A NEW APPROACH TO ENHANCE INCIDENCE ANGLE BASED SPECTRUM TUNING CAPABILITY OF ONE-DIMENSIONAL TERNARY PHOTONIC BAND GAP STRUCTURE

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Abstract—This paper demonstrates a novel and superior approach to enhance the incident angle based spectrum tuning capability of 1D ternary Photonic Band Gap (PBG) structure. The incidence angle sensitive wavelength band shift of a ternary periodic structure was significantly enhanced when the refractive index of sandwiched layers in each period was changed to 1.5 from 2.04. The ranges of enhancements for TE and TM wavelength band shifts were 0.5–1.5 nm and 5.5–20.5 nm respectively at different angles of incidence of light on the structure. Unlike previous approach, this approach not only enhances the incidence angle based spectrum tuning capability of 1D ternary PBG structure, but, it also ensures that the size of structure does not increase and temperature immunity of the structure does not decrease to enhance spectrum tuning capability.

1. INTRODUCTION

Periodic media like one-dimensional (1D) photonic band gap structures (PBGs) [1–15] have attracted great interest of researchers due to their remarkably useful capabilities to guide and control light propagation. In these 1D PBG structures, generally, two or more different optical materials are periodically arranged. Actually, these 1D PBG structures are classified into the categories like binary and ternary, depending on the number of different material layers present in one period. 1D binary PBG structure contains two layers in a period and 1D ternary
PBG structure contains three layers in a period. Finally, these periods are repeated several times to form PBG structures. The approach is simple; this ternary structure can be fabricated by depositing a thin layer of a third material between the two layers of the binary structure periodically. However, this sandwiching of a thin layer of a third material between every two layers binary PBG structure periodically has remarkable consequences i.e., it drastically enhances the performance of 1D binary PBG structure to guide and control light propagation [10–15]. Recently, in a research work [15], it was found that the incidence angle based spectrum tuning capability of 1D binary PBG structure enhances when the structure is modified to 1D ternary PBG structure by sandwiching a thin layer of third material, between every two layers, constituting a period of lattice. It was also found that the incidence angle based spectrum tuning capability of 1D ternary PBG structure further enhances with increase in thickness of the sandwiched layers. However, the increase in thickness of the sandwiched layers to further enhance spectrum tuning capability [15] of 1D ternary PBG structure increases the overall size of the structure and decreases the temperature immunity [13] of transmission spectrum, which are drawbacks. To overcome these drawbacks and still enhance incidence angle based spectrum tuning capability of a 1D ternary PBG structure [15], an alternative new approach is demonstrated in this paper. In this approach the incidence angle based spectrum tuning capability of a 1D ternary PBG structure [15] was further enhanced by changing the refractive index of sandwiched layers from 2.04 to 1.5 in 1D ternary PBG structure. This not only ensured that the size of PBG structure does not increase and temperature immunity of the spectrum does not decrease, but also enhanced the incidence angle based spectrum tuning capability of 1D ternary PBG structure.

2. THEORY

The 1D binary PBG structure consisting of alternate layers of refractive indices $n_1$ and $n_2$ with thicknesses $d_1$ and $d_2$ respectively is depicted in Fig. 1. $d = d_1 + d_2$ is the period of the lattice.

The binary structure shown in Fig. 1 can be modified by sandwiching a thin layer of a third material of refractive index $n_B$ and thickness $d_B$ between two layers of the binary structure periodically to form a ternary PBG structure as depicted in Fig. 2. Now this modified periodic structure (i.e., ternary structure) has alternate layers of refractive indices $n_1$ (first material layer), $n_B$ (second material layer) and $n_2$ (third material layer) with thicknesses: $d_1$, $d_B$ and $d_2$ respectively. $d = d_1 + d_B + d_2$ is the period of the lattice.
According to the optical transmission matrix theory \[16\], for light incident from air on the multilayer at an angle \( \theta_0 \); for the \( s \) wave, the characteristic matrix \( M[d] \) of one period is given by

\[
M[d] = \prod_{i=1}^{l} \begin{bmatrix} \cos \beta_i & \frac{-i \sin \beta_i}{p_i} \\ -ip_i \sin \beta_i & \cos \beta_i \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \tag{1}
\]

where \( l = 3 \) (1, 2 and 3 signify the layers of refractive indices \( n_1, n_B \) and \( n_2 \) respectively),

\[
\beta_1 = \frac{2\pi n_1 d_1 \cos \theta_1}{\lambda_0}, \quad \beta_2 = \frac{2\pi n_B d_B \cos \theta_2}{\lambda_0} \quad \text{and} \quad \beta_3 = \frac{2\pi n_2 d_2 \cos \theta_3}{\lambda_0}.
\]

\( \lambda_0 \) is the free space wavelength, \( p_1 = n_1 \cos \theta_1 \), \( p_2 = n_B \cos \theta_2 \) and \( p_3 = n_2 \cos \theta_3 \). \( \theta_1, \theta_2 \) and \( \theta_3 \) are the ray angles inside the layers 1, 2
and 3 respectively and are related to the angle of incidence $\theta_0$ by

$$
\cos \theta_1 = \left[ 1 - \frac{\sin^2 \theta_0}{n_1^2} \right]^{\frac{1}{2}}, \quad \cos \theta_2 = \left[ 1 - \frac{\sin^2 \theta_0}{n_B^2} \right]^{\frac{1}{2}}
$$

and

$$
\cos \theta_3 = \left[ 1 - \frac{\sin^2 \theta_0}{n_2^2} \right]^{\frac{1}{2}}. \quad (2)
$$

The matrix $M[d]$ in Equation (1) is unimodular as $|M[d]| = 1$.

For an $N$ period structure, the characteristic matrix of the medium is given by

$$
[M (d)]^N = \begin{bmatrix}
M_{11}U_{N-1} (a) - U_{N-2} (a) & M_{12}U_{N-1} (a) \\
M_{21}U_{N-1} (a) & M_{22}U_{N-1} (a) - U_{N-2} (a)
\end{bmatrix}
$$

$$
\equiv \begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix},
$$

where

$$
M_{11} = \left( \cos \beta_1 \cos \beta_2 \cos \beta_3 - \frac{p_2 \sin \beta_1 \sin \beta_2 \cos \beta_3}{p_1} \right) - \frac{p_3 \cos \beta_1 \sin \beta_2 \sin \beta_3}{p_2} - \frac{p_3 \sin \beta_1 \cos \beta_2 \sin \beta_3}{p_1},
$$

$$
M_{12} = -i \left( \frac{\sin \beta_1 \cos \beta_2 \cos \beta_3}{p_1} + \frac{\cos \beta_1 \sin \beta_2 \cos \beta_3}{p_2} + \frac{\cos \beta_1 \cos \beta_2 \sin \beta_3}{p_3} - \frac{p_2 \sin \beta_1 \sin \beta_2 \sin \beta_3}{p_1 p_3} \right),
$$

$$
M_{21} = -i \left( p_1 \sin \beta_1 \cos \beta_2 \cos \beta_3 + p_2 \cos \beta_1 \sin \beta_2 \cos \beta_3 + p_3 \cos \beta_1 \cos \beta_2 \sin \beta_3 - \frac{p_1 p_3 \sin \beta_1 \sin \beta_2 \sin \beta_3}{p_2} \right),
$$

$$
M_{22} = \left( \cos \beta_1 \cos \beta_2 \cos \beta_3 - \frac{p_1 \sin \beta_1 \sin \beta_2 \cos \beta_3}{p_2} \right) - \frac{p_2 \cos \beta_1 \sin \beta_2 \sin \beta_3}{p_3} - \frac{p_1 \sin \beta_1 \cos \beta_2 \sin \beta_3}{p_3},
$$

$U_N$ are the Chebyshev polynomials of the second kind

$$
U_N (a) = \frac{\sin[(N + 1) \cos^{-1} a]}{[1 - a^2]^{\frac{1}{2}}}, \quad (3)
$$

where

$$
a = \frac{1}{2} [M_{11} + M_{22}] . \quad (4)
$$
The transmission coefficient of the multilayer is given by

\[ t = \frac{2p_0}{(m_{11} + m_{12}p_0)p_0 + (m_{21} + m_{22}p_0)} \] (5)

and the transmissivity for this structure can be written in terms of transmission coefficient as

\[ T = |t|^2, \] (6)

where

\[ p_0 = n_0 \cos \theta_0 = \cos \theta_0 \quad \text{as} \quad (n_0 = 1). \]

The transmissivity of the structure, for \( p \) wave can be obtained by using expressions (1)–(6) with the following values of \( p \)

\[ p_1 = \frac{\cos \theta_1}{n_1}, \quad p_2 = \frac{\cos \theta_2}{n_B}, \quad p_3 = \frac{\cos \theta_3}{n_2} \quad \text{and} \quad p_0 = \cos \theta_0. \]

3. RESULTS AND DISCUSSIONS

Figures 3–6, show the output spectra for 1D ternary PBG structure at different angles of incidence with \( n_1 = 3.3, n_B = 2.04 \) and 1.5, \( n_2 = 1.46, d_1 = 117 \text{nm}, d_B = 40 \text{nm}, d_2 = 265 \text{nm} \) and \( N = 10 \). Solid curves are for TE waves and dotted curves are for TM waves.

![Figure 3](image-url)  
**Figure 3.** Transmission spectra for 1D ternary PBG structure with \( n_1 = 3.3, n_B = 2.04 \) and 1.5, \( n_2 = 1.46, d_1 = 117 \text{nm}, d_B = 40 \text{nm}, d_2 = 265 \text{nm} \) and \( N = 10 \) at 0° angle of incidence of light.
Figure 4. Transmission spectra for 1D ternary PBG structure with $n_1 = 3.3$, $n_B = 2.04$ and 1.5, $n_2 = 1.46$, $d_1 = 117$ nm, $d_B = 40$ nm, $d_2 = 265$ nm and $N = 10$ at $30^\circ$ angle of incidence of light.

Figure 5. Transmission spectra for 1D ternary PBG structure with $n_1 = 3.3$, $n_B = 2.04$ and 1.5, $n_2 = 1.46$, $d_1 = 117$ nm, $d_B = 40$ nm, $d_2 = 265$ nm and $N = 10$ at $45^\circ$ angle of incidence of light.
Considering the transmission peak to the right of photonic band edge, from these figures, it is clear that the transmission peak shifts toward lower wavelengths with increase in angle of incidence of the light on the ternary PBG structure. It is also clear from these figures that the incidence angle sensitive output spectrum transmission peak shifts of 1D ternary PBG structure significantly enhance when the refractive index of the sandwiched layers is decreased from 2.04 to 1.5.

As shown in Fig. 4, the incidence angle sensitive TE and TM wave transmission peak shifts were enhanced by 0.5 nm and 5.5 nm respectively when the refractive index of sandwiched layers was changed to 1.5 from 2.04. As evident from Fig. 5, the incidence angle sensitive TE and TM wave transmission peak shifts were enhanced by 1.5 nm and 12.5 nm respectively when the refractive index of sandwiched layers were changed to 1.5 from 2.04. As visible in Fig. 6, the incidence angle sensitive TE and TM wave transmission peak shifts were enhanced by 1.5 nm and 20.5 nm respectively when the refractive index of sandwiched layers were changed to 1.5 from 2.04. Table 1 summarizes the effect of refractive indices of sandwiched layers on the incidence angle sensitive output spectrum transmission peak shift enhancement, at angles of incidence 30°, 45° and 60°. From Figs. 3–6 and Table 1, it is clear that the incidence angle sensitive output spectrum transmission peak shift reasonably enhances on decreasing

![Figure 6](image-url) Figure 6. Transmission spectra for 1D ternary PBG structure with $n_1 = 3.3$, $n_B = 2.04$ and 1.5, $n_2 = 1.46$, $d_1 = 117$ nm, $d_B = 40$ nm, $d_2 = 265$ nm and $N = 10$ at 60° angle of incidence of light.
Table 1. Effect of refractive index of sandwiched layers on the incidence angle sensitive output spectrum transmission peak shift enhancement.

<table>
<thead>
<tr>
<th>Angle of incidence (degree)</th>
<th>$n_B$</th>
<th>Enhancement in incidence angle sensitive output spectrum transmission peak shift (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TE wave</td>
</tr>
<tr>
<td>30</td>
<td>2.04</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.5</td>
</tr>
<tr>
<td>45</td>
<td>2.04</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>1.5</td>
</tr>
<tr>
<td>60</td>
<td>2.04</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 2. Thicknesses of sandwiched layers for given transmission peak shift enhancements.

<table>
<thead>
<tr>
<th>Enhancement in incidence angle sensitive output spectrum transmission peak shift (nm)</th>
<th>Required increase in thickness of the sandwiched layer using approach demonstrated in reference [15] (nm)</th>
<th>Increase in overall thickness of the structure using approach demonstrated in reference [15] (nm)</th>
<th>Increase in overall thickness of the structure using approach proposed in this paper (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE</td>
<td>0.5</td>
<td>18.33</td>
<td>183.3</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>41.66</td>
<td>416.6</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>68.33</td>
<td>683.3</td>
</tr>
</tbody>
</table>

the refractive index of the sandwiched layers from 2.04 to 1.5 in this 1D ternary PBG structure. This decreases the tuning angle, tuning time and tuning power required and increases spectrum tuning efficiency. The TE and TM wave transmission peak shift enhancements in this approach is obtained by changing refractive index of the sandwiched layers from 2.04 to 1.5 without any increase in size of the structure. However, by using the approach demonstrated in reference [15] the TE and TM wave transmission peak shift enhancements can be obtained by increasing the thickness of sandwiched layers only in the 1D ternary PBG structure. Therefore, the approach proposed in this paper is superior to the approach demonstrated in reference [15].

Table 2 shows the required increase in thicknesses of the
sandwiched layers if previous approach [15] is used to obtain transmission peak shift enhancements given in Table 1. From Table 2, it is clear that by using the approach suggested in reference [15] given transmission peak shift enhancements (considering the higher enhancements only, i.e., column 2 in Table 2) can be obtained by increasing the thickness of the sandwiched layers, which increases the overall size of the PBG structure and also makes the output spectrum of this structure highly temperature sensitive (recently, it was found [13] that the temperature sensitive transmission spectrum shift in 1D ternary PBG structures increases (i.e., temperature immunity of the spectrum decreases) linearly with the increasing thickness of the sandwiched layers), which are major drawbacks. While, using the approach demonstrated in this paper these drawbacks can be overcome. Further, in a suitably engineered [17] ternary PBG structure in which external applied electric field may change the refractive index of a layer through electro-optic effect, the tuning range and hence the tuning capability of the structure can also be tuned in a desired way.

4. CONCLUSIONS

A novel way to enhance the incident angle based spectrum tuning capability of a 1D ternary PBG structure has been suggested. This approach is superior to the approach suggested in reference [15]. This approach not only enhances the incidence angle based spectrum tuning capability of 1D ternary PBG structure further, but, also ensures that the size of structure does not increase and temperature immunity of the structure does not decrease. A decrease in refractive index of sandwiched layers in a 1D ternary PBG structure to 1.5 from 2.04 caused a sufficiently large transmission peak shift enhancements ranging from 0.5 to 1.5 nm and 5.5 to 20.5 nm for TE and TM waves respectively at different angles of incidence of light on the structure. This increases the tuning range, tuning speed of optical filters and in turn reduces the tuning power required. The tuning capability of a suitably designed ternary PBG structure can be controlled by tuning the refractive index of the sandwiched layers through electro-optic effect.

ACKNOWLEDGMENT

I am grateful to Sri. Aseem Chauhan, Maj. Gen. K. K. Ohri, Prof. S. T. H. Abidi, Brig. U.K. Chopra and Prof. N. Ram of Amity University, Lucknow for their constant encouragement and support during this research work.
REFERENCES

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