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ANALYSIS OF THE FIELD FOCUSED BY HYPERBOLIC LENS EMBEDDED IN CHIRAL MEDIUM

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Abstract—This paper is a theoretical investigation and analysis of the focal region fields of a hyperbolic focusing lens embedded in chiral medium. Chiral-dielectric and dielectric-chiral interfaces are studied and the behavior of waves after passing through these interfaces are discussed. Geometric optics (GO) is used initially. However, it fails around the focal region because it gives non-realistic singularity in this region. So, Maslov's method is used in the caustic region and the field analysis is made. The effect of chirality variation on the amplitude of the fields around the focal region is given and discussed.

1. INTRODUCTION

Geometrical optics is a technique which is used to determine the radiation and scattering from objects with dimensions greater than the wavelength. It is simple to apply and can be used to solve complicated problems that don't have exact solutions. The general expression for the field calculation using GO is given as following [1]:

$$u(\mathbf{r}) = E_0(\xi,\zeta) J^{-1/2} \exp\left\{-jk\left(s_0 + \int_{t_0}^{t} n^2 dt\right)\right\} \tag{1}$$

In the above equation, $E_0(\xi,\zeta)$ is the initial value of the field amplitude and $J = D(t)/D(0)$, where $D(t) = \frac{\partial(x,z)}{\partial(\xi,\zeta)}$, is the Jacobian transformation from ray coordinates (ξ, ζ) to cartesian coordinate (x, z). In the focal region, GO solution of Equation (1) fails because the tube of rays in which the intensity is being conserved has zero cross section, i.e., $J(t) = 0$. Thus the GO field expression yields an infinite field at this point. The failure of GO around the focal region

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is compensated by using the Maslov's method [2]. This method uses the simplicity of ray theory and the generality of Fourier transform to predict the field around the caustic region. It uses a combination of spatial domain and wave vector domain and gives rise to a hybrid space. This eliminates the possibility of occurrence of singularity around the caustic. General expression of Maslov's method for field calculation is given as [3]:

$$u(r) = \sqrt{\frac{-k}{2j\pi}} \int_{-\infty}^{\infty} A_0(\xi) \left(\frac{D(t)}{D(0)} \frac{\partial q_x}{\partial x} \right)^{-\frac{1}{2}} e^{-jk\{ns_0 + t - x(q_x,z)q_x + xq_x\}} dq_x$$

The expression $\frac{D(t)}{D(0)} \frac{\partial q_x}{\partial x}$ can be calculated more simply as

$$\frac{D(t)}{D(0)} \frac{\partial q_x}{\partial x} = \frac{1}{D(0)} \frac{\partial(q_x, z)}{\partial(\xi, t)} = \frac{1}{D(0)} \frac{\partial q_x}{\partial \xi} \frac{\partial z}{\partial t} \tag{2}$$

GO and Maslov's method has been utilized by many authors to study different focusing systems in the caustic region [10–19]. Focusing system in this problem is the hyperbolic lens which is placed inside a chiral medium. When chiral objects are placed together in such a way that they are uniformly distributed and randomly oriented, they form a homogeneous chiral medium. A chiral object is a three dimensional object which can not be brought into congruence with its mirror image by simple translation or rotation [4]. Chirality causes handedness and handedness is responsible for optical activity in chiral medium. Optical activity in chiral medium is the ability of its objects for rotating the plane of polarization of linear polarized plane wave traveling through the medium [4]. It causes the splitting of the incident wave into right circularly polarized (RCP) and left circularly polarized (LCP) waves. Optical activity depends upon the amount of samples of the handed molecules. Thus, optical activity depends upon the distance traveled inside the chiral medium [5]. Drude-Born-Fedorov (DBF) is mostly utilized in describing the waves inside a chiral medium. This representation is given as follows:

$$\mathbf{D} = \epsilon(\mathbf{E} + \beta \nabla \times \mathbf{E}), \quad \mathbf{B} = \mu(\mathbf{H} + \beta \nabla \times \mathbf{H}) \tag{3}$$

where ϵ, μ and β are the permittivity, permeability and the chirality parameter, respectively. Solution of Maxwell's equations by using Equation (3) results in coupled differential equations. This complication is resolved by using uncoupled differential equations for **E** and **H**, which are obtained using the following transformation [6]:

$$\mathbf{E} = \mathbf{Q}_L - j\sqrt{\frac{\mu}{\epsilon}} \mathbf{Q}_R, \quad \mathbf{H} = \mathbf{Q}_R - j\sqrt{\frac{\epsilon}{\mu}} \mathbf{Q}_L \tag{4}$$

where $\mathbf{Q}_L$ and $\mathbf{Q}_R$ are LCP and RCP waves respectively which satisfies the following equations:

$$(\nabla^2 + n_1^2 k^2)\mathbf{Q}_L = 0, \quad (\nabla^2 + n_2^2 k^2)\mathbf{Q}_R = 0 \tag{5}$$

where, $n_1 = 1/(1 - k\beta)$ and $n_2 = 1/(1 + k\beta)$ are equivalent refractive indices for LCP and RCP waves respectively and $k = \omega\sqrt{\epsilon\mu}$ [5].

In Section 2, phenomenon of refraction by two different interfaces in chiral medium has been explained. Hyperbolic lens placed in chiral medium is analyzed in Section 3. In Section 4, the discussion about the fields around the focal region of hyperbolic focusing lens, placed in chiral medium, has been studied using GO and Maslov's method. All the results are summarized and concluded in Sections 5 and 6, respectively.

2. REFRACTION IN CHIRAL MEDIUM

In chiral medium, incident wave is divided into LCP and RCP waves. They both have different refractive indices and have different angle of refraction as well. Placing of hyperbolic focusing lens inside a chiral medium has two issues regarding the refraction. When a combination of RCP and LCP waves are incident then these two waves experience two different boundaries. One is chiral-dielectric interface and the other (after traveling through the lens) is dielectric-chiral interface. Both of these boundaries are discussed latter in this section. Consider a dielectric medium as a sandwiched between two chiral mediums as shown in Figure 1.

2.1. Refraction at Chiral-dielectric Interface

Figure 2 shows that a combination of RCP and LCP waves are obliquely incident on chiral-dielectric interface at an angle of θ_1 and θ_2

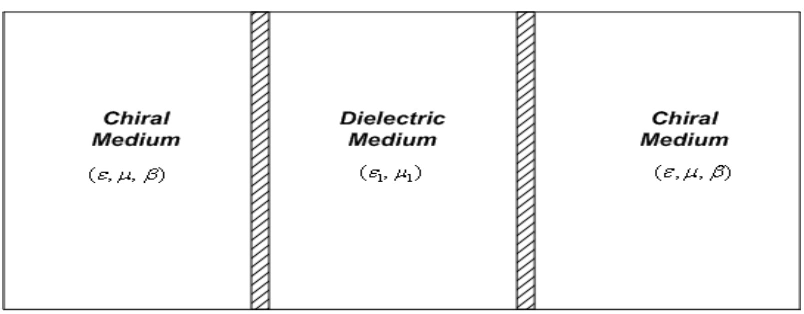

Figure 1. Chiral-dielectric-chiral interface.

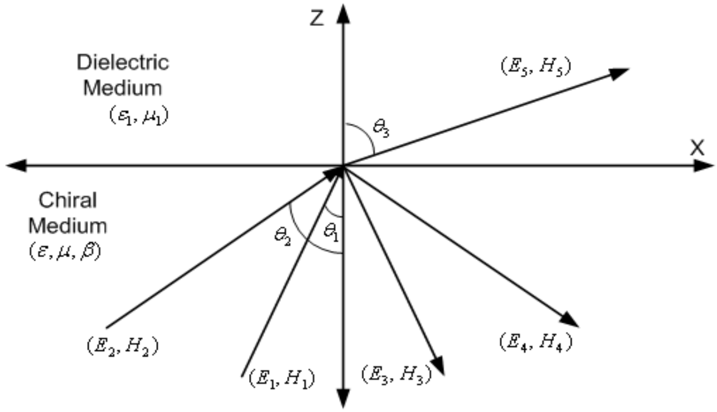

Figure 2. Waves traveling from chiral to dielectric medium [7].

with apparent wave number n_1 and n_2, respectively. These waves are partially transmitted to the dielectric medium and partially reflected back into the chiral medium. The refracted ray makes an angle θ_3 with the normal (i.e., z-axis) in this case. Applying Snell's law at the boundary of both the interfaces results in following expression:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = k_1 \sin\theta_3, \tag{6}$$

where $k_1 = \omega\sqrt{\epsilon_1\mu_1}$. Both RCP and LCP are refracted as a single wave inside the dielectric. This transmitted wave has less magnitude than the incident RCP and LCP waves. Perfect transmission is considered for desired analysis which occurs only when $\eta = Z$. Where

$$Z = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt{1 + \frac{\mu}{\epsilon}\beta^2}}, \quad \eta = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{7}$$

and μ, ϵ, β are the permeability, permittivity and chirality parameter. While μ_1, ϵ_1 are permeability and permittivity of the dielectric Medium. This above condition of absolute transmission occurs (for $\mu = \mu_1$) when $\epsilon_1 = (\epsilon + \mu\beta^2)$. Neglecting the reflection coefficients. The transmission coefficients for general case of oblique incidence are given as following [7]:

$$\begin{pmatrix} E_{5\parallel} \\ E_{5\perp} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{8}$$

In present case of the hyperbolic focusing lens vertical incidence occurs, i.e., the angles $\theta_1 = \theta_2 = \theta_3 = 0$. The transmission coefficients then becomes:

$$T_{11} = -T_{22} = \frac{2\eta}{\eta + Z}, \tag{9a}$$

$$T_{12} = T_{21} = \frac{2\eta}{\eta + Z} \tag{9b}$$

The perpendicular and parallel components of the transmitted field inside the dielectric medium is given as follows:

$$E_{5\parallel} = \frac{2\eta}{\eta + Z}(E_1 + E_2) \tag{10a}$$

$$E_{5\perp} = \frac{2\eta}{\eta + Z}(E_1 - E_2) \tag{10b}$$

The parallel and perpendicular field components of the transmitted field given in Equations (10a) and (10b) will act as incident to the dielectric-chiral interface.

2.2. Refraction at Dielectric-chiral Interface

When a plane wave is incident on the dielectric-chiral interface, it is partially transmitted to the chiral medium and partially reflected back to the dielectric medium as shown in Figure 3. By ignoring the

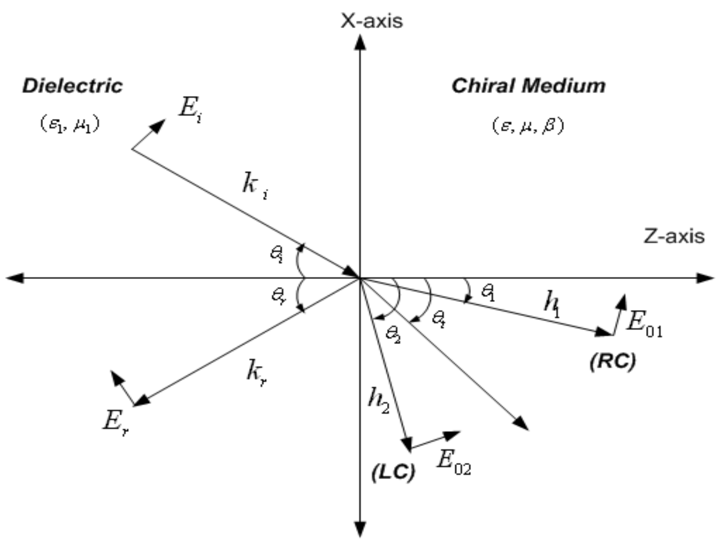

Figure 3. The incident, reflected and transmitted waves at an oblique incidence at dielectric-chiral interface [8].

reflection (i.e., taking the perfect transmission), the transmitted wave into the chiral medium will split into two waves designated as LCP and RCP waves, which will make an angle of θ_1 and θ_2 with the normal, respectively. The magnitude of RCP and LCP waves, E_{01} and E_{02} respectively, are given as [8].

$$\begin{pmatrix} E_{01} \\ E_{02} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\parallel} \end{pmatrix} \tag{11}$$

The values of $E_{i\perp}$ and $E_{i\parallel}$ are given in Equations (10a) and (10b), respectively. The values of all four transmission coefficients are given as follows [8]:

$$T_{11} = \frac{-2i\cos\theta_i(g\cos\theta_i + \cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2)+2g(\cos^2\theta_i+\cos\theta_1\cos\theta_2)}, \tag{12a}$$

$$T_{12} = \frac{2\cos\theta_i(\cos\theta_i + g\cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2)+2g(\cos^2\theta_i+\cos\theta_1\cos\theta_2)}, \tag{12b}$$

$$T_{21} = \frac{2i\cos\theta_i(g\cos\theta_i + \cos\theta_1)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2)+2g(\cos^2\theta_i+\cos\theta_1\cos\theta_2)}, \tag{12c}$$

$$T_{22} = \frac{2\cos\theta_i(\cos\theta_i + g\cos\theta_1)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2)+2g(\cos^2\theta_i+\cos\theta_1\cos\theta_2)} \tag{12d}$$

where g in the above expressions is given as:

$$g = \sqrt{\frac{\mu_1}{\epsilon_1}\beta^2 + \frac{\mu_1\epsilon}{\epsilon_1\mu}} \tag{13}$$

The angle of incidence and refraction for LCP and RCP waves, using Snell's law of refraction, are given as:

$$\theta_2 = \sin^{-1}\left(\frac{k\sin\theta_i}{n_2}\right), \quad \theta_1 = \sin^{-1}\left(\frac{k\sin\theta_i}{n_1}\right) \tag{14}$$

So, RCP and LCP waves upon entering into the dielectric medium through a chiral dielectric interface will become a single wave. This wave will neither be RCP or LCP. Similarly, a wave passing through a dielectric when enters chiral medium through dielectric chiral interface, is divided into two waves of opposite handedness. The output waves can be obtained by multiplying parallel and perpendicular components of field inside dielectric with transmission coefficient matrix.

3. HYPERBOLIC LENS PLACED IN CHIRAL MEDIUM

A hyperbolic lens is a two dimensional structure with one of its surface is plane and the other is hyperbolic. Consider a hyperbolic lens with

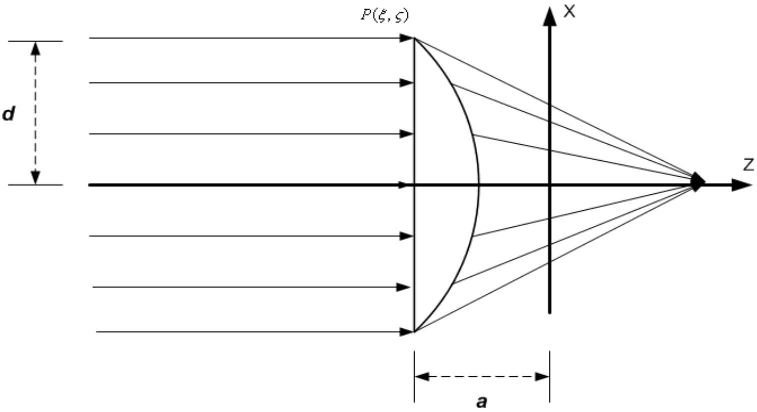

Figure 4. Fields and parameters of a hyperbolic lens.

height d and placed at a distance a from x-axis as shown in Figure 4. Waves are incident at the planar surface of the lens. They are refracted by the lens and are converged at focal point of the lens. The surface profile of the hyperbolic focusing lens is given as following [3]:

$$g(\xi) = \zeta = \frac{a}{b}\sqrt{\xi^2 + b^2} \tag{15}$$

where

$$\xi = \frac{b^2 \sin(\alpha)}{\sqrt{a^2 \cos^2(\alpha) - b^2 \sin^2(\alpha)}}, \quad \zeta = \frac{a^2 \cos(\alpha)}{\sqrt{a^2 \cos^2(\alpha) - b^2 \sin^2(\alpha)}} \tag{16}$$

and (ξ, ζ) are the cartesian coordinates on the lens. A hyperbolic lens, with one surface as hyperbolic and the other as planar, placed inside the chiral medium (ϵ, μ, β) is shown in Figure 5. A combination of LCP and RCP waves is normally incident on the hyperbolic lens. The incident field is a combination of LCP and RCP waves. It has unit amplitude and is traveling in chiral medium along positive z-axis as given below [9]:

$$Q_L = \exp(-jkn_1 z), \quad Q_R = \exp(-jkn_2 z) \tag{17}$$

Vertical incidence occurs which makes incidence and refraction angle equal to zero degree. Also, both RCP and LCP waves get unified into one wave traveling through dielectric medium towards dielectric-chiral interface [8]. The transmitted wave into the chiral medium makes an angle α with normal to the hyperbolic surface. Lens will refract this wave and chiral molecules will split it into two waves, LCP and RCP, each making an angle α_1 and α_2 with normal to the hyperbolic surface. A magnified view of the hyperbolic lens is shown in Figure 6.

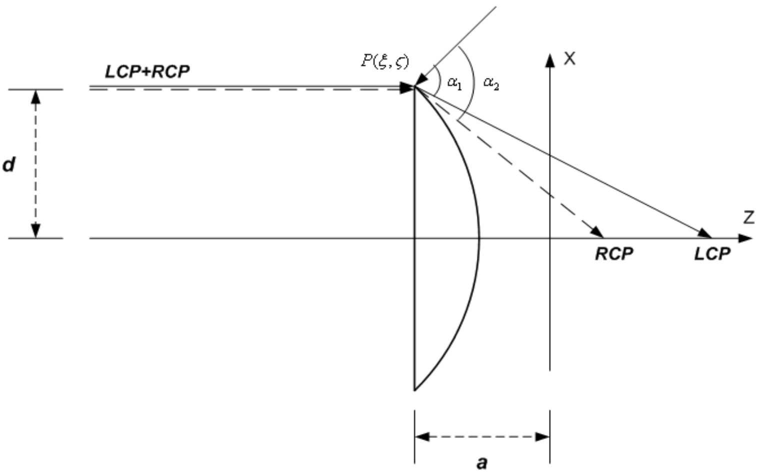

Figure 5. Hyperbolic lens placed in chiral medium.

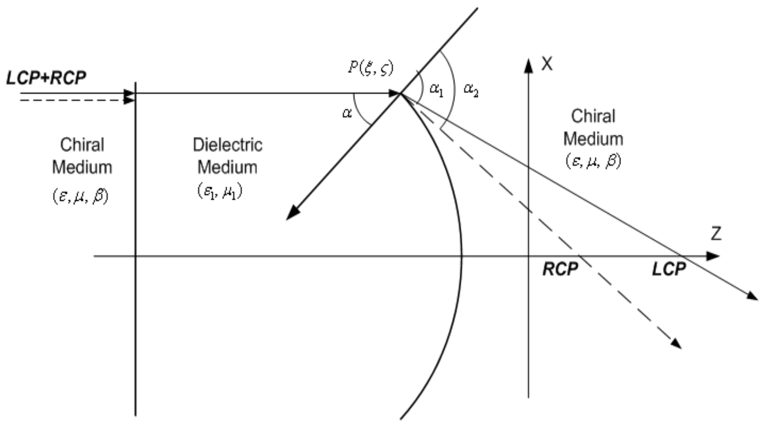

Figure 6. Internal magnified view of the hyperbolic lens.

where

$$\alpha_1 = \sin^{-1}\left(\frac{n}{n_1}\sin\alpha\right), \quad \alpha_2 = \sin^{-1}\left(\frac{n}{n_2}\sin\alpha\right) \tag{18}$$

4. CALCULATIONS OF FOCAL REGION FIELD USING GEOMETRICAL OPTICS AND MASLOV'S METHOD

General solution for field using GO is given in Equation (1). To evaluate the expression, the wave vector for two waves, i.e., LCP and

RCP are to be calculated first. The wave vector of the refracted wave can be calculated by using Snell's Law of refraction. Its general form is given as [3]:

$$\mathbf{q} = n\mathbf{p}^i + \sqrt{1 - n^2 + n^2(\mathbf{p}^i\mathbf{N})^2}\mathbf{N} - n(\mathbf{p}^i\mathbf{N})\mathbf{N}, \tag{19}$$

where $\mathbf{N}$ is normal to the hyperbolic surface, $\mathbf{p}^i$ is the wave vector of the incident wave and n is the refractive index of the lens material which is normally a constant and is greater than unity for natural dielectric lens. The wave vector of the incident waves, for both LCP and RCP, is given as:

$$\mathbf{p}^i = \mathbf{i}_z \tag{20}$$

The general expression for the normal to hyperbolic lens is given as following:

$$\mathbf{N} = \sin\alpha\mathbf{i}_x + \cos\alpha\mathbf{i}_z \tag{21}$$

Now the wave vector of Equation (19) can be calculated using Equations (20) and (21) for LCP and RCP waves separately. The values of $\mathbf{q}_L$ (for LCP) and $\mathbf{q}_R$ (for RCP) waves are given as follows:

$$\mathbf{q}_L = n_1 K(\alpha)\sin(\alpha)\mathbf{i}_x + n_1(n + K(\alpha)\cos(\alpha))\mathbf{i}_z \tag{22a}$$

$$\mathbf{q}_R = n_2 K(\alpha)\sin(\alpha)\mathbf{i}_x + n_2(n + K(\alpha)\cos(\alpha))\mathbf{i}_z \tag{22b}$$

where,

$$K(\alpha) = \sqrt{1 - n^2\sin^2\alpha} - n\cos(\alpha) \tag{23}$$

It should be noted here that the apparent wave numbers of both waves, LCP and RCP, are making the difference. LCP and RCP waves make different impact on the focal region because of the difference in the phase velocities. Phase velocity is dependent on the wave number which in turn is dependent on the chirality parameter (β) of the chiral medium. Jacobian transformation is used to transform ray coordinates to the cartesian coordinates. The value of Jacobian $J(t) = \frac{D(t)}{D(0)}$ is given as

$$J(t) = 1 + \frac{t}{\Xi}\left(q_z\frac{\partial q_x}{\partial\xi} - q_x\frac{\partial q_z}{\partial\xi}\right), \quad \Xi = q_z - q_x\tan\alpha \tag{24}$$

where q_x and q_z have different values for LCP and RCP waves. The Jacobian for both the waves is calculated from Equation (24) using respective values of unknowns for LCP and RCP. Unknowns in the Jacobian of Equation (24) for LCP wave:

$$\frac{\partial q_x}{\partial\xi} = n_1 AB \tag{25}$$

and for RCP wave:

$$\frac{\partial q_x}{\partial\xi} = n_2 AB \tag{26}$$

A and B are given as follows:

$$A = \left(\frac{\cos\alpha(1-2n^2\sin^2\alpha)}{\sqrt{1-n^2\sin^2\alpha}} - n\cos 2\alpha\right) \tag{27a}$$

$$B = \frac{\partial}{\partial\xi}\alpha = \frac{ab\cos^2\alpha}{(\xi^2+b^2)^{\frac{3}{2}}} \tag{27b}$$

The phase of the field on a ray is equal to the product of the optical path length of the ray from some reference point and the wave number of the medium [1]. Hence the initial phases of both LCP and RCP waves are given as following:

$$S_{0L} = n_1(c-\zeta), \quad S_{0R} = n_2(c-\zeta) \tag{28}$$

where c is the hyperbolic lens parameter. Substituting Equations (24)–(27b) in Equation (1), the fields around the focal region of a hyperbolic focusing lens using GO are obtained:

$$E_L(r) = E_{0L}^t J_L^{-1/2} e^{-jk(S_{0L}+t)} \tag{29a}$$

$$E_R(r) = E_{0R}^t J_R^{-1/2} e^{-jk(S_{0R}+t)} \tag{29b}$$

GO fails around the focal region. So, Maslov's method is used to calculate the fields around the focal region. The fields for LCP and RCP waves are given as following:

$$E_L(r) = \sqrt{\frac{k}{j2\pi}} \int_{-T/2}^{T/2} E_{0L}^t(\xi)\left(J_L\frac{\partial q_{xL}}{\partial x}\right)^{-1/2} e^{-jk\phi_L} dq_{xL} \tag{30a}$$

$$E_R(r) = \sqrt{\frac{k}{j2\pi}} \int_{-T/2}^{T/2} E_{0R}^t(\xi)\left(J_R\frac{\partial q_{xR}}{\partial x}\right)^{-1/2} e^{-jk\phi_R} dq_{xR} \tag{30b}$$

4.1. Evaluation of Amplitude

The unknown terms in Equations (30a) and (30b) are given as following:

$$dq_{xL} = n_1 A d\alpha, \tag{31a}$$

$$dq_{xR} = n_2 A d\alpha, \tag{31b}$$

$$J_L\frac{\partial q_{xL}}{\partial x} = \frac{n_1 A B q_{zL}}{\Xi}, \tag{31c}$$

$$J_R\frac{\partial q_{xR}}{\partial x} = \frac{n_2 A B q_{zR}}{\Xi}, \tag{31d}$$

$$D = n_2\left(\frac{\cos\alpha(1-2n^2\sin^2\alpha)}{\sqrt{1-n^2\sin^2\alpha}} - n\cos 2\alpha\right) \tag{31e}$$

where the values of A and B are given in Equations (27a) and (27b). The magnitudes of transmitted fields, for LCP and RCP waves, are given as following:

$$E^t_{0L} = \frac{nn_1 \cos\alpha}{\cos\alpha + nn_1\sqrt{1 - n^2 \sin^2\alpha}} \tag{32a}$$

$$E^t_{0R} = \frac{nn_2 \cos\alpha}{\cos\alpha + nn_2\sqrt{1 - n^2 \sin^2\alpha}} \tag{32b}$$

4.2. Phase Function

The phase of LCP and RCP waves are given as:

$$\begin{aligned}\phi_L &= nn_1(c - 2\zeta) + n_1K(\alpha)\sin\alpha(x - \xi) \\ &\quad + z(nn_1 + n_1K(\alpha)\cos\alpha) - n_1\cos\alpha K(\alpha)\zeta \end{aligned} \tag{33a}$$

$$\begin{aligned}\phi_R &= nn_2(c - 2\zeta) + n_2K(\alpha)\sin\alpha(x - \xi) \\ &\quad + z(nn_2 + n_2K(\alpha)\cos\alpha) - n_2\cos\alpha K(\alpha)\zeta \end{aligned} \tag{33b}$$

The subtending angle T of lens is given by:

$$T = \tan^{-1}\left(\frac{d}{2c}\right) \tag{34}$$

5. RESULTS AND DISCUSSION

Results of Equations (32a) and (32b) are plotted and analyzed for different values of the chirality parameter (β). The values of different parameters of hyperbolic lens are: $ka = 14$, $kb = 18$ and $kd = 20$. Field intensity of LCP and RCP waves are given for $\beta = 0$, 0.1, 0.5 and for $\beta = 1.3$, 1.5, 1.7 in Figures 7–10.

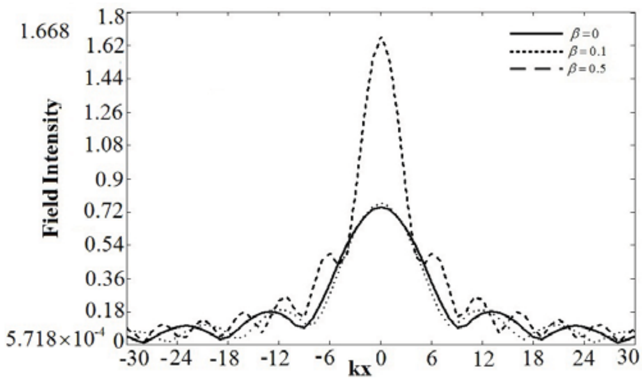

Figure 7. Field intensity of LCP wave for $\beta = 0$, 0.1, 0.5.

It is seen that the amplitude of LCP and RCP waves increases by increasing the chirality, in both the cases of weak ($\beta < 1$) and strong ($\beta > 1$) chiral medium. It is because of the chiral-dielectric-chiral interface. As we are considering only transmitted waves, while

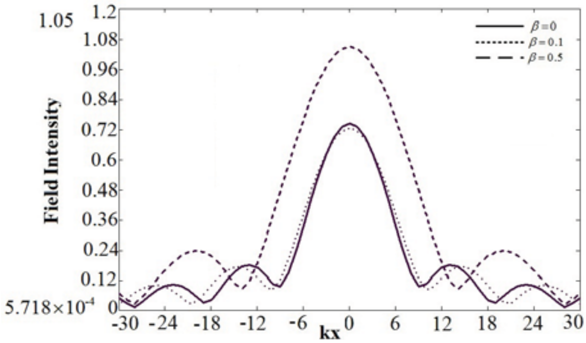

Figure 8. Field intensity of RCP wave for $\beta = 0$, 0.1, 0.5.

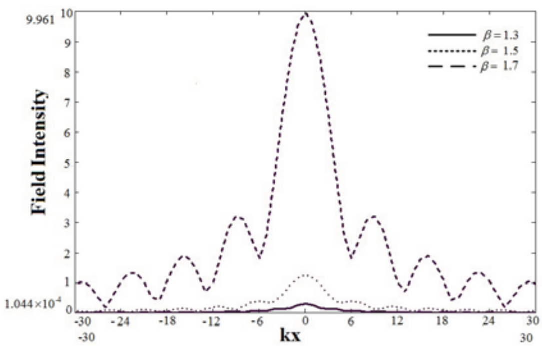

Figure 9. Field intensity of LCP wave for $\beta = 1.3$, 1.5, 1.7.

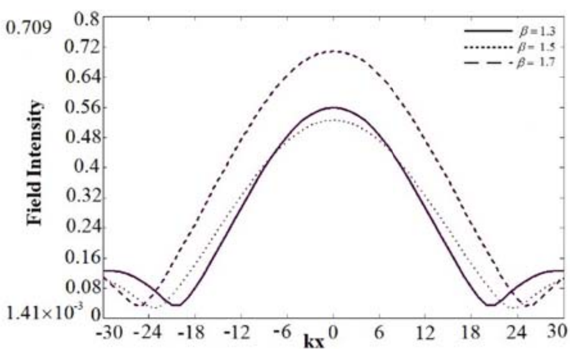

Figure 10. Field intensity of RCP wave for $\beta = 1.3$, 1.5, 1.7.

reflections are being ignored for the sake of simplification. Whereas the reflection is also dependent upon the chirality as given in [7, 8]. So, by increasing or decreasing the chirality parameter causes decrease or increase in reflections. In other words transmission is being increased or decreased.

6. CONCLUSION

Field patterns around the caustic region of a hyperbolic focusing lens placed in chiral medium are obtained. Effect of chiral-dielectric-chiral interface is studied and mathematical recipe of Maslov is used to find the fields in the caustic region of hyperbolic lens. The effect of chirality on the amplitude of the refracted fields around the focal region is shown and discussed. It is observed that the intensity of field in the focal region of hyperbolic focusing lens increases by increasing the chirality (β) of chiral medium.

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