

ANALYSIS AND SYNTHESIS OF RADAR CROSS SECTION OF ARRAY ANTENNAS

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Abstract—Our previous work has proved that the Monostatic Radar Cross Section (MRCS) of array antennas can be decomposed into the multiplication of array MRCS factor and element MRCS factor. The principle was derived in a special case that the array only had dipole antenna elements. However, many array antennas have more general antenna elements whose current is aperture distributed along the antenna structure. Obviously it encounters limited application problem when the principle is used to analyze more general array antennas other than dipole arrays. Therefore, the principle is extended into the more general array with arbitrary aperture antenna elements in this paper. In deriving the principle, the devices in the feed are assumed to have identical transmission and reflection coefficients. In order to validate the principle the scattering pattern of a waveguide slot array and an array with helix antenna elements are synthesized utilizing the array RCS factor. The simulation and calculation results prove that the principle is correct for the RCS pattern synthesis of general arrays with aperture antenna elements.

1. INTRODUCTION

The scattering property of array antennas has been recognized as one of the most important performances of antennas in many applications such as guidance, tracking, and navigation. Hence how to control the scattering property of array antennas is a crucial problem for the above applications [1].

Most previous work pays more attention to the accurate and approximate scattering calculation of different kinds of array antennas

such as microstrip patch array antennas [2–6], waveguide slot array antennas [7–9], and conformal array antennas [9–11]. However, optimizing the RCS of array antennas with the above numerical algorithm will be a complex and time consuming work. Consequently many other works move their focus toward the optimization of the shapes and materials of various objects themselves to minimize the RCS, e.g., multilayer dielectric cylinders [12], perfect electrical conductors [13–15], and impedance strip [16]. But few of them paid attention to the optimization of the RCS of array antennas. The reason may be that there is insufficient scattering theory to utilize to optimize the RCS of array antennas. Though Coe and Ishimar [1] successfully realized the RCS optimization of an array having only two half-wave dipoles, his method might not afford the design of larger arrays. Thors and Josefsson realize a design of a low RCS conformal array antenna through the tradeoff design of radiation and scattering performance [11]. The radiation and scattering tradeoff is implemented with a suitable choice of impedance loads. However, the tradeoff design essentially is not a synthesis method. Hence a scattering theory which can be used to simply optimize the RCS of array antennas is urgently needed.

To solve the aforementioned issue, our previous work introduced a principle that the scattering of array antennas should contains two parts: the array scattering factor and element scattering factor [17]. However in our previous work the principle was derived in a special case that the element of the array is dipole antenna. To extend the applications, the principle is derived for the array with arbitrary aperture antennas in this paper.

Utilizing the principle, a RCS simple and effective synthesis method was introduced in our report [18]. Similar with our previous work, the RCS pattern of a waveguide slot array, which used as a computational example in [8], is synthesized to restrain the grating-lobes of the RCS pattern. The simulated and calculated results validate the principle as well as the synthesis method for the array has any antenna elements.

2. THEORY AND FORMULATION FOR THE SCATTERING OF ARRAY ANTENNAS

In this section, the known theory of scattered field from single element is introduced first. Then the array RCS factor is extracted from the structural mode MRCS and antenna model RCS of the array antenna, respectively.

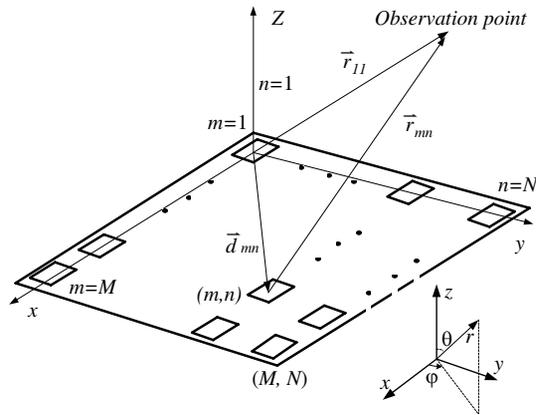


Figure 1. Geometry of an array antenna with $M \times N$ aperture antenna elements.

2.1. Scattered Field of Single Element

As references [19]–[21] discussed when antenna load impedance (Z_L) is equal to the characteristic impedance (Z_c) of the transmission line which connects the antenna and load, the reflection seen from the transmission line to the load is $\Gamma^l = (Z_L - Z_c)/(Z_L + Z_c) = 0$. Then there is no reflected energy back to the antenna leaving only induced current on the antenna body. Therefore, the antenna acts as a general passive scattering object. A planar array with arbitrary elements form is shown in Fig. 1, in which each square represents the body of one element composed of only metallic conductor. Applying the conclusion to the element in the array, the scattered field of single antenna in the array is given as follows

$$\vec{E}_{mn}(Z_L) = \vec{E}_{s(mn)}^s(Z_c) + \vec{E}_{a(mn)}^s(Z_L) \quad (1)$$

Thus Equation (1) separates the scattered field from the element into two components. The first term is called the structural mode scattering, and the second term is called antenna mode scattering. The superscript s represents the scattered field, while the subscript s and a represent the structural mode and antenna mode scattering respectively.

2.2. Extracting the Array MRCS Factor from the Structural Mode Scattering of Array Antennas

$\vec{E}_{s(mn)}^s(Z_c)$ in (1) is the structural mode scattering which represents the scattered field generated by the induced current on the antenna physical structure. This scattered field can be determined by method of moment as follows [22]

$$\vec{E}_{s(mn)}^s(Z_c) = -\frac{j\omega\mu_0}{4\pi r_{mn}} e^{-j\vec{k}_0 \cdot \vec{r}_{mn}} \sum_{l=1}^L I_{l(mn)} \int_{S'_{mn}} (\hat{\theta}\hat{\theta} + \hat{\varphi}\hat{\varphi}) \vec{f}_l(\vec{r}'_{mn}) e^{j\vec{k}_0 \cdot \vec{r}'_{mn}} dS' \quad (2)$$

where \vec{k}_0 is the vector wave number of the free space, ω is the angular frequency of scattered field. The permeability of free space is μ_0 . \vec{r}_{mn} represents the observation point with spherical coordinates (r, θ, φ) , \vec{r}'_{mn} is the location of the induced current on the element (m, n) , S'_{mn} and $\vec{f}_l(\vec{r}'_{mn})$ represent the source range and basis function of element (m, n) respectively. If there is no mutual coupling between the elements in the array, the current coefficients should have equal value $I_{l(mn)} = I_l (l = 1, 2, \dots, L)$. Considering far field observation and referring to Fig. 1

$$\vec{r}_{mn} = \vec{r}_{11} - \vec{d}_{mn} \quad (3)$$

$$\vec{r}'_{mn} = \vec{r}'_{11} + \vec{d}_{mn} \quad (4)$$

$$r_{mn} \approx r_{11} \quad (5)$$

Then Equation (2) reduces to

$$\vec{E}_{s(mn)}^s(Z_c) = \vec{E}_{s(11)}^s(Z_c) \cdot e^{2j\vec{k}_0 \cdot \vec{d}_{mn}} \quad (6)$$

where

$$\vec{E}_{s(11)}^s(Z_c) = -\frac{j\omega\mu_0}{4\pi r_{11}} e^{-j\vec{k}_0 \cdot \vec{r}_{11}} \sum_{l=1}^L I_l \int_{S'_{11}} (\hat{\theta}\hat{\theta} + \hat{\varphi}\hat{\varphi}) \vec{f}_n(\vec{r}'_{11}) e^{j\vec{k}_0 \cdot \vec{r}'_{11}} dS' \quad (7)$$

$$\vec{d}_{mn} = \hat{x} \cdot dx_m + \hat{y} dy_n$$

Then the total structural model scattering of the array is obtained by summing over all the elements

$$\vec{E}_s(\vec{k}) = \vec{E}_{s(11)}^s(Z_c) \sum_{m=1}^M \sum_{n=1}^N e^{2j\vec{k}_0 \cdot \vec{d}_{mn}} \quad (8)$$

2.3. Extracting the Array MRCS Factor from the Antenna Mode Scattering of Array Antennas

When the incident wave frequency falls into the operating band of an antenna, the incident energy collected by the antenna travels through a feed system, and is reradiated due to mismatch at the terminating load of the antenna. Then the reradiated field represented by antenna mode scattering is given by [19]

$$\vec{E}_{a(mn)}^s(Z_L) = \frac{\Gamma_{mn}^l}{1 - \Gamma_{mn}^A \Gamma_{mn}^l} b_{mn} \vec{E}_r \quad (9)$$

where $\Gamma_{mn}^l = (Z_L - Z_c)/(Z_L + Z_c)$, $\Gamma_{mn}^A = (Z_A - Z_c)/(Z_A + Z_c)$, \vec{E}_r is the radiated pattern of the element excited by a unit amplitude source with $a = 1(\text{watt})^{1/2}$ at the frequency of the incident wave, b_{mn} is the receiving amplitude of the element (m, n) when the element terminates a match load impedance. Then b_{mn} is given as [23]

$$b_{mn} = \frac{1}{2a} \oint \left(\vec{E}_{mn}^i \times \vec{H}_{mn}^r - \vec{E}_{mn}^r \times \vec{H}_{mn}^i \right) \cdot d\vec{S} \quad (10)$$

where $(\vec{E}_{mn}^i, \vec{H}_{mn}^i)$ and $(\vec{E}_{mn}^r, \vec{H}_{mn}^r)$ are the incident field and scattered field of the element, respectively. Following a similar procedure (substituting Equations (3)–(5) into Equation (10)) to that used to obtain (8), b_{mn} can be couched in terms of b_{11} as follows

$$b_{mn} = b_{11} \cdot e^{j2\vec{k}_0 \cdot \vec{d}_{mn}} \quad (11)$$

Substituting (11) into (9), the antenna mode scattering is given by

$$\vec{E}_{a(mn)}^s(Z_L) = \frac{\Gamma_{mn}^l}{1 - \Gamma_{mn}^A \Gamma_{mn}^l} e^{2j\vec{k}_0 \cdot \vec{d}_{mn}} b_{11} \vec{E}_r$$

Then the total scattered field of the array is obtained by summing over all the elements

$$\vec{E}_a^s(\vec{k}) = b_{11} \vec{E}_r \sum_{m=1}^M \sum_{n=1}^N \frac{\Gamma_{mn}^l}{1 - \Gamma_{mn}^A \Gamma_{mn}^l} e^{2j\vec{k}_0 \cdot \vec{d}_{mn}} \quad (12)$$

Assuming that each element in the array is connected with the identical feed structure and neglecting the mutual coupling, the elements should have the equal load impedance so that reflections from them should all have the same amplitude and phase. Consequently, Equation (12) can be couched as follows

$$\vec{E}_a^s(\vec{k}) = \vec{E}_{a(11)}^s(Z_L) \sum_{m=1}^M \sum_{n=1}^N e^{2j\vec{k}_0 \cdot \vec{d}_{mn}} \quad (13)$$

2.4. Radar Cross Section of Array Antennas

From Equations (1), (8) and (13), the scattered field of an array antenna can be determined as

$$\vec{E}^s(\vec{k}) = \vec{E}_e^s(\vec{k}) \cdot E_{ay}^s(\vec{k}) \quad (14)$$

where $E_{ay}^s(\vec{k}) = \sum_{m=1}^M \sum_{n=1}^N e^{2j\vec{k} \cdot \vec{d}_{mn}}$ is defined as array scattering factor

and $\vec{E}_e^s(\vec{k})$ is defined as element scattering factor. Since scattered fields are proportional to the square root RCS, so (14) can easily be recast in the form

$$\sigma = \sigma_a \cdot \sigma_e \quad (15)$$

where $\sigma_a = \left| \sum_{m=1}^M \sum_{n=1}^N e^{2j\vec{k} \cdot \vec{d}_{mn}} \right|^2$ and $\sigma_e = 4\varpi R^2 \left| \frac{\vec{E}_e^s(\vec{k})}{\vec{E}_i(\vec{k})} \right|^2$ are the RCS values associated with each of the two terms in (14).

3. SCATTERING PATTERN OPTIMIZATION OF WAVEGUIDE SLOT ARRAY ANTENNAS

3.1. Optimization Algorithm

The Particle Swarm Optimization (PSO) algorithm is one of promising optimization techniques in recently years. It requires only primitive mathematical operators, and is computationally inexpensive in terms of both memory requirements and speed. Hence the PSO algorithm is employed to optimize the radiation and scattering of the array antenna. The formulation of traditional PSO is given by

$$v_{id}(t+1) = \omega v_{id}(t) + c_1 r_1 (p_{id} - x_{id}(t)) + c_2 r_2 (p_{gd} - x_{id}(t)) \quad (16)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (17)$$

where $i = 1, 2, \dots, N$, $d = 1, 2, \dots, M$. In this paper r_1 and r_2 are random real numbers in the region of $[0, 1]$. $c_1 = c_2 = 1.492$ are the accelerating constants, the inertia weights is $\omega = 0.729$, p_{gd} is the optimum position found so far by any member of i 's topological neighborhood. p_{id} is the optimum position found so far when individual i is at its current position x_{id} in the searching space with velocity v_{id} .

3.2. Optimization of Scattering Pattern

The goal of scattering optimization is to reduce the scattering field in those directions which have strong scattering energy. The fitness function is hence determined as follows

$$\text{fitness}(d_1, \dots, d_{M \times N}) = \min[\max(10 \cdot \lg \sigma_a(\theta))] \quad (18)$$

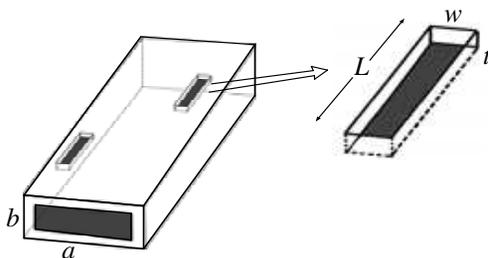


Figure 2. Geometric structure of two slots, where $l = 16.38$ mm, $w = 2.66$, $t = 1$ mm, $a = 21.2$ mm, $b = 5.22$ mm.

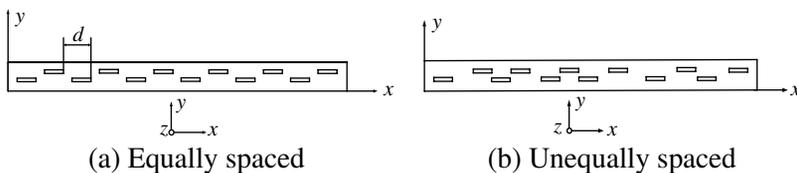


Figure 3. Spatial arrangement of slot elements in the array aperture. (a) Before optimization. (b) After optimization.

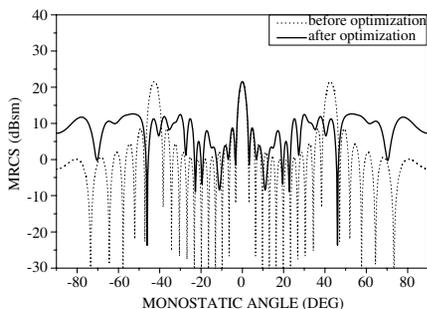


Figure 4. Calculated MRCS of array factor with Equation (15).

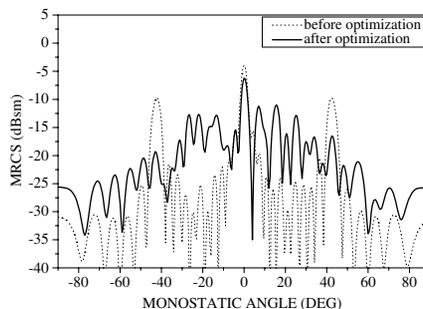


Figure 5. Simulated RCS of slot array for v - v polarization.

where $\sigma_a(\theta)$ is the array RCS factors with respect to the incident angle. It needs to be noted that the aperture of the array is not changed during the optimization process.

4. NUMERICAL RESULTS

A waveguide slot array is firstly used as an example. The aperture of the waveguide slot array whose aperture is 266.2 mm (8.13λ) is composed of 12 uniform slots. The explicit structure of the slot is shown in Fig. 2. Assume a plane wave whose frequency is 9.16 GHz incident on the waveguide slots array. When the polarization of incident electromagnetic waves is along y -axis (v -polarized), the slots on the waveguide will be excited. Then the main component of the scattered field may be that scattered from the periodically arranged waveguide slots. Then it can be predicted with (15) that there are two grating-lobes at the angle $\pm 42.6^\circ$ in the v - v polarized RCS pattern. In Fig. 4 these two grating-lobes can be found at the angle $\pm 42.6^\circ$ in the calculated RCS pattern of array RCS factor. In Fig. 5 these two grating-lobes also can be found in the simulated RCS pattern of waveguide slot array for v - v polarization at the same angle $\pm 42.6^\circ$. Therefore Equation (15) is proved to be right for the array with aperture antenna elements.

When the polarization of the incident waves is along x -axis (H -polarized), the slots will not be excited, then the scattering is mainly comprised with the scattering of the conductive surface of the waveguide. Therefore the scattering pattern should be similar with that of metal planes. The simulated RCS pattern in Fig. 6 proved the inference.

The unequally spaced slot array, which is shown in Fig. 3(b), is obtained when the optimization is carried out. As discussed above, the optimum spatial arrangement of the slots restrains the grating-lobes of the RCS pattern of v - v polarization due to the incident wave excites the slots, which is shown in Fig. 4 and Fig. 5. Meanwhile, the

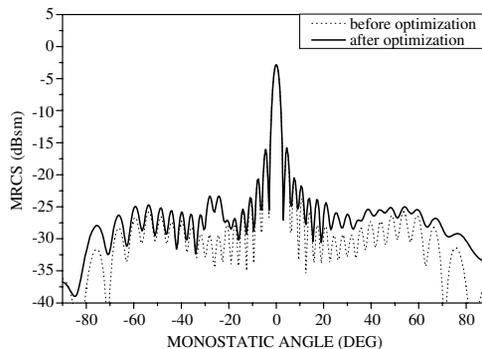


Figure 6. Simulated RCS of H - H polarization of slot array.

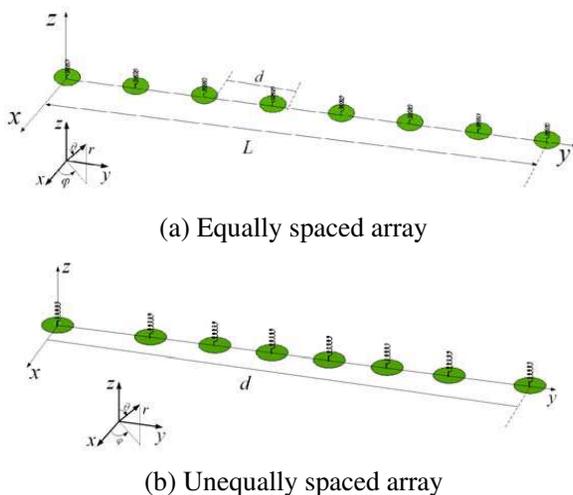


Figure 7. Physical models of the array antennas with 1×8 helix antenna elements.

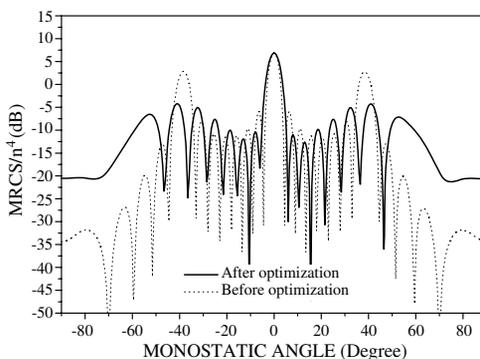


Figure 8. Simulated MRCS of the array antennas with 1×8 helix antenna elements.

RCS at some other incident angles increase approximately 15–20 dB since the nature limitation of the synthesis method. In contrast since the incident wave can not excite slots when the polarization of the incident wave is along x -axis, the spatial arrangement of the slots has little effects on the H - H RCS pattern.

Because the MRCS of the array factor is independent on the element MRCS, which is indicated in (15) the proposed method can be applied for larger arrays and for the arrays with other kind of antenna

elements. Hence, in order to illustrate the general application of the proposed synthesis method, an array with 1×8 helix antenna elements is used as an example. Fig. 7(a) shows the geometry of the array with helix elements. The spacing between adjacent elements is $d = 0.65\lambda$. Then the aperture of the array is $L = 4.55\lambda$.

After carrying out the proposed synthesis method, the unequally spaced array shown in Fig. 7(b) is obtained. Fig. 8 shows the simulated MRCS of the equally and unequally spaced arrays by FEKO. It can be seen from Fig. 8 that the two grating lobes of equally spaced array is restrained by the proposed synthesis method. Then the MRCS of the unequally spaced array is lower than -6 dB for all angles except the normal direction.

5. CONCLUSION

The proposed principle, that the RCS of array antennas can be decomposed into the array RCS factor and element RCS factor, is extended into more general case in this paper. Once the principle has been proved to be validated in general case, we can synthesis the RCS pattern for the general array with most kinds of antenna elements.

The waveguide slot array and an array antenna with helix antenna elements are used as two examples. The simulated and calculated results show that the synthesis method can help to suppress the grating-lobes of the array RCS pattern. The proposed method affords an effective way to design low RCS array antennas.

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