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ON THE DESIGN OF CONCENTRIC RING ARRAY FOR ISOFLUX RADIATION IN MEO SATELLITES BASED ON PSO

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Abstract—The design of concentric ring arrays for isoflux radiation is presented in this paper. This design considers the reduction of the side lobe level and isoflux radiation requirements for Medium Earth Orbit (MEO) satellites. The optimization problem considers the spacing among rings and levels of amplitude excitations. The well-known method of Particle Swarm Optimization (PSO) is utilized for this design case. The obtained results could cause the satellite hardware to be reduced significantly even more than that presented previously in the literature.

1. INTRODUCTION

Nowadays, a variety of applications of the well-known antenna arrays for different communication systems exist because of their great radiating capabilities. The considerations for their radiating synthesis commonly are to reduce certain issues presented in the antenna systems. Essential issues could be found in the case of satellite applications. For instance, the electromagnetic fields transmitted from the onboard antenna to the earth are attenuated on the coverage area far from the nadir direction and/or near the horizon due to path loss variation as well as atmospheric conditions [1]. This factor is unwanted in the transmission for weather forecast, radio, television or global positioning services. These circumstances may be mitigated by the

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antenna system with an isoflux radiation [2]. Usually, large antennas as reflectors or horns are utilized for this purpose [3, 4]. Nevertheless, these antennas are not favorable in terms of the volume occupation for the complete antenna system. Furthermore, the heat dissipations as well as power consumptions are other important factors that must be addressed in the design of a antenna system for satellite applications. The implementation of non-uniform antenna arrays with an annular aperture could be a natural solution because of their capabilities for shape radiation in a compact size.

The state of the art research for antenna arrays includes the design for antenna arrays with uniform spacing [5, 6]. These recent designs provide proper results for different satellite systems as LEO (Low Earth Orbit) and GEO (Geostationary Earth Orbit) satellites by optimizing the antenna excitations. However, these uniform designs complicate the feeding network. Consequently, it implies higher cost of hardware due to the great number of excitations to illuminate each cell in the coverage area. As a rule, the hardware in the satellite has to be reduced to the degree possible. To do so, an antenna array with non-uniform distribution of the elements could be considered because a non-uniform antenna array implies less cost and volume occupation. For the case of satellite systems, a design for an aperiodic antenna array considering a deterministic approach to optimize the inter-element spacing with no side lobe level reduction for LEO satellite systems is presented in [7]. The synthesis for antenna arrays with unequal distribution of antenna elements is increasingly suitable for different satellite systems. Recently, relevant contributions have been proposed in the literature. Those works use some deterministic approaches for synthesizing uniform and sparse planar antenna arrays in order to obtain a desirable isoflux radiation in MEO (Medium Earth Orbit) and GEO satellites [8, 9]. Thus, the aurhors' objective has been focused on synthesizing antenna arrays to satisfy isoflux radiation in a compact size with fewer device excitations in order to improve the performance of the antenna system. In this case, we select the well-known geometry of concentric ring arrays for the following reasons:

- This geometry has important characteristics such as a symmetric distribution of the antenna elements in the four quadrants [11, 12]. Hence, the antenna elements are able to radiate or receive electromagnetic fields with the same amount of energy as well as the same phase value for different directions. It is very important to obtain a symmetric isoflux radiation in both azimuth and elevation planes in order to globally illuminate the earth surface. In fact, proper results have been obtained for the case of GEO satellite systems using a radial geometry [10]. In this case, the results revealed a better performance of the isoflux radiation than the results presented in [8, 9].

- As mentioned previously, the behavior of a sparse planar antenna array with rectangular geometry has been studied in order to provide global isoflux radiation in [9]. However, this geometry provides a considerable deviation of isoflux radiation in a wide FOV angle range.

Therefore, this study focuses on the radial geometry of an antenna array, i.e., the design of a concentric ring array is focused on reduction of the hardware of MEO satellite system. Specially, in this paper, the behavior of the radiation pattern is investigated by means of optimization of the spacing among rings and the levels of amplitude excitations. The spacing among rings is selected as an optimization variable to obtain an unequal distribution of the radius of the rings. Therefore, the main contribution of this research is a global optimization using an evolutionary algorithm for synthesis of a concentric ring array in MEO satellite systems with fewer excitations devices to accomplish a desirable shape pattern with side lobe level reduction. It is important to state that there are several works in the literature for synthesizing concentric rings arrays. However, they are oriented to different objectives of study [13, 14]. These works do not consider any satellite application for the synthesis of the antenna arrays. In this case, these works consider different approaches to synthesize concentric ring arrays for a pencil beam radiation and/or flat radiation, not an isoflux radiation and to reduce SLL, without any consideration of the reduction of amplitude levels because it is important to satisfy the requirement in MEO satellites. This synthesis is performed by using the well-known Particle Swarm Optimization (PSO) [15]. This particular optimization of the concentric ring antenna arrays for the requirements in MEO satellite systems has not been considered yet in the literature.

2. CONCENTRIC RING ANTENNA ARRAY MODEL

Now, let us consider a radial geometry with a concentric ring array of N_T elements spaced on plane X-Y given by:

$$N_T = 1 + \sum_{p=1}^{N_r} N_p \tag{1}$$

The array factor for this concentric ring antenna array shown in

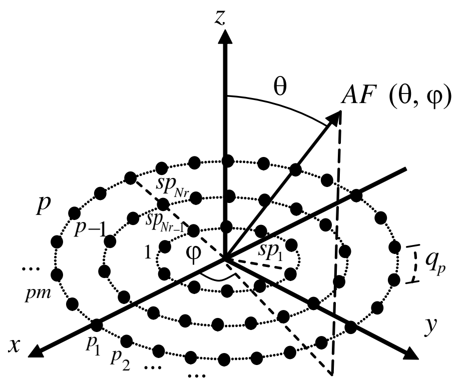

Figure 1. Concentric ring antenna array.

Figure 1 is now given by [11]:

$$AF\left(\theta, \varphi, SP, WE\right) = 1 + \sum_{p=1}^{N_r} \sum_{m=1}^{N_p} w_{pm} e^{jkr_p[u\cos\varphi_m + v\sin\varphi_m]} \qquad (2)$$

where $u = \sin\theta\cos\varphi$, $v = \sin\theta\sin\varphi$; N_r represents the number of the rings; N_p represents the number of elements on the ring p; $\varphi_m = 2\pi(m-1)/N_p$ represents the angular position of element m on ring p; r_p is the radius of ring p. The radius of each ring defines the spacing among the rings as follows: $sp_1 = r_1$, $sp_2 = (r_2 - r_1)$, $sp_3 = (r_3 - r_2), \ldots, sp_{Nr} = (r_{Nr} - r_{Nr-1})$ which are arranged in a set of real numbers $SP = \{sp_1, sp_2, \ldots, sp_{Nr}\}$. In addition, w_{pm} represents the amplitude excitation of element m of ring p defined in a set of real numbers $WE{=}\{w_{11}, w_{12}, \ldots, w_{1N1}, \ldots, w_{21}, w_{22}, \ldots, w_{2N2}, \ldots, w_{NrNp}\}$. This model regards the center of each ring as the phase reference in the array factor.

In this design, the optimization is constrained by the minimum spacing among rings of $sp_{\min} \geq 0.5\lambda$ and the spacing among the antenna elements in the ring p as $q_p = 2\pi r_p/N_p$. In the case of $sp_1 = sp_2 =, \ldots, = sp_{Nr} = sp_{\max}$, the maximum aperture might become $A_{\max} = N_r sp_{\max}$ where $sp_{\max}$ is the boundary in the search space of the elements in the vector *SP*. Hence, the spacing among the rings is established to be in the range of $0.5\lambda \leq sp_p \leq 1\lambda$. The levels of amplitude excitations are arranged in a set of NA real numbers $LE = \{le_1, le_2, \ldots, le_g\}$ where le_g is the excitation value for the elements in groups of rings of the array, i.e., excitation value forming a sub-array.

3. PROBLEM STATEMENT

The shape of the earth causes path loss variation as well as undesirable atmospheric conditions for satellite communications. This is an unwanted satellite communication factor in the transmission for television, weather forecast or global positioning services etc. Commonly, uniform illumination on the earth surface is required to compensate the attenuations of the electromagnetic fields in the coverage area far from the nadir direction. For this purpose, MEO satellite antenna systems are usually based on isoflux radiation. These systems provide radiation without variation in the strength power density to any point of the illuminated earth surface [2] as long as the satellites are spinning around the earth. For this requirement, it is necessary to establish a pattern shape for the synthesis of the radiation. In this research, an accurate isoflux prescribed pattern is proposed. Firstly, consider an illumination in the equatorial pole as illustrated in Figure 2. Since this framework assumes an elliptical symmetric shape for the earth, we propose an accurate prescribed pattern that can be calculated as a function of $R_s\left(\theta\right)$ in the coverage area for the elevation plane as follows (see Appendix A):

$$R_s^2\left(\theta\right)\left(\frac{\sin^2\theta}{b^2}+\frac{\cos^2\theta}{a^2}\right)+R_s(\theta)\left(\frac{-2\left(h+a\right)\cos\theta}{a^2}\right)+\left(\frac{(h+a)^2}{a^2}-1\right)=0 \tag{3}$$

The function $R_s(\theta)$ indicates the relative distance of the satellite to any point of the illuminated earth surface in any cut of the azimuth plane. Assuming any sweep in the azimuth plane, the function $R_s(\theta)$ should be identical according to the elliptical symmetric shape of the earth. h represents the height of the satellite; R_e is the distance of the earth center to any point of the illuminated earth surface; a is the equatorial radius of the earth; b is the polar radius of the earth; y' and x' are the variables of the canonical implicit equation of an ellipse. We consider an elliptical shape of the earth in order to deeply study the behavior of a concentric ring array for an isoflux radiation in order to mimic the real shape of the earth.

Now, the fitness function of this design problem is formulated as follows:

$$of=\left|AF\left(\theta_r,\phi\right)-R_s\left(\theta_r\right)\right|^2+\left|AF\left(\theta_{SLL},\phi_{SLL}\right)/\max\left(AF\left(\theta,\phi\right)\right)\right| \tag{4}$$

where θ_r is the range of the elevation plane in the coverage area above the earth for each cut in the azimuth plane of the array factor, and $(\theta_{SLL},\varphi_{SLL})$ is the angle where the maximum side lobe level is attained. The optimization is to minimize two aspects: firstly, the sum of the square error for each angle in the FOV zone between the real-elliptical shape of the earth and the array factor, secondly, the maximum side lobe level. Furthermore, as observed in this formulation, it is considered to achieve a constant power density in the illuminated earth surface, and the antenna directivity must increase as a function of the angle θ away from the nadir direction. In this case, the problem design of this research is to optimize the spacing among rings and semi-uniform excitations for a concentric ring array considering the zone of coverage. This optimization assumes the conditions of the prescribed pattern $R_s(\theta)$ and the reduction of the side lobe level. Depending on the performance requirements, the concentric ring array could cause the satellite hardware to be reduced significantly due to the use of fewer excitation devices. This study uses PSO to find out the optimal solution. We selected PSO for its effectiveness in antenna array design. Next, the main characteristics and the procedure for this evolutionary algorithm are described in the next subsections.

4. PARTICLE SWARM OPTIMIZATION

In this work, PSO is utilized to synthesize the concentric ring antenna array to achieve the desirable isoflux shape pattern. In this case, each particle is represented by optimization variables of spacing among rings and levels of amplitudes. Then, the particles within the swarm move influenced by its current position, its memory and the cooperation or social knowledge of the swarm, using only one operator, the so-called velocity operator. Let us suppose a swarm of Z particles, in which each particle $X_Z = (x_{z1}, \ldots, x_{zE})$ representing a potential solution is defined as a point in a E-dimensional space. The limits of the parameters x_{ze} to be optimized define the search space in E-dimensions. Iteratively, each particle z within the swarm flies over the solution space to a new position X_Z with a velocity $V_Z = (v_{z1}, \ldots, v_{zE})$,

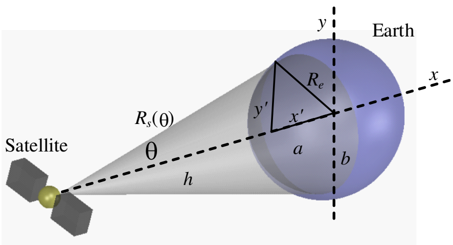

Figure 2. Electromagnetic Illumination in the equatorial pole of the earth.

both updated along each dimension e, by the following:

$$v_{ze} = iw \cdot v_{ze} + c_1 r_1 (pbest_{z,e} - x_{ze}) + c_2 r_2 (gbest_e - x_{ze}), \ v_z \leq v_{e,\max} \forall e \quad (5)$$

$$x_{ze} = x_{ze} + v_{ze} \Delta t \quad (6)$$

where iw is known as the inertial weight; c_1 and c_2 are the acceleration constants and determine how much the particle is influenced by its best location (usually referred as memory, nostalgia or self-knowledge) and by the best position ever found by the swarm (often called shared information, cooperation or social knowledge), respectively. Moreover, r_1 and r_2 represent two separate calls to a random number function $U[0, 1]$. $v_{e,\max}$ is the maximum allowed velocity for each particle used as a constraint to control the exploration ability of the swarm and usually set to the dynamic range of each dimension [16], and Δt is a time-step usually chosen to be one. In PSO, the population size, inertial weight and acceleration constants summarize the parameters to be selected and tuned. The simulation results of utilizing this evolutionary procedure for the optimization of the concentric ring array are discussed in the next section.

5. SIMULATION RESULTS

The synthesis of a concentric ring array for an isoflux radiation was implemented by the method of PSO. Pursuant to the *WGS84* (World Geodetic System 84) coordinate system, the dimensions of the earth are: the equatorial pole $b = 6356752.314$ meters and the polar pole $a = 6378137$ meters. In the prescribed pattern for MEO satellites with an altitude of 20000 km, an angle of elevation of $\theta_0 \approx 14^\circ$ is enough to illuminate the earth with an attenuation of -2.1 dB in the nadir direction $\theta = 0^\circ$. In order to compare the results in terms of number of elements N_T, maximum aperture $A_{\max}$ and isoflux shape with respect to the sparse planar array design presented previously in the literature [9], we choose the following specifications: $N_T = 37$ elements distributed in $N_r = 3$ rings. Thus, the number of elements for each ring is $N_1 = 6$, $N_2 = 12$ and $N_3 = 18$. This distribution also considers a central element placed on the origin. In order to decrease to the degree possible, the number of antenna excitations, the design for 3 levels of amplitude excitations is also proposed where level l_1 is for the central element and rings $p = 1$, l_2 for rings $p = 2$ and l_3 for ring $p = 3$. The use of two amplitudes in a concentric ring array for MEO satellite is not favorable in terms of SLL and isoflux shape. The parameters of PSO are set as follows: the maximum number of iterations $i_{\max} = 500$; number of particles $p_{size} = 200$; inertial weight w varies downward in the range of [0.95–0.4] through the optimization process; the acceleration constants are $c_1 = c_2 = 2$. We select these parameters for their proper performance that has been studied in previous works [17, 18]. Under these specifications, the behavior of the isoflux radiation was rigorously studied. In this case, we implemented PSO twice as: a first case with the isoflux shape for the range of the field of view (FOV) of $-14° \leq \theta \leq 14°$ and a second case with the isoflux shape with a deviation in the FOV of $\pm 2°$ for this altitude.

The optimization cases were run in MATLAB using a computer with an Intel CPU-i7 860 and 8Gb of memory RAM. The execution time was *3.65 hours* for each optimization case. This rigorous study obtains an optimal concentric ring array for MEO satellites. In this case, it is important to mention that the designs of concentric rings array shown in [10] are for GEO satellites and utilize only two amplitudes in the best case in terms of reducing the antenna hardware. This new study contributes to obtaining the best number of amplitude excitations for MEO satellites which is three amplitudes. Figure 3 shows the obtained isoflux radiation for both optimization cases in the elevation plane and in both elevation and azimuth planes. It could be appreciated that the PSO obtained an isoflux radiation with a reduction of the SLL for both cases. Note that there exist a trade-off between the range of FOV and SLL. Case 1 has a better accuracy in the range of FOV with a value of $SLL \leq -19\,\text{dB}$ as long as Case 2 has a tiny deviation of $\pm 2°$ in the range of FOV with a bigger reduction of $SLL \leq -31\,\text{dB}$. If these results are compared with the design presented in [9], it could be mentioned that the results presented in [9] require many excitation devices (amplitudes and phases) and a bigger aperture $A_{\max} = 8\lambda$ in both axes, i.e., physical dimensions to achieve an isoflux radiation for the same application. From the point of view of the need to reduce the size of the antenna hardware in the satellites, the obtained results for concentric ring arrays are suitable solutions for this particular application. Figures 3(a) and (b) show that the suppression of $-2.1\,\text{dB}$ in the nadir direction of $\theta = 0°$ is reached for both cases. The discrepancy between the prescribed pattern and obtained isoflux shape in the remaining range FOV zone of $14° \leq \theta \leq 14°$ is about $|1\,\text{dB}|$ for Case 1 and $|0.5\,\text{dB}|$ for Case 2. Observe that this discrepancy is due to the prescribed pattern taking the elliptical shape of the earth. In fact, assuming a hypothetical-spherical shape of the earth, the discrepancy is practically zero.

In Figure 4, the obtained distribution of the concentric ring array is presented. For both cases, the rings were distributed increasingly to the maximum obtained aperture. Observe that the aperture for Case 2 is slightly greater than that for Case 1 because of the deviation of $\pm 2°$ in the range of FOV. In this case, there exists a trade-off between

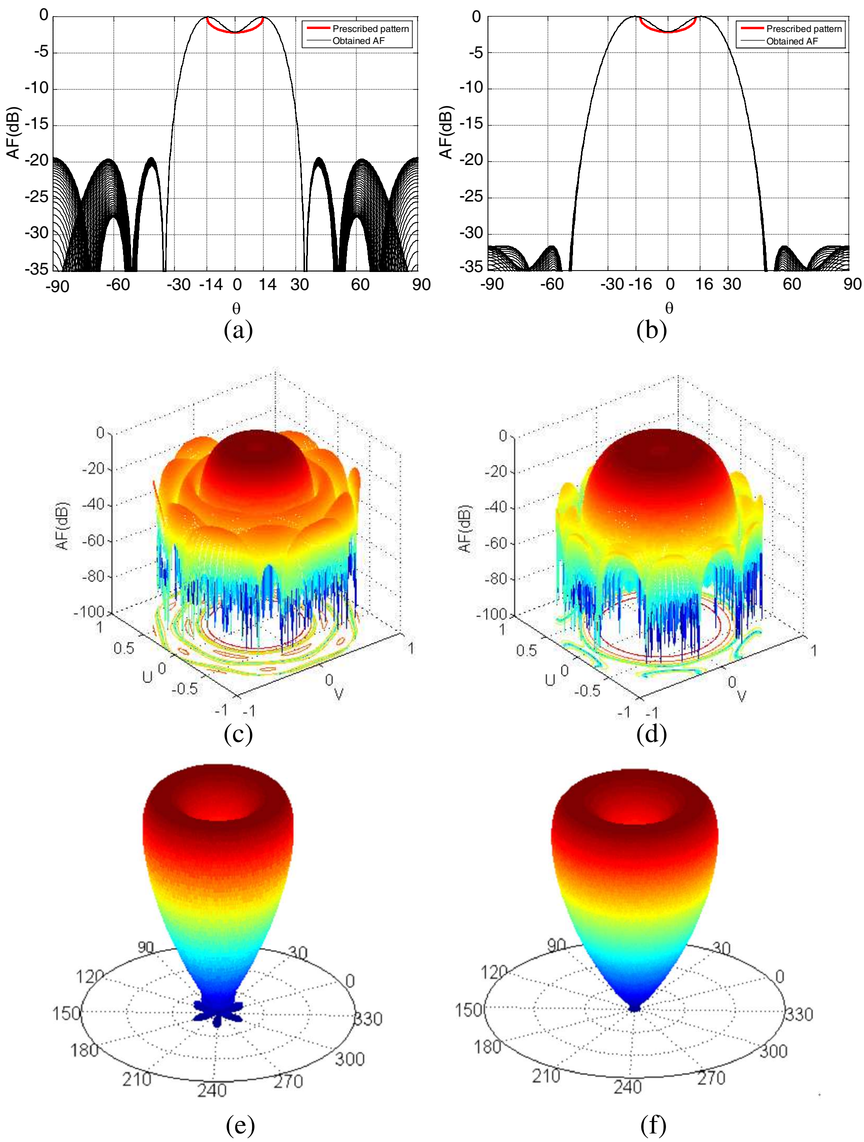

Figure 3. Isoflux radiation of the obtained concentric ring array. Elevation plane: (a) Case 1 and (b) Case 2. Elevation and azimuth planes in dB: (c) Case 1 and (d) Case 2. And elevation and azimuth planes in the power density of the $AF(\theta, \varphi)$ in spherical coordinates: (e) Case 1 and (f) Case 2.

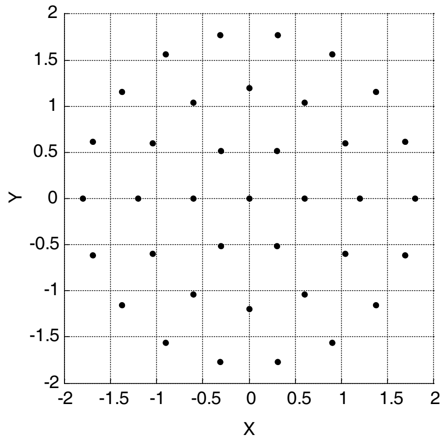

Figure 4. Geometry of the obtained concentric ring antenna array for: (a) Case 1 and (b) Case 2.

the range of FOV and the $A_{\max}$. In this case, for the altitude of MEO satellites, the angle range of FOV and the required suppression are satisfied by any aperture considering the range $4\lambda \leq A\max \leq 5\lambda$ by using a concentric ring array. Thus, the suitable number of rings or number of elements, which corresponds to that range of aperture is 37 antenna elements distributed in three rings because the appropriate sequence of the number of elements for each ring is as follows: 6, 12, 18, ..., as described in one of our previous studies [17], in order to constrict a minimum spacing of 0.5λ among elements. The optimization cases have the maximum aperture of $A_{\max} = 3.6\lambda$ for Case 1 and $A_{\max} = 3.1\lambda$ for Case 2. Both apertures determine a smaller antenna array than the design in [9]. Furthermore, it can be mentioned that it requires a bigger amplitude excitation in the center of the array to achieve the desirable isoflux shaped pattern. The numerical values of side lobe level for the optimized array factors shown in Figure 3 are presented in Table 1. This table shows the excitations required for the optimized designs depicted in Figure 4 for providing the isoflux radiation. Note that the optimizations provide a better *SLL* reduction with a reduction of excitation devices, which could considerably simplify the feeding network for an isoflux radiation, which is desired for satellite applications.

Table 1. Numerical values of SLL and excitations values for the optimization cases.

Design	**Case**	$\mathbf{N}_T$	**SLL (dB)**	**Levels of amplitudes (L)**	**Spacing rings (S)**
Concentric ring	1	37	−19.3	0.6000 0.6009 0.6000	−3.9513 −0.8048 0.8282
	2	37	−31.6	0.5000 0.5003 0.5513	−3.4996 −0.2726 0.5274
Sparse				**Levels of amplitudes**	**Levels of phases**
	Ref. [9]	100	Not specified	Not specified	Not specified

6. CONCLUSION

This investigation reports a concentric ring array for MEO satellite system with an isoflux radiation for a reduction of the side lobe level by using particle swarm optimization. The optimized designs perform a considerable reduction of the side lobe level of $SLL < -19\,\mathrm{dB}$ and $SLL < -31\,\mathrm{dB}$. Under the assumption of the requirement of reducing the volume occupation of hardware in the antenna system and isoflux radiation, the concentric ring array could provide an acceptable solution for MEO satellites. Although only simulation results are presented in the paper, the obtained designs show a better performance in terms of isoflux shape, side lobe level reduction, aperture and number of excitation devices than the designs presented previously in the literature under the same conditions and MEO satellite requirements. In fact, the possibility to build this type of feeding network is realistic because the number of amplitudes is limited to the maximum number of the rings, which permits to define the nonuniformly spacing among the rings. Undoubtedly, this study is fundamental to building the antenna array in future works. Moreover, other future works could be focused on the applications of the evolutionary algorithm for the simplification of the feeding network.

APPENDIX A.

From Figure 1, trigonometric math functions are defined as:

$$\begin{aligned} \sin\theta &= \frac{y'}{R_s(\theta)} \\ R_s(\theta) &= \frac{y'}{\sin\theta} \end{aligned} \tag{A1}$$

$$\cos\theta = \frac{(h+a) - x'}{R_s(\theta)} \qquad \text{(A2)}$$
$$x' = (h-a)\,R_s(\theta)\cos\theta$$

Besides, consider the canonical implicit equation of an ellipse as follows:

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1, \quad y' = b\sqrt{1 - \frac{x'^2}{a^2}} \qquad \text{(A3)}$$

Then, the math calculation for the prescribed radiation pattern $R_s(\theta)$ is obtained by substituting Equations (A2) and (A3) in (A1).

$$R_s^2(\theta)\left(\frac{\sin^2\theta}{b^2} + \frac{\cos^2\theta}{a^2}\right) + R_s(\theta)\left(\frac{-2\,(h+a)\cos\theta}{a^2}\right) + \left(\frac{(h+a)^2}{a^2} - 1\right) = 0 \quad \text{(A4)}$$

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