the width of air gap. On the other hand, the optical bistability is achieved at very low values of input intensity due to the enhancement of Kerr nonlinearity near the frequency of the defect mode. It is shown that transmission of the structure is relatively insensitive to incident angle and losses.

1. INTRODUCTION

Photonic crystals (PCs) are artificial materials having periodic modulation in dielectric constant which can create a range of forbidden frequencies, the so-called photonic band gap (PBG) [1–4]. In the photonic crystals with defects, the discrete localized modes will appear inside the PBG because of the multiple (Bragg) scattering in the structures. In a one-dimensional (1D) case, the localized modes as well as the Bragg gaps will shift in frequency noticeably when the incident angles and polarizations vary since the Bragg resonant condition has changed. To overcome this limit, some researchers have realized PBG in metamaterials [5–8]. Metamaterial or double-negative (DNG) material is a composite material, for which the effective permittivity \( \varepsilon \) and the effective permeability \( \mu \) are both negative over a finite frequency band [9, 10]. People indeed obtained the localized modes with a weak dependence on incident angles and polarizations in the photonic structures containing double-negative...
materials [5, 6]. However, the above localized modes are not strictly omnidirectional. On the other hand, since the localized modes inside the Bragg gaps originate from the interference, the waves will have phase accumulation after they transmit through the structures [11]. So it is not easy to realize (near) zero phase delay for a localized mode inside a Bragg gap unless strong localization occurs, particularly in a 1D case. Fortunately, there is another kind of metamaterials in which only one of the material parameters has a negative value [12–18]. The single-negative (SNG) metamaterials include the epsilon-negative (ENG) media (with negative $\varepsilon$ and positive $\mu$) and the mu-negative (MNG) media (with positive $\varepsilon$ and negative $\mu$). In the periodic structure composed of single-negative materials, there exists a new gap (zero-$\phi$ gap) resulting from a zero total phase [5, 16, 19–21]. It was shown that the heterostructures constituted by ENG and MNG materials can possess tunneling modes inside the forbidden gap, which are independent of incident angles and polarizations and have zero phase delay [22, 23]. In the recent years, the nonlinear responses of the periodic structure containing DNG or SNG materials were theoretically studied [24–29]. And the defect-induced bistable switching in the defect structure formed by inserting a nonlinear defect layer into the periodic structure composed of DNG or SNG materials are also investigated [24, 30]. Moreover, if the defect layer is inserted into an asymmetric structure, the bistable diode can be found [30]. However, the frequency of bistability of such a structure [31] are non-tuneable which inevitably limits its application. In this paper, we design a nonlinear heterostructure with which we can achieve a tuneable bistable structure. The results show that by simply adjusting the thickness of the defect layer of air, we can fulfill the adjustability of the frequency of the bistable structure.

The following paper is organized as follows. In Section 2, the theoretical analysis of the nonlinear heterostructure is presented. Numerical results are shown in Section 3 on the nonlinear transmission property of structure and the distribution of the electric field. In the end, the conclusion and discussion will be given in Section 4.

2. THEORETICAL MODEL

Consider the 1D heterostructure $(AB)_nC(\hat{A}\hat{B})_n$ shown in Fig. 1. Here, $(AB)_n$ and $(\hat{A}\hat{B})_n$ are two different 1D periodic structure composed of SNG materials. $A(\hat{A})$ and $B(\hat{B})$ indicate ENG materials and MNG materials, respectively. $n$ is the period number. $C$ is the defect layer of air. We suppose that relative permittivity and permeability in the
ENG materials are given by
\[
\varepsilon_{A(\hat{A})} = 1 - \frac{\omega_{ep}^2}{\omega^2 + i\omega\Gamma_e} + \lambda|E|^2, \quad \mu_{A(\hat{A})} = a, \tag{1}
\]
and those in the MNG materials are given by
\[
\varepsilon_{B(\hat{B})} = b, \quad \mu_{B(\hat{B})} = 1 - \frac{\omega_{mp}^2}{\omega^2 + i\omega\Gamma_m}. \tag{2}
\]
where \(a\) and \(b\) are positive constants, and \(\omega_{ep}\) and \(\omega_{mp}\) are the electronic and the magnetic plasma frequencies, respectively. Both \(\Gamma_e\) and \(\Gamma_m\) are the dissipation factors. Note that these kinds of dispersion may be realized in special microstrips [32–34]. For instance, a metallic microstructure containing a regular arrays of thin wires is able to behave like a plasma with the resonant frequency \(\omega_{ep}\) at GHz region. As consequence, the dielectric response of such a plasma can be described by the first equation. On the other hand, loops of conducting wires have the magnetic plasma properties and the permeability is described by the second equation. The angular frequency \(\omega\) in the above equations is measured in GHz. To simplify the computational work the electric plasma frequency \(\omega_{ep}\) in the ENG layer and the magnetic plasma frequency \(\omega_{mp}\) in the MNG layer are taken to have the same values. The values of the magnetic permeability, \(a\) in the ENG layer and the electric permittivity, \(b\) in the MNG layer are taken to be the same. The values of dissipation factors \(\Gamma_e\) and \(\Gamma_m\) are also taken to have the same values. However, these values could be different.
and do not affect the main features of the results. Here, we assume $a = b = 3$, $\omega_{ep} = \omega_{mp} = 10\text{GHz}$ and $\Gamma_e = \Gamma_m = \Gamma = 0.002\omega$. It should be mentioned that the parameters $\varepsilon_A(\tilde{\omega})$ and $\mu_B(\tilde{\omega})$ in Eqs. (1) and (2) diverge under the limit of $\omega \to 0$, so it has no physical meanings in the low frequency limit. We use dimensionless units $\tilde{\omega} = \omega d/c$ and $\tilde{d}_i = d_i/d$ in the computational work. Here, $i = A, \dot{A}, B, \dot{B}, C$, $d = d_A + d_\dot{A} = d_B + d_\dot{B}$ and $c$ is the velocity of light. In these dimensionless units, $\tilde{\omega}_{ep} = 0.7333$, $\tilde{d}_A = 0.3636$, $\tilde{d}_{\dot{A}} = 0.6364$, $\tilde{d}_B = 0.5455$ and $\tilde{d}_{\dot{B}} = 0.4545$. The advantage of using these dimensionless units is that the results apply to any length scale and the corresponding values of frequencies. For simplicity, the nonlinear coefficient $\lambda$ is chosen to be negative, so that $\varepsilon_A(\tilde{\omega})$ is always negative for any input intensity in the zero-$\phi$ gap. As a result, the input electromagnetic wave does not change the quality of the negative permittivity material. For a conventional dielectric medium, positive $\lambda$ corresponds to a self-focusing nonlinear material, whilst negative $\lambda$ characterizes defocusing effects in the beam propagation. However, this classification becomes reversed in the case of metamaterial and, for example, a self focusing metamaterial corresponds to negative $\lambda$ [35]. Although most naturally occurring materials have positive Kerr coefficient, the occurrence of negative Kerr nonlinearity in different composite crystals has been experimentally demonstrated [36–39]. So, as in other theoretical studies [25, 28, 31], here we take the sign of Kerr coefficient to be negative. We consider the nonlinear coefficient $\lambda$ is absorbed in the dimensionless control parameter, which represents the intensity of the incident light, i.e., $|\lambda||\tilde{E}_i|^2$. Let a plane wave be injected from the vacuum into the 1D heterostructure at an incident angle $\theta$ with $+z$ direction, as show in Fig. 1. For the transverse electric (TE) wave, the electric field $E$ is in the $x$ direction (the dielectric layers are in the $xy$ plane and the $z$ direction is normal to the interface of each layer). In general, the electric and magnetic fields at any two dimensionless positions $\tilde{z}$ and $\tilde{z} + \Delta \tilde{z}$ in the same layer can be related via a transfer matrix [40, 41]

$$M_j(\Delta \tilde{z}, \tilde{\omega}) = \begin{pmatrix} \cos(\tilde{k}_z^j \Delta \tilde{z}) & i \frac{1}{q_j} \sin(\tilde{k}_z^j \Delta \tilde{z}) \\ iq_j \sin(\tilde{k}_z^j \Delta \tilde{z}) & \cos(\tilde{k}_z^j \Delta \tilde{z}) \end{pmatrix},$$

(3)

where $\tilde{k}_z^j$ is the $z$ component of the dimensionless wave vector $\tilde{k}_j = k_j d$ in the $j$th layer, and $q_j = \sqrt{\varepsilon_j}/\sqrt{\mu_j} \sqrt{1 - (\sin^2(\theta)/\varepsilon_j \mu_j)}$. We have used the nonlinear transfer matrix [28, 42] approach to calculate the transmission coefficient of the structure. In this case, the values of
\(k_j, q_z\) and \(\varepsilon_j\) for the nonlinear layers (ENG layers) are determined by the value of electric field intensity at the corresponding interfaces. The tangential components of electric and magnetic field at the incident side \(\tilde{z} = 0\) and at the transmitted side \(\tilde{z} = \tilde{L} = 2n + \tilde{d}_C\) are related by the following matrix equation:

\[
\begin{bmatrix}
E_1 \\
H_1
\end{bmatrix}_{\tilde{z}=0} = \left(\prod_{j=1}^{2n+1} M_j (\tilde{d}_j, \tilde{\omega})\right)^{-1} \begin{bmatrix}
E_{2n+1} \\
H_{2n+1}
\end{bmatrix}_{\tilde{z}=\tilde{L}},
\]

(4)

Then, the transmission coefficient \(t(\tilde{\omega})\) can be obtained from the transfer matrix method \([40, 41, 43]\)

\[
t(\tilde{\omega}) = \frac{2}{|x_{22}(\tilde{\omega}) + x_{11}(\tilde{\omega})| - [q_0x_{12}(\tilde{\omega}) + q_0^{-1}x_{21}(\tilde{\omega})]}.
\]

(5)

Here, \(q_0 = \cos(\theta)\) and \(x_{ij}(\tilde{\omega})\) \((i, j = 1, 2)\) are the matrix elements of total transfer matrix \(X(\tilde{\omega}) = \begin{bmatrix} x_{11}(\tilde{\omega}) & x_{12}(\tilde{\omega}) \\
x_{21}(\tilde{\omega}) & x_{22}(\tilde{\omega}) \end{bmatrix} = \prod_{j=1}^{2n+1} M_j (\tilde{d}_j, \tilde{\omega})\), which connects the fields at the incident end and those at the exit end. Hence, the transmissivity follows

\[
T(\tilde{\omega}) = |t(\tilde{\omega})|^2.
\]

(6)

3. RESULTS AND DISCUSSION

We first consider the linear limit case, i.e., the nonlinear coefficient \(\lambda\) is zero. The transmission spectra of the heterostructure \((AB)_{12}C(\bar{A}\bar{B})_{12}\) with different incident angles \((\theta)\) and width of air gap \(\tilde{d}_C\) are shown in the left panel of Fig. 2. Here, the dissipation factors \(\bar{\Gamma} = \Gamma d/c\) is set to zero. In the absence of losses, we can see that the defect modes have unit transmission. Moreover, the position of the defect mode in the zero-\(\phi\) gap moves much slowly with the incident angle changing from \(0^\circ\) to \(15^\circ\) to \(30^\circ\), while the defect modes in the Bragg gaps move significantly, which has been discussed and reported in Refs. [16, 19]. This merit of the defect mode in the zero-\(\phi\) gap will make the single-negative defect structure much useful. The formation of this defect modes can be explained based on the effective medium theory [22]. For the heterostructure without air gap, when the condition \(\bar{\varepsilon} = 0\) and \(\bar{\mu} = 0\) for the average \(\varepsilon = 0\) and \(\mu = 0\) are satisfied, the interface modes at different interfaces of ENG and MNG layers can resonantly couple each other, leading to the emergence of the tunneling mode. Moreover, the heterostructure is designed in order that the tunneling mode can lie within the forbidden gap. However, for the heterostructure with the air
Figure 2. Transmission spectra as a function of dimensionless frequency $\tilde{\omega}$ with $d_C = 0$ (solid lines), $d_C = 0.68$ (dash lines) and $d_C = 1.36$ (dash-dot lines) at the incident angle (a) $\theta = 0^\circ$, (b) $\theta = 15^\circ$ and (c) $\theta = 30^\circ$. Left panel for $\tilde{\Gamma} = 0.0$ and right panel for $\tilde{\Gamma} = 0.002\tilde{\omega}$.

gap, the condition is not $\tilde{\varepsilon} = 0$ and $\tilde{\mu} = 0$ anymore. In fact, the $(AB)_n$ in the heterostructure can be taken as an effective MNG material while the $(A\hat{B})_n$ an effective ENG material. So, the whole heterostructure can be seen as a sandwiched structure MNG/air/ENG [44]. So, the structure can be seen as a FabryPerot cavity with electric and magnetic walls on its two sides. When the sum of reflection phase on electric wall adding that on magnetic wall together with the phase accumulated in the round trip of the air gap is $2m\pi$ ($m$ is integer), tunneling mode will appear. To show the effect of losses, the linear transmission spectra of the heterostructure is plotted in the right panel of Fig. 2 for $\tilde{\Gamma} = 0.002\tilde{\omega}$. As on can see from the figure, in the presence of the losses the transmission of the defect modes are slightly less than unit.

In order to investigate the nonlinear responses of the defect structure near the defect mode frequency, the dimensionless output intensity $|\lambda||E_t|^2$ of the nonlinear structure are plotted as a function of dimensionless input intensity $|\lambda||E_i|^2$ in Fig. 3.

The typical S-shaped response indicates that the nonlinear defect structure can exhibit bistability. The bistability of the electromagnetic wave can also be observed from the transmission curves. In the absence of the losses ($\Gamma = 0$), the transmission spectra of the nonlinear structure is shown for different width of the defect layer and hence, different frequencies in the left panel of Fig. 4. Here, we considered three incident angles $\theta = 0^\circ$ (solid lines), $\theta = 15^\circ$ (dash lines) and $\theta = 30^\circ$ (dash-dot lines), in which the defect modes are nearly insensitive
Figure 3. Dimensionless output intensity $|\lambda||E_i|^2$ of the nonlinear structure as a function of dimensionless input intensity $|\lambda||E_i|^2$ at the incident angles $\theta = 0^\circ$ (solid lines), $\theta = 15^\circ$ (dash lines) and $\theta = 30^\circ$ (dash-dot lines) for (a) $\tilde{d}_C = 0$, (b) $\tilde{d}_C = 0.68$ and (c) $\tilde{d}_C = 1.36$. Here, $\tilde{\Gamma} = 0$.

Figure 4. Transmission spectra of the nonlinear structure as a function of dimensionless input intensity $|\lambda||E_i|^2$ with $\tilde{\Gamma} = 0$, at the incident angles $\theta = 0^\circ$ (solid lines), $\theta = 15^\circ$ (dash lines) and $\theta = 30^\circ$ (dash-dot lines) for (a) $\tilde{d}_C = 0$, (b) $\tilde{d}_C = 0.68$ and (c) $\tilde{d}_C = 1.36$. Left panel for $\tilde{\Gamma} = 0.0$ and right panel for $\tilde{\Gamma} = 0.02\tilde{\omega}$.

to the incident angle. As one can see, the transmission curves do not change very much with the increasing of the incident angle. Due to the localization property of the defect mode and the enhancement of the Kerr nonlinearity near the frequency of defect mode, the threshold
intensity to achieve the bistability is relatively low. To one’s interest, the three curves reach the transmission peaks at almost the same dimensionless incident intensity. This useful property can provide a potential application for a nonlinear structure which can present not only transmission bistability but also a strong characteristic of high transmission under oblique incidence. Moreover, we see that the easily tuneable frequency of optical bistability (by changing the width of air gap) dose not significantly affect the characteristics of the optical bistability associated with the structure. In order to show the effect of losses on the optical bistability, we plotted the transmission spectra of the nonlinear structure for $\tilde{\Gamma} = 0.002\tilde{\omega}$ in the right panel of Fig. 4. It is seen that there is no significant effect on the characteristics of optical bistability in response to the presence of a realistic choice of damping frequency for the single-negative materials working in the GHz frequency range.

To show the localization properties of the defect modes, the spatial distribution of the electric field in the nonlinear structure are plotted as a function of dimensionless distance $\tilde{z} = z/d$ in Fig. 5. The investigation of spatial distribution of the electric field indicates that the electric fields for different width of air gap form solitons in the nonlinear defect structure, with the peaks locating at the interfaces of the defect layer. From Fig. 5, we can see that although the electric

![Figure 5](image-url)

**Figure 5.** The spatial distribution of the electric field in the nonlinear structure as a function of dimensionless distance $\tilde{z}$, with $\tilde{\Gamma} = 0, \theta = 0^\circ$ for (a) $\tilde{d}_C = 0$, (b) $\tilde{d}_C = 0.68$ and (c) $\tilde{d}_C = 1.36$. 
field are continuous at the interfaces between the layers, due to the sign of $\mu$, the slopes of the fields at two side of the interfaces have different signs. Here, the solitons are formed by interaction between the evanescent waves, whose magnitude varies (increase or decrease) exponentially away from the interface. Since our investigations show that the solitons in the nonlinear heterostructure have an insensitive characteristic to the incident angle, we considered only the case of normal incident ($\theta = 0$) in Fig. 5. When the incident angle is not equal to zero, the solitons have the similar shape and tendency with that of normal incidence. The only difference between solitons with different incident angles is the amplitudes of the electric fields at the same position in the defect structure.

4. CONCLUSION

In summary, the nonlinear response of a 1D heterostructure containing two kinds of single-negative materials with an air gap is investigated. It is shown that the frequency of zero-$\phi$ gap bistable heterostructure can be tuned simply by adjusting the width of air gap. It is found that the bistability in the transmission in the zero-phase gap region near the defect mode frequency is insensitive to the incident angle. Due to the high field localization at the defect mode, the optical bistability is seen to occur at extremely low values of input intensity. It is hoped that these useful properties can provide a promising application in designing extremely low input and highly compact optical switches requiring strong property for oblique incidence and frequency tuning.

REFERENCES


