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COMPUTATION OF THE RCS OF 3D CONDUCTOR WITH ARBITRARY SHAPE BY USING PIECEWISE SIBC AND FORWARD BACKWARD ITERATIVE SCHEME

A. Bouzidi and T. Aguili

Syscom Laboratory, Engineer National School
B.P 37 Le Belvedere, Tunis 1002, Tunisia

Abstract—In this paper, we propose a computational method for computing RCS of 3D conductor, by using piecewise surface impedance boundary conditions and forward backward iterative scheme. In our previous work, we have reported a numerical method combining Rytov's perturbation method and level set technique to construct a piecewise surface impedance, we showed that by using level set technique, we could model an arbitrarily shaped conductor by a piecewise distribution of low- and high-order SIBCs. The method proposed in this article postulates the use of local "buffer regions" to suppress spurious edge effects introduced by the abrupt termination of each SIBC and ensure stability of RCS computing.

1. INTRODUCTION

The RCS prediction and analysis has been a requirement of many engineering fields such as Aeronautics and defence industry, to gain important information about a system before it is built, thus saving time and resources. For these reasons, it is imperative to develop efficient methods for RCS computing. Numerical techniques based on rigorous formulation such as the method of moments (MOM) give very accurate results, but in some cases, the computational cost is prohibitive.

A popular alternative uses surface impedance boundary conditions to eliminate the conducting volume from the numerical implementation and focus only on exterior field [1]. Judicious choice of SIBC's order

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Corresponding author: Afif Bouzidi (afif.bouzidi@gmail.com).

provides good compromise between accuracy and implementation cost, but for complicated geometry conductors it is difficult to have such opportunity by using only one SIBC's order. Indeed, application of high order SIBC on entire conductor's surface makes numerical solution very expensive, without necessarily providing significant improvement in accuracy. On the other hand, the improper use of low SIBC's orders degrades numerical accuracy, So, it will be interesting to apply a variable SIBC to have a good compromise between accuracy and implementation cost. In our previous work [2], we have reported a numerical method combining Rytov's perturbation method and level set technique to construct a piecewise surface impedance, we showed that by using level set technique, we could model an arbitrarily shaped conductor by a piecewise distribution of low- and high-order SIBCs. The proposed method [2] leads to a SIBC discontinuity between adjacent regions, which produces spurious edge effects when calculating the scattered fields. The method proposed in this article postulates the use of the forward-backward iterative scheme and the local "buffer regions" to suppress these unwanted effects and ensure stability of RCS computing.

2. THREE-DIMENSIONAL MOM FORMULATION

Let consider Ω an open subset of $\mathbb{R}^3$, occupied by a homogeneous conducting medium, and let Γ be a closed surface in Ω. ε and σ denote respectively the relative electric permittivity and the conductivity of the medium. The conductor is illuminated by a plan linearly polarized electromagnetic wave (E_i, H_i):

$$\vec{E^i} = \exp\left(jk\hat{k}\cdot\vec{r}\right)\hat{a} \tag{1}$$

$$\vec{H^i} = \frac{1}{Z_0}\exp\left(jk\hat{k}\cdot\vec{r}\right)\hat{b} \tag{2}$$

where $\hat{k}$, $\hat{a}$ and $\hat{b}$ are unit vectors specifying the directions of incidence, the electric field and the magnetic field, respectively.

The propagation constant, permittivity and intrinsic impedance of the surrounding medium are k, ε_0 and Z_0 respectively, and a time factor $\exp(-j\omega t)$ has been assumed and suppressed.

The total electric and magnetic fields are written as the sum of the incident and scattered fields.

$$\vec{E} = \vec{E^i} + \vec{E^s} \tag{3}$$

$$\vec{H} = \vec{H^i} + \vec{H^s} \tag{4}$$

The tangential components of the surface field are interpreted as electric and magnetic currents:

$$\vec{J} = \vec{n} \times \vec{H} \tag{5}$$

$$\vec{M} = -\vec{n} \times \vec{E} \tag{6}$$

The surface impedance boundary conditions can be written in the form:

$$\vec{n} \times \vec{M} - jkZ_0 Z\left(\vec{J}\right) = 0 \tag{7}$$

where $\vec{n}$ denotes the normal to Γ directed to the exterior of Ω and where Z is a local boundary operator acting on tangential vector Fields on Γ.

The electric current $\vec{J}$ and the magnetic current $\vec{M}$ verify the EFIE equation [3]:

$$\vec{n} \times \vec{E^s} \times \vec{n} = jkZ_0 \vec{n} \times \tau\left(\vec{J}\right) \times \vec{n} + \vec{n} \times \kappa\left(\vec{M}\right) \times \vec{n} + \frac{1}{2}\vec{n} \times \vec{M} \tag{8}$$

where

$$\tau\left(\vec{J}\right) = \int_\Gamma G\left(r, r'\right) \vec{J}\left(r'\right) + \frac{1}{k^2} \vec{\bigtriangledown}_r G\left(r, r'\right) \vec{\bigtriangledown}_\Gamma \cdot J\left(\vec{r'}\right) d\Gamma\left(r'\right) \tag{9}$$

$$\kappa\left(\vec{M}\right) = \int_\Gamma \vec{\bigtriangledown}_{r'} G\left(r, r'\right) \times \vec{M}\left(r'\right) d\Gamma\left(r'\right) \tag{10}$$

$G(r, r')$ is the Green function

$$G\left(r, r'\right) = \frac{1}{4\pi \left|r - r'\right|} \exp\left(jk\left|r - r'\right|\right) \tag{11}$$

By using the Equation (7), the EFIE leads to the following equation

$$\begin{aligned} &\frac{1}{2} jkZ_0 Z\left(\vec{J}\right) - ikZ_0 \vec{n} \times \tau\left(\vec{J}\right) \times \vec{n} \\ &-ikZ_0 \vec{n} \times \kappa\left(Z\left(\vec{J}\right) \times \vec{n}\right) \times \vec{n} = \vec{n} \times \vec{E^i} \times \vec{n} \end{aligned} \tag{12}$$

The variational formulation for the Equation (12) is:

$$\begin{aligned} &\frac{1}{2} jkZ_0 \int_\Gamma Z\left(\vec{J}\right) \cdot \vec{J'} d\Gamma - jkZ_0 \int_\Gamma \tau\left(\vec{J}\right) \cdot \vec{J'} d\Gamma \\ &-jkZ_0 \int_\Gamma \kappa\left(Z\left(\vec{J}\right) \times \vec{n}\right) \cdot \vec{J'} d\Gamma = \int_\Gamma \vec{E^i} \cdot \vec{J'} d\Gamma \end{aligned} \tag{13}$$

where $\vec{J'}$ is a tangential vector on Γ.

By discretizing the conductor surface into triangular panels, the electric current $\vec{J}$ is approximated using Rao-Wilton-Glisson (RWG) basis functions $(\vec{\varphi_n})_{1\leq n\leq N}$ as follow:

$$\vec{J} = \sum_{n=1}^{N} J_n \vec{\varphi_n} \tag{14}$$

then Equation (13) can be converted into a linear system

$$MI = V \tag{15}$$

where M is an $N \times N$ system matrix, I is a column vector with the coefficients of the unknown currents and V is a column vector associated with the incident fields evaluated at conductor's surface.

3. LEVEL SET BUFFER REGION

Let ψ a level set function defined by:

$$\psi(r) = \begin{cases} \text{distance } (r, \Gamma) & \text{if } r \in \Omega \\ -\text{distance } (r, \Gamma) & \text{if } r \notin \Omega \end{cases} \tag{16}$$

Γ divides the domain omega into two parts, and then the level set function ψ is positive inside and negative outside.

$$\Omega = \left\{ r \in \mathbb{R}^3,\ \psi(r) > 0 \right\} \tag{17}$$

$$\Gamma = \left\{ r \in \mathbb{R}^3,\ \psi(r) = 0 \right\} \tag{18}$$

Let ζ_1 and ζ_2 be a closed curves defined on Γ and represent the boundaries of regions Γ_1^z, Γ_2^z, Γ_3^z and Γ_4^z in the conductor's surface Γ. We represent ζ_1 and ζ_2 respectively by the intersection of the zero level set of a real valued functions φ_1 and φ_2 on $\mathbb{R}^3$ with the zero level set of $\psi(r)$ [4].

$$\begin{aligned} \Gamma_1^z &= \text{support of } [H(\varphi_1)H(\varphi_2)] \\ \Gamma_2^z &= \text{support of } [H(\varphi_1)(1 - H(\varphi_2))] \\ \Gamma_3^z &= \text{support of } [H(\varphi_2)(1 - H(\varphi_1))] \\ \Gamma_4^z &= \text{support of } [(1 - H(\varphi_1))(1 - H(\varphi_2))] \end{aligned}$$

$H(\varphi)$ is the Heaviside function, defined by:

$$H(\varphi) = \begin{cases} 1 & \text{if } \varphi > 0 \\ 0 & \text{if } \varphi < 0 \end{cases} \tag{19}$$

Let ζ_1^+, ζ_1^-, ζ_2^+ and ζ_2^- four curves defined on Γ, as shown in the Figure 1, such that:

$$\begin{aligned} \text{distance}\left(\zeta_1^+, \zeta_1\right) &= \text{distance}\left(\zeta_1^-, \zeta_1\right) = d_\varepsilon \\ \text{distance}\left(\zeta_2^+, \zeta_2\right) &= \text{distance}\left(\zeta_2^-, \zeta_2\right) = d_\varepsilon \end{aligned} \tag{20}$$

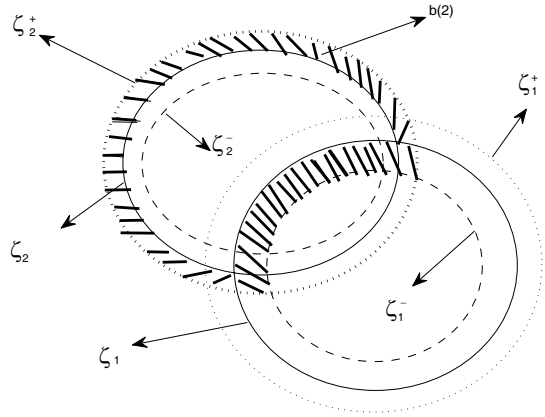

Figure 1. Level set buffer region.

ζ_1^+, ζ_1^-, ζ_2^+ and ζ_2^- can be represented respectively by four level set functions φ_1^+, φ_1^-, φ_2^+ and φ_2^- where [5]

$$\begin{aligned} \varphi_1^+ &= \varphi_1 - d_\varepsilon \left\| P_{\bigtriangledown\psi} \bigtriangledown \varphi_1 \right\| \\ \varphi_1^- &= \varphi_1 + d_\varepsilon \left\| P_{\bigtriangledown\psi} \bigtriangledown \varphi_1 \right\| \\ \varphi_2^+ &= \varphi_2 - d_\varepsilon \left\| P_{\bigtriangledown\psi} \bigtriangledown \varphi_2 \right\| \\ \varphi_2^- &= \varphi_2 + d_\varepsilon \left\| P_{\bigtriangledown\psi} \bigtriangledown \varphi_2 \right\| \end{aligned} \tag{21}$$

where $P_{\bigtriangledown\psi}$ is the projection matrix on the surface Γ and defined as [5]:

$$P_{\bigtriangledown\psi} = I - \frac{\bigtriangledown\psi}{\|\bigtriangledown\psi\|} \cdot \text{transpose}\left(\frac{\bigtriangledown\psi}{\|\bigtriangledown\psi\|}\right) \tag{22}$$

I is the 3×3 identity matrix.

We call the areas $b(1)$, $b(2)$, $b(3)$ and $b(4)$ the "buffer sub-regions" of the regions Γ_1^z, Γ_2^z, Γ_3^z and Γ_4^z respectively, as shown in Figure 1, where

$b(1)$=support of $\left[H(\varphi_1^+)H(\varphi_2^+)\left(1 - H(\varphi_1)H(\varphi_2)\right)\right]$

$b(2)$=support of $\left[H(\varphi_1^+)(1 - H(\varphi_2^-))\left(1 - H(\varphi_1)(1 - H(\varphi_2))\right)\right]$

$b(3)$=support of $\left[H(\varphi_2^+)(1 - H(\varphi_1^-))\left(1 - H(\varphi_2)(1 - H(\varphi_1))\right)\right]$

$b(4)$=support of $\left[(1-H(\varphi_1^-))(1-H(\varphi_2^-))\left(1-(1-H(\varphi_1))(1-H(\varphi_2))\right)\right]$

By generalizing, we see that n level set functions split the surface Γ in 2^n regions Γ_i^z, $1 \leq i \leq 2^n$.

The buffer sub-region $b(i)$ of the region Γ_i^z is defined by:

$$b(i) = \text{support of} \prod_{j=1}^{n} R_i^b(\varphi_j) \left[1 - \prod_{j=1}^{n} R_i(\varphi_j) \right] \tag{23}$$

where

$$R_i^b(\varphi_j) = \begin{Bmatrix} H(\varphi_j^+) \text{ if } b_j^i = 0 \\ 1 - H(\varphi_j^-) \text{ if } b_j^i = 1 \end{Bmatrix} \tag{24}$$

$$R_i(\varphi_j) = \begin{Bmatrix} H(\varphi_j) \text{ if } b_j^i = 0 \\ 1 - H(\varphi_j) \text{ if } b_j^i = 1 \end{Bmatrix} \tag{25}$$

And b_j^i is an element of the binary representation of $i-1$ [5].

$$bin(i-1) = \left(b_1^i, b_2^i, \ldots, b_m^i\right), \quad b_j^i \in \{0,1\}.$$

4. METHODOLOGY

In our previous work [2], we have reported a numerical scheme combining Rytov's perturbation method and level set technique to construct a piecewise surface impedance for an arbitrarily shaped conductor. By using n level set functions $\varphi_{1\leq i\leq n}$, we can subdivide the conductor's surface into 2^n sub-regions Γ_i^z, $1 \leq i \leq 2^n$, where each sub-region Γ_i^z is characterised by its own local SIBC $Z_{i-1}(\vec{J})$ [2].

$$Z\left(\vec{J}\right) = \sum_{i=1}^{2^n} Z_{i-1}\left(\vec{J}\right) \prod_{j=1}^{n} R_i\left(\varphi_j\right) \tag{26}$$

For example, let Z_0, Z_1, Z_2 and Z_3 denote respectively the PEC, Leontovich, Mitzner and Rytov SIBC, by using two level set functions (φ_1, φ_2) the conductor's surface is divided into four parts as shown in Figure 2.

$$\begin{aligned} Z \;=\; & Z_0 H(\varphi_1)H(\varphi_2) + Z_1(1-H(\varphi_1))H(\varphi_2) \\ & + Z_2 H(\varphi_1)(1-H(\varphi_2)) + Z_3(1-H(\varphi_1))(1-H(\varphi_2)) \end{aligned} \tag{27}$$

the method [2] leads to a SIBC discontinuity between adjacent regions which produces spurious edge effects when calculating the scattered fields, to circumvent this problem, we used forward-backward and quasiedge buffer iterative scheme.

The introducing of local buffer area between adjacent sub-regions suppresses the singularities introduced by the abrupt termination of each sub-region and ensures stability and accuracy.

By using the forward-backward approach, the scattered electromagnetic fields due to an incident wave are calculated separately in

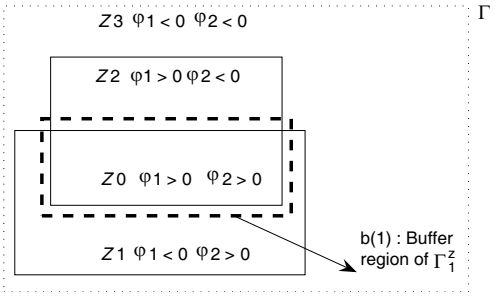

Figure 2. Two-dimensional piecewise SIBC.

each sub-region by the EFIE equation [6]. Then, electric current J is calculated in each sub-region $\Gamma^z_{i,1\leq i\leq 2^n}$. The electromagnetic fields radiated by these currents are calculated at the other sub-regions $\Gamma^z_{j,1\leq j\leq 2^n, j\neq i}$. These fields are considered as the new excitation for that sub-region. Then the cycle of calculation of electric current is repeated as a new iteration, the iteration process between sub-regions continues until a convergence criterion is achieved.

The algorithm [2] involves decomposing the M matrix into 2^n blocks, the M_{ij} block containing the interactions between basis functions residing in the ith and jth subregions on conductor's surface.

The iterative scheme can be written in a form of matrix as [6]:

$$\hat{M}_{ii}\hat{J}_i^{(k)} = \hat{W}_i - \sum_{j\neq i, j\notin b(i)}^{2^n} \hat{M}_{ij}\hat{I}_j, \quad i = 1, 2, \ldots 2^n \tag{28}$$

where

$$\hat{M}_{ii} = \begin{bmatrix} M_{ii} & M_{ib(i)} \\ M_{b(i)i} & M_{b(i)b(i)} \end{bmatrix}, \quad \hat{J}_i^{(k)} = \begin{bmatrix} I_i^{(k)} \\ I_{b(i)} \end{bmatrix}, \quad \hat{I}_j = \begin{cases} I_j^{(k)}, & j < i \\ I_j^{(k-1)}, & j > i \end{cases} \tag{29}$$

and

$$\hat{M}_{ij} = \begin{bmatrix} M_{ij} \\ M_{b(i)j} \end{bmatrix}, \qquad \hat{W}_i = \begin{bmatrix} V_i \\ V_{b(i)} \end{bmatrix} \tag{30}$$

$b(i)$ represents the buffer region between the region Γ_i^z and it's adjacent sub-regions.

$I_j^{(k)}$ is the surface current of Γ_j^z after the kth iteration, M_{ii} is the self-impedance matrix in the Γ_i^z, M_{ij} is the impedance matrix between Γ_i^z and Γ_j^z, $M_{b(i)b(j)}$ is the self-impedance matrix in the buffer region $b(i)$, $M_{b(i)j}$ is the impedance matrix between the buffer region $b(i)$ and

Γ_i^z, $M_{b(i)j}$ is the mutual-impedance matrix between the buffer region $b(i)$ and Γ_j^z.

The error on the current is used for convergence criterion:

$$e_k = \frac{\left\| \vec{J}^{(k)} - \vec{J}^{(k-1)} \right\|}{\left\| \vec{J}^{(k)} \right\|} \tag{31}$$

5. NUMERICAL EXAMPLE

In order to illustrate the accuracy of the proposed method, numerical results are presented for an aluminum finite cylinder (1 m Diameter × 2 m High), The relevant conductor geometry is modelled by piecewise surface impedance boundary conditions, as shown in Figure 3.

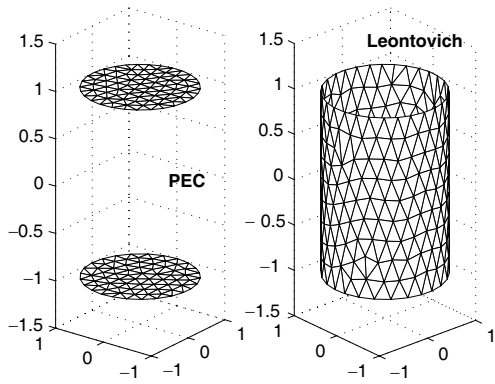

Figure 3. Piecewise surface impedance boundary conditions.

We represented the cylinder by a level set function, by using [2], we divided the cylinder's surface into two sub regions (Leontovich and PEC) such that the error on the surface impedance don't exceed $10^{-2}\%$.

The conductor is illuminated by an incident plane wave at 10 MHz and 100 MHz, the bistatic RCS has been studied for the incidence directions $(\theta = \frac{\pi}{2}, \phi = 0)$, $(\theta = \frac{\pi}{4}, \phi = 0)$ and for the scattering plane $\phi = 0$.

The proposed scheme has been developed and implemented within MATLAB environment, the level set Toolbox of Ian Mitchell [7] and distmesh Toolbox [8].

Figure 4 shows a comparison between the bistatic RCS values obtained from the proposed scheme and those obtained from the FEKO

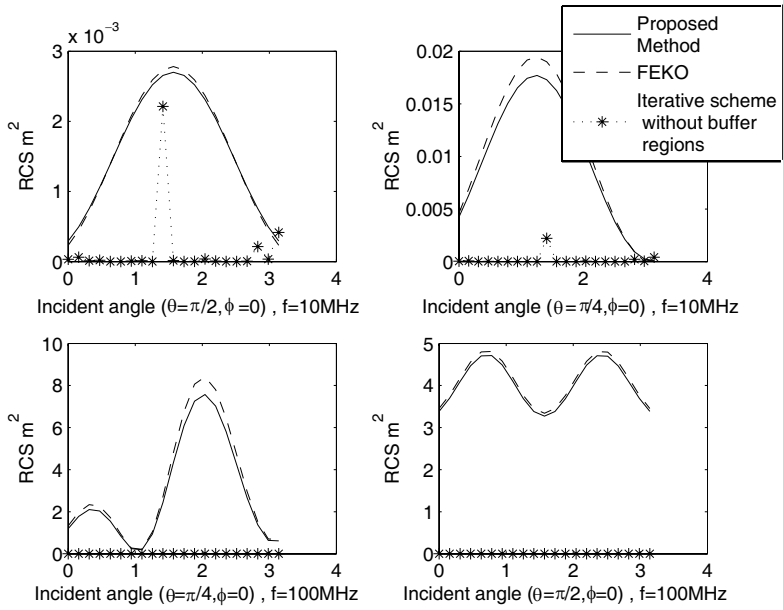

Figure 4. RCS of an aluminium finite cylinder.

electromagnetic simulation software, we observe the good agreement between both results.

With FEKO environment, the whole cylinder surface is modelled by Leontovitch impedance and the RCS is calculated by the Method of Moments.

From results, we deduce that the proposed method is able to give the same accuracy in the RCS computing as that of FEKO electromagnetic simulation software, with less SIBC's order. We show also the important role played by the buffer regions in eliminating spurious edge effects and in the stability of the proposed method.

6. CONCLUSION

We have reported a numerical scheme to compute the bistatic RCS of a 3D conductor with arbitrarily shape, by using piecewise surface impedance boundary conditions and forward backward iterative algorithm. Stability was achieved by the adoption of buffer regions which suppresses spurious edge effects and ensures accuracy of the RCS computing. Comparison between the RCS values obtained from the proposed scheme and those obtained from the FEKO electromagnetic simulation software, showing the good agreement between both results.

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