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PROPAGATION OF ELECTROMAGNETIC FIELDS IN NEAR AND FAR ZONES: ACTUALIZED APPROACH WITH NON-ZERO TRACE ELECTRO-MAGNETIC ENERGY-MOMENTUM TENSOR

A. L. Kholmetskii[1], O. V. Missevitch[2], R. Smirnov-Rueda[3,*], and T. Yarman[4,5]

[1]Department of Physics, Belarus State University, 4 Nezavisimosti Avenue, Minsk 220030, Belarus

[2]Institute of Nuclear Problems, 11 Bobruyskaya Street, Minsk 220030, Belarus

[3]Applied Mathematics Department, Faculty of Mathematics, Complutense University, Madrid 28040, Spain

[4]Department of Engineering, Okan University, Akfirat, Istanbul, Turkey

[5]Savronik, Organize Sanayii Bölgesi, Eskisehir, Turkey

Abstract—The present work is motivated by our recent experimental results [2–4] that indicate on anomalously small retardation of bound (or velocity-dependent) electromagnetic (EM) fields in the near zone of an emitter, whereas in the far zone the retardation tends to the standard value determined by the velocity of light c. Such anomaly is specific only for bound field component, while EM radiation has the constant propagation velocity c in the entire space. One possible explanation of these experimental results can be linked to our earlier finding [6, 8] that conventional EM energy-momentum (EMEM) tensor describes bound and radiative EM fields only in spatial regions free of charges and currents. In this work we show that an additional term has to be included into the standard EMEM tensor in order to make viable the description of the whole system of *"charges plus fields"*. Such approach to the EMEM tensor actually admits anomalously small retardation of bound EM fields in regions very close to a field source, providing the standard propagation in the far zone. Some special implications are also discussed.

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* Corresponding author: Smirnov-Rueda (smirnov@mat.ucm.es).

1. INTRODUCTION

According to the modern standpoint on EM field structure there are two components with different dependence on a distance form the source: velocity-dependent (bound) component falls off usually as inverse square of a distance R whereas acceleration-dependent (radiation) fields fall off as R^{-1}. As a consequence, bound fields are dominant within the *near zone* ($R < \frac{\lambda}{2\pi}$, where λ is EM radiation wavelength) and EM radiation components prevail in the *far zone* ($R > \frac{\lambda}{2\pi}$). However, it should be specially noted that the energy transmission velocity of EM fields turns out to be scarcely studied on empirical level within the regions very close to the source. Strangely enough this fundamental gap seems to have troubled nearly no one up to present days even though just the extrapolation of classical EM fields to very small distances leads to well-known infinities within the framework of classical electrodynamics.

In fact, it is generally recognized that the origin of these difficulties can be traced to that part of EM field which is not related to the radiation but remains bound to the source (velocity-dependent EM fields). The latter part contains self-interaction and eventually leads to the infinities of self-energy that can be removed by re-normalization procedures. Hence it appears that the first logical step in treating the above-mentioned problems is a decomposition of EM field in two parts: velocity and acceleration dependent components, respectively. As a result, the energy-momentum tensor can be identified with respect to both parts with no ambiguity [1] and it provides the study of EM energy-momentum relation as close to the source as desired up to quantum limits. It is obvious that this relationship should be compatible with observable properties of bound velocity-dependent EM fields (dominant in the near zone), including its propagation velocity, which is commonly assumed to be equal to c, but without a special rigorous experimental confirmation.

Only very recently we proposed an experimental approach for the measurement of propagation velocity of bound EM field component [2–4], as a response to the above-mentioned lack of empirical information on EM field propagation properties very close to the source. Several procedures implemented in [2–4] provided unambiguous identification of retarded positions of bound EM fields on the time scale as functions of a distance R from the emitting source. Contrarily to CED predictions, *a negligible retardation of bound EM fields was observed in the near zone* whereas at larger distances (the far zone) bound field had already standard propagation velocity c equal to that of radiation fields.

Thus, in contrast to the well-known CED prediction, bound EM field nearly have no retardation in the near zone. To be more specific, the latest experimental results [4] indicated that an area of anomalously small retardation of bound EM field can be scaled exactly as the near zone size in an emitting antenna for different current frequencies. In Fig. 3 of Ref. [4] the retardation of bound EM field is plotted versus the distance between emitting and receiving antennas for three different current frequencies corresponding to three different near zone sizes R_n. It shows that the region of anomalously small retardation is about $0.6R_n$, whereas at distances larger than $\approx 1.5R_n$ the retardation of bound EM field becomes standard.

Being at odd with the conventional CED viewpoint on EM field propagation (see, for instance [5–8]), these experimental results need theoretical interpretation. In the present work we propose a possible explanation based on the analysis of EM energy-momentum (EMEM) tensor and corresponding conservation laws. We start with this analysis since the standard expression for symmetric EMEM tensor (see e.g., [5]) as well as corresponding conservation laws for EM field seem to be in no way compatible with the dependence shown in Fig. 3 of Ref. [4]. In view of this incompatibility, in the next Section 2 we first come across the inconsistency present in the conventional derivation of the symmetric EMEM tensor, which, in fact, restricts its application only to spatial regions with no charges and currents. Removing this limitation leads us to the generalized form of EMEM tensor entirely applicable to the general case of the whole system of *"particles plus their fields"*. In Section 3 we will show that for dominant bound EM field (i.e., in regions very close to a field source) the generalized EM energy-momentum tensor admits additional gauge modification, providing the elimination of the divergent terms present in the standard CED framework (infinite self-energy, infinite self-force). As we will see, it also allows the absence of retardation for bound EM field (i.e., the zero slope of retardation curves in Fig. 3 of Ref. [4]). In Section 4 we will consider the general case: EM field as a superposition of bound and radiative contributions. It will be shown that in the far zone, where radiative EM fields prevail, the generalized EM energy-momentum tensor yields the standard continuity equation (Poynting theorem), hence implying equal propagation (retardation) rates for both bound and radiative EM field components. It is also in line with the experimental data of [4], since the retardation curves of bound EM fields asymptotically approaches the standard retardation slope at large distances (far zone). Given interpretations, as well as some specific views on CED structure based on recent experimental data [2–4] will be discussed in Section 4.

2. GENERALIZED ELECTROMAGNETIC ENERGY-MOMENTUM TENSOR FOR A SYSTEM OF CHARGED PARTICLES

In this section we show the EMEM tensor derivation, removing limitations present in the conventional approach. It already had been discussed in [9, 10] and for the readers convenience we will reproduce here only main results.

We start our analysis with the canonical EM energy momentum (EMEM) tensor written in its standard form [5]:

$$T_{\mathrm{EM}}^{\mu\nu} = -\frac{1}{4\pi}(\partial^\mu A^\gamma)F_\gamma^\nu + \frac{1}{16\pi}g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}, \tag{1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the tensor of EM field; A^μ is the four-potential; $g^{\mu\nu}$ is the metric tensor; μ, $\nu = 0$, 1, 2, 3.

In order to transform Eq. (1) into a symmetric form (which is only meaningful for spinless classical charges), the following gauge transformation has to be applied for tensor entities:

$$T^{\mu\nu} \to T^{\mu\nu} + \partial_\gamma \Psi^{\mu\nu\gamma}, \tag{2}$$

where $\Psi^{\mu\nu\gamma}$ satisfies the requirement $\Psi^{\mu\nu\gamma} = -\Psi^{\mu\gamma\nu}$.

If we choose $\Psi_\gamma^{\mu\nu}$ as

$$\Psi_\gamma^{\mu\nu} = \frac{1}{4\pi}A^\mu F_\gamma^\nu, \tag{3}$$

then we can explicitly calculate the derivative

$$\partial^\gamma \Psi_\gamma^{\mu\nu} = \frac{1}{4\pi}\left(\partial^\gamma A^\mu\right)F_\gamma^\nu + \frac{1}{4\pi}A^\mu\left(\partial^\gamma F_\gamma^\nu\right). \tag{4}$$

Further, using *homogeneous* Maxwell's equation

$$\partial_\gamma F^{\nu\gamma} = 0 \tag{5}$$

and combining Eqs. (1), (2), (4) and (5), we arrive at symmetric representation of the EMEM tensor:

$$T_{\mathrm{EM}}^{\mu\nu} = \frac{1}{4\pi}\left(-F^{\mu\gamma}F_\gamma^\nu + \frac{1}{4}g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}\right). \tag{6}$$

Now we point out a restriction related to the standard derivation of EMEM tensor (6): it is based on the homogeneous Maxwell Eq. (5), which, in general, is not valid for systems that include charged particles. Hence the extension of Eq. (6) to the general case of fields plus particles is not fully justified in the framework of the standard CED. To extend the description to systems with charges, one ought to require the compatibility of EMEM tensor with *inhomogeneous* Maxwell's equation

$$\partial_\gamma F^{\nu\gamma} = -\frac{4\pi}{c} j^\nu. \tag{7}$$

As a result, in order to get a symmetric EMEM tensor for a system with charges, the corresponding gauge transformation (3) should be based on Eq. (7) instead of Eq. (5), providing that [9]

$$T^{\mu\nu}_{\mathrm{EM}} = \frac{1}{4\pi}\left(-F^{\mu\gamma}F^\nu_\gamma + \frac{1}{4}g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}\right) - \frac{1}{c}A^\mu j^\nu. \tag{8}$$

This modified form of EMEM tensor (8) differs from the conventional one (6) by the additional term $A^\mu j^\nu$, which, in general, is not symmetric, when the four-potential A^μ and four-current j^ν are taken from the different source particles. Thus, in order to achieve the symmetric representation of the tensor (8), we further assume that in the product $A^\mu j^\nu$ both A^μ and j^ν are always issue from the same charge, supplying this product by the subscript "*pr*" (proper). In this case the four-potential A^μ is proportional to j^μ, and the symmetry of the term $(A^\mu j^\nu)_{pr}$ is straightforward.

The Eq. (8) can be approached alternatively starting from the definition of the energy-momentum tensor given by Hilbert (see Appendix A).

For further analysis, it is convenient to separate the tensor (8) in two complementary parts:

$$T^{\mu\nu}_{\mathrm{EM}} = T^{\mu\nu}_{(\mathrm{EM})in} + T^{\mu\nu}_{\mathrm{EEM}}, \tag{9}$$

where we will refer

$$T^{\mu\nu}_{(\mathrm{EM})\mathrm{in}} = \frac{1}{4\pi}\left(-F^{\mu\gamma}F^\nu_\gamma + \frac{1}{4}g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}\right)_{\mathrm{in}} \tag{10}$$

as *interaction part* of EMEM tensor, where the first and second factors in each of the products $-F^{\mu\nu}F^\nu_\gamma$, $F_{\gamma\alpha}F^{\gamma\alpha}$ correspond to different particles, while in the tensor

$$T^{\mu\nu}_{\mathrm{EEM}} = \frac{1}{4\pi}\left(-F^{\mu\gamma}F^\nu_\gamma + \frac{1}{4}g^{\mu\nu}F_{\gamma\alpha}F^{\gamma\alpha}\right)_{pr} - \frac{1}{c}\left(A^\mu j^\nu\right)_{pr} \tag{11}$$

the products $-F^{\mu\nu}F^\nu_\gamma$, $F_{\gamma\alpha}F^{\gamma\alpha}$ are taken for the same charged particle. In what follows, the tensor (11) will be regarded as *Eigen Electromagnetic energy-momentum* (EEM) of a system of charged particles.

The given expression for the symmetric EMEM tensor (8) and its separation into two complementary parts (10), (11) has a general validity and is equally applicable to bound EM field, radiative EM field and their combination. At the same time, this separation turns out to be especially convenient for the case when only bound EM fields are present, i.e., when EM radiation is absent. In the next Section we will consider this particular case implying accelerations of all charged particles as negligible and will show that the corresponding EMEM tensor (8) can be subjected to an appropriate gauge transformation, which allows elimination of the divergent terms in its structure and logically brings into the absence of bound EM field retardation within the near zone. It provides a basis for further analysis of the dependence presented in Fig. 3 of Ref. [4].

3. VELOCITY-DEPENDENT (BOUND) EM FIELDS AND ELECTROMAGNETIC MASS OF CLASSICAL CHARGED PARTICLES

First of all, we remind that the total energy-momentum tensor $T^{\mu\nu}$ of any macroscopic system of charged particles has to be understood as a sum of two parts: (1) EMEM tensor $T^{\mu\nu}_{\text{EM}}$ and (2) mechanical energy-momentum tensors $T^{\mu\nu}_{\text{mech}}$:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{EM}} + T^{\mu\nu}_{\text{mech}}, \tag{12}$$

where

$$T^{\mu\nu}_{\text{mech}} = m\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}, \tag{13}$$

and m is a mechanical mass density.

Now we notice the important property of EMEM tensor (8): in contrast to the conventional EMEM tensor (6), its trace is not equal to zero due to the contribution $(A^\mu j^\mu)_{pr}$. Hence using (8) we get a possibility to describe the EM mass contribution into the total mass of charged particle which in not viable using the standard tensor (6).

The *EM mass tensor* can be introduced by analogy with the mechanical tensor (13) as

$$T^{\mu\nu}_{\text{EM mass}} = m_{\text{EM}}\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}, \tag{14}$$

here m_{EM} is EM mass density to which is added the *Poincaré stresses tensor*

$$T^{\mu\nu}_P = -m_P\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}, \tag{15}$$

where m_P is the mass density associated with the energy of "Poincaré stresses" necessary for the stability of the classical electron [10]. Thus the total (observable) mass density of charged particle becomes

$$m_t = m + m_{\text{EM}} - m_P \tag{16}$$

and as a consequence, the concept of EM mass acquires a special significance when a system of charged particles produces bound EM field alone. Since accelerations of all particles are negligible in this case (implying their velocities to be constant during a sufficiently large lapse of time), then the four-divergence of the total mass tensor $T^{\mu\nu}_{\text{mass}} = (m + m_{\text{EM}} - m_P)\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}$ should be vanishing. The same is also valid for the EM mass tensor

$$\partial_\mu T^{\mu\nu}_{\text{EM mass}} = \partial_\mu m_{\text{EM}} \frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau} = 0 \tag{17}$$

due to independence of EM and mechanical masses. Moreover, for non-radiating charges the four-divergence of Eigen EMEM tensor is also equal to zero:

$$\partial_\mu T^{\mu\nu}_{\text{EEM}} = 0. \tag{18}$$

We stress here that the fact of zero divergence of two different tensors $\partial_\mu T^{\mu\nu}_{\text{EEM}} = 0$ and $\partial_\mu T^{\mu\nu}_{\text{EM mass}} = 0$ implies the existence of some tensorial gauge transformation (2) (there is no need here to determine this gauge explicitly) which provides the mathematical relationship between both tensors $T^{\mu\nu}_{\text{EEM}}$ and $T^{\mu\nu}_{\text{EM mass}}$. Hence in the expression for the generalized EMEM tensor (9), $T^{\mu\nu}_{\text{EEM}}$ can be replaced by $T^{\mu\nu}_{\text{EM mass}}$:

$$T^{\mu\nu}_{\text{EM}} = m_{\text{EM}} \frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau} + T^{\mu\nu}_{(\text{EM})\text{in}}. \tag{19}$$

Without influencing its four-divergence. This procedure was named in [9] as *gauge renormalization.*

Thus the total energy-momentum tensor $T^{\mu\nu}$ can be written as follows:

$$T^{\mu\nu} = (m + m_{\text{EM}} - m_P)\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau} + T^{\mu\nu}_{(EM)\text{in}}. \tag{20}$$

where we highlight again that the suggested gauge renormalization which leads to Eq. (20) is rigorously applicable only to bound EM fields, i.e., when source particles move with constant velocities and the equalities (17), (18) take place. Moreover, in the absence of EM radiation one can define the total EM mass of a particle through straightforward integration of EM mass density over the entire space. Taking into account that any gauge transformation does not change the total energy-momentum of a system, one immediately obtains an explicit expression for a total mass difference $M_{\text{EM}} - M_P$ of an isolated classical charged particle at rest:

$$M_{\text{EM}} - M_P = \int\limits_V (m_{\text{EM}} - m_P)\, dV = \frac{1}{c^2}\int\limits_V \frac{E^2}{8\pi} dV - \frac{1}{c^2}\int\limits_V \rho\varphi dV, \tag{21}$$

where V denotes the entire space domain; E is electric field produced by a charged particle; φ is its scalar potential; ρ is particle's charge density.

The resulting Eqs. (20), (21) allow one to remove two divergent terms usually present in the conventional classical EM theory. In fact, charged particle self-forces due to its own bound EM fields do not already take place since the interaction part of the total energy-momentum tensor $T^{\mu\nu}$ (20) is formed only by $T^{\mu\nu}_{(\mathrm{EM})\mathrm{in}}$. On the other hand, Eq. (21) for the difference of masses $M_{\mathrm{EM}} - M_P$ can take a finite value even if the limit case $r \to 0$ is considered (r denotes the radius of a classical charged particle) [11].

It is of interest to compare internal structures of Eqs. (16) and (21). The first term in *rhs* of Eq. (21) represents EM mass of a charge with the density

$$m_{\mathrm{EM}} = \frac{E^2}{8\pi c^2}, \tag{22}$$

(or $\frac{1}{8\pi c^2}(E^2 + B^2)$ in the case of a moving charge, where $\mathbf{B}$ is its magnetic field), and the second term of *rhs* of Eq. (21) corresponds to the mass parameter

$$M_P = \frac{1}{c^2}\int\limits_V \rho\varphi dV, \tag{23}$$

which is associated with *Poincaré stresses*. Thus, the mass density due to *Poincaré stresses* is

$$m_p = \frac{\rho\varphi}{c^2} = \frac{q\varphi}{c^2}\delta(\mathbf{r} - \mathbf{r}_0), \tag{24}$$

where q is the particle charge. For further convenience, we will take the mechanical mass density of a point particle as

$$m = M\delta(\mathbf{r} - \mathbf{r}_0), \tag{25}$$

where M is mechanical mass; $\mathbf{r}_0$ and $\mathbf{r}$ are position vectors of a point-particle and a point of observation, respectively.

Thus, combining Eqs. (16), (22) and (24), (25) we get explicitly the total observable mass density m_t of a charged particle:

$$m_t = \left(M - \frac{\rho\varphi}{c^2}\right)\delta(\mathbf{r} - \mathbf{r}_0) + \frac{E^2 + B^2}{8\pi c^2}. \tag{26}$$

Notice that according to the mathematical definition $m_{\mathrm{EM}} = \frac{1}{8\pi c^2}(E^2 + B^2)$, EM mass density is always distributed over the entire space domain. Therefore, the circumstance that it is linearly added to the mass density $(M - \frac{\rho\varphi}{c^2})\delta(\mathbf{r} - \mathbf{r}_0)$ localized at a point $\mathbf{r}_0$ means that, generally speaking, $\frac{1}{8\pi c^2}(E^2+B^2)$ should be considered as some *rigid entity.* From the physical viewpoint it is possible only in the case, when the retardation of bound EM field is supposed to be *negligible* in the entire space. This property appears as soon as the divergent self-energy and self-interaction turn out to have been removed from the standard CED framework according to the above general procedure.

Coming back to the definition of a mechanical mass which implies contributions of non-electromagnetic nature, it is natural to consider the mass density $m_p \frac{q\varphi}{c^2}\delta(\mathbf{r}-\mathbf{r}_0)$ of Poincaré stresses as some part of the mechanical mass density of one individual charged particle. In this case, the mechanical contribution $T^{\mu\nu}_{\text{mech}}$ (13) into the total energy-momentum tensor $T^{\mu\nu}$ (12) can be extended as

$$T^{\mu\nu}_{\text{mech}} = (m - m_P)\,\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau}. \tag{27}$$

As a consequence, we arrive at the final expression for the EMEM tensor of a macroscopic system of N interacting charged particles, producing bound EM field alone:

$$T^{\mu\nu}_{\text{EM}} = m_{\text{EM}}\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau} + T^{\mu\nu}_{\text{(EM)in}} = \frac{E^2+B^2}{8\pi c^2}\frac{dx^\mu}{dt}\frac{dx^\nu}{d\tau} + T^{\mu\nu}_{\text{(EM)in}}, \tag{28}$$

where E and B are electric and magnetic fields produced by charged particles.

For uniformly moving charges, EMEM tensor (28) can be associated with a particular solution to Maxwell's equations given by Heaviside's formula for electric and magnetic fields [5]:

$$\mathbf{E} = \frac{q\left(1-\frac{v^2}{c^2}\right)\mathbf{r}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2} r^3}; \quad \mathbf{B} = \frac{1}{c}\left(\mathbf{v}\times\mathbf{E}\right) \tag{29}$$

where θ is polar angle; particle position vector $\mathbf{r}$ is a present time function (implicit time dependence).

The expression (29) for EM fields of one uniformly moving charge can be viewed as a particular case of Lienard-Wiechert solutions to Maxwell's equations that implies retardation of both bound (velocity-dependent) and radiation (acceleration-dependent) components being determined by the velocity of light (explicit time dependence). There are well-known arguments to accept an apparent rigidity of electric and magnetic fields (29) for uniformly moving charge as a mere mathematical coincidence (with no special physical content) that solution (29) can be expressed at present time coordinates (implicit time dependence). By contrast, within the present framework, one implies the absence of retardation of bound EM field in the entire space, otherwise, the definition of total mass of charged particle (26) would have no physical meaning.

Finalizing the analysis of the case of purely bound EM field, it is of interest to explore the equation of motion and continuity equation generated by the tensor (28). In this respect we refer to our recent work [10], where the same expression (28) for EMEM tensor had been derived (though based on a different arguments). It had been shown that the tensor (28) yields the standard equation motion of a charged particle, where, however, the term of self-interaction in the proper bound EM field is excluded. We also had shown that the tensor (28) yields three different continuity equations respectively for the *proper*, *interaction* and *total* EM energy density. For more details we refer the reader to Ref. [10]. Here we only mention the fact that the form of the continuity equation for proper EM energy coincides with the form of the continuity equation used in fluid mechanics and the rate of EM energy transfer turns out to be equal to the instant particle velocity. This result reenforce again the viewpoint that bound EM fields (in the absence of EM radiation) move rigidly with the particle with no observable retardation. The above-discussed results are meaningful for further qualitative analysis of the general case when arbitrarily moving charged particles produce both bound and radiative EM fields.

4. ELECTROMAGNETIC FIELDS IN NEAR AND FAR ZONES

Let us consider again an isolated system of charged particles with their accelerations no longer negligible. EM fields generated by particles have already both bound (velocity-dependent) and radiative (acceleration-dependent) components with different dependence on a distance r between a source charge and point of observation. We remind that radiative components usually scale as r^{-1}, while bound component fall off as r^{-2} or faster. Thus, regardless any particular relationship between velocities and accelerations of charges, there always exist a spatial region close to a charged particle, where bound EM fields are dominant within the near zone on which higher limit the intensities of bound EM field and EM radiation are equal to each other. At larger distance (intermediate zone) EM radiation prevails, though the contribution of bound EM field remains essential. Finally, at distances large enough (far zone), the relative intensity of bound EM field becomes very small and one deals with EM radiation alone. For charged particles under harmonic oscillations, and for wavelength of EM field λ much larger than the size of the system of charges, the boundary of near zone is determined by the relationship $r_n = \lambda/2\pi$, the intermediate zone can be defined as the region $\lambda/2\pi < r < \lambda$, while the far zone corresponds to the inequality $r > \lambda$.

Within the conventional CED framework different zones are distinguished only by the relative intensities of bound and radiative EM fields. However, as it was found in experimental works [2–4], the actual situation is more complicated, and one needs to take into account variation of retardation rates of bound EM field in different spatial regions.

Let us assume that an observed retardation dependence of bound EM field is directly related to the fact that a specific gauge renormalization can be applied to the EMEM tensor (8) only within the near zone (where bound fields are dominant). In fact, just in regions very close to a source the gauge renormalization allows elimination of divergent terms present in the conventional CED framework. More specifically, this gauge renormalization is rigorously applicable only for present time dependence of EM fields. For source charges moving with constant velocities (see Eqs. (17), (18)) the near zone have an infinite size and the present time dependence for bound EM field takes place in the entire space. In the general case, according to the referred experimental data [4], within the near zone bound EM fields show present time dependence, providing the validity of the above-mentioned renormalization procedure. On the other hand, it restricts the volume of integration in the expression for EM mass $M_{\mathrm{EM}} = \int\limits_V m_{\mathrm{EM}} dV$ by the size of the near zone.

Let us estimate a fraction of the total EM energy of a charged particle (the classical electron) which lays inside the near zone. First one needs to calculate approximately the near zone size of a radiating classical electron. The smallest wavelength of EM radiation emitted by an electron is determined by the Compton wavelength $\lambda_c = \frac{h}{m_0 c} = \frac{2\pi r_0}{\alpha}$, where h is Plank's constant, m_0 is electron rest mass, r_0 (about 10^{-13} m) is the classical radius of electron, and $\alpha = 1/137$ is the fine structure constant. This relationship reflects a well-known fact that EM radiation with Compton's wavelength has the total EM energy just equal to electron rest energy, i.e., $h\nu = m_0 c^2 = 511\,\mathrm{keV}$. A corresponding near zone size for this radiation wavelength is $\frac{\lambda_c}{2\pi} = \frac{r_0}{\alpha}$. Hence, even for this limited wavelength of EM radiation, the electron must *feel* rigidly more than 99% of its bound EM energy.

Thus, remaining within domains of applicability of classical electrodynamics and assuming that the total EM energy is small in comparison with the rest mass energy, one concludes that a charged particle *feels* rigidly almost the entire bound EM field energy. It provides a straightforward interpretation of its EM mass as a part of inertial mass. In other words, the above gauge renormalization procedure can be considered as a valid approximation in regions close to source particles providing a sound treatment of mass parameters related to classical radiating charges since it implies a linear superposition of electromagnetic, mechanical and Poincaré mass densities.

Following the same line of reasoning it becomes clear that the gauge renormalization based on the equalities $\partial_\mu T^{\mu\nu}_{\text{EEM}} = 0$ and $\partial_\mu T^{\mu\nu}_{\text{EM mass}} = 0$ is not applicable within the far zone, where EM radiation dominates. In this case we have to use the general expression (8) for EMEM tensor in order to explore its physical implications. Let us use it in the derivation of continuity equation

$$\partial_\mu T^{\mu 0} = 0 \tag{30}$$

In connection with it we note that the appearance of the extra-term $-\frac{1}{c}(A^\mu j^\nu)_{pr}$ in Eq. (8) in comparison with the standard expression for EMEM tensor (6) does not influence the conventional Poynting theorem (e.g., Ref. [5]) derived on the basis of Eq. (30). This statement follows from the fact that the four-divergence of time-like components of the extra-term is equal to zero (see Appendix B).

Thus we conclude that the expression for the electromagnetic energy-momentum tensor (8) should be valid in far zone and does not imply any changes in the formulation of the standard Poynting theorem. As a consequence both bound and radiative EM field components should have the same propagation (or retardation) rates determined by the velocity of light c. This conclusion strictly agrees with the experimental data reported in [3, 4].

Finally, it can be added that the general expression (8) for EMEM tensor yields the standard equation of motion just implying that the mass of a charged particle is interpreted as its total observable mass according to the definition (26) [10].

5. CONCLUSION

Having eliminated in the present work the inconsistency in the derivation of the symmetric EMEM tensor (i.e., the use of homogeneous Maxwell Eq. (5) for a system of charges) we obtained the general expression (8) for EMEM tensor compatible with inhomogeneous Maxwell Eq. (7). As a consequence it was possible to explain two main features in the behavior of bound EM fields reported in the recent experimental works [2–4]: (a) disappearance of bound EM field retardation in regions close to a source ($r \to 0$), and (b) asymptotic approximation to the standard retardation in the far zone. This explanation involves the fact that a special gauge renormalization turns out to be applicable to a general EMEM tensor only in spatial regions close to a source particle. It allows an appropriate elimination of infinite terms present in the conventional CED framework in a mathematically correct way and not as a premeditate omission of such terms which usually takes place in the standard renormalization procedure. On one hand, it provides a solid interpretation for anomalously small retardation observed for bound EM fields in the near zone and, on the other, gives a real physical mechanism to how the divergent terms for point-like charges can be eliminated: no retardation of the bound EM fields in a close vicinity of a charge is required.

One can assume that a similar physical behavior may take place in quantum electrodynamics (QED) in which framework the infinite self-energy of the electron is also treated by the standard renormalization procedure (e.g., [12]). In fact, it is well known in QED that real photons (EM radiation) propagate at the light velocity c, whereas there are some reasons to interpret the propagation of virtual photons as being instantaneous (e.g., [13]).

Finally, we would like to stress that the present theoretical approach is entirely motivated by experimental results [2–4] that suggest a more complicated EM field properties than it is usually implied in the conventional CED: the absence of retardation of bound EM fields within the near zone and equality of propagation rates of bound and radiation EM fields in the far zone. The main result of the present work provides a qualitative explanations of bound EM field retardation rate asymptotic behavior in the near and the far zones. Nevertheless, a further more detailed exploration of these issues is necessary. In particular, it would be of special interest to proceed within a direct field approach for inhomogeneous EM wave equation which solution requires two boundary conditions: at infinity (vanishing fields) and on a source. However, the latter one is a problematic issue in the conventional CED due to existent divergences. The present tensorial approach attacks the problem in a way that EM field propagation properties become compatible with the recent experimental data. Further extension of this analysis to intermediate zone will be considered elsewhere.

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APPENDIX A. GENERALIZED ELECTROMAGNETIC ENERGY-MOMENTUM TENSOR VIA THE HILBERT APPROACH

As known, the definition of the energy-momentum tensor given by Hilbert [5] reads:

$$\frac{1}{2}\sqrt{-g}T_{\mu\nu} = \frac{\partial\sqrt{-g}L}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^{\gamma}}\frac{\partial\sqrt{-g}L}{\partial(\partial g^{\mu\nu}/\partial x^{\gamma})}, \tag{A1}$$

where L is electromagnetic Lagrangian density. The conventional classical electrodynamics provides the Lagrange function for EM field L in explicit form as

$$L = -\frac{1}{16\pi}F_{\gamma\alpha}F^{\gamma\alpha}. \tag{A2}$$

As a consequence, the standard representation (6) for the symmetric EMEM tensor follows from (A1) if the definition (A2) is considered as adequate. However, it should be especially stressed here that expression (A2) for the Lagrangian density makes sense only for spatial regions free of charged particles, which may include both EM radiation and bound EM field. However, the latter component of EM field is undividable from its source charges and therefore, the use of (A2) (where the omission of the Lagrange density for source charges is obvious) in the derivation of symmetric EMEM tensor can not be considered as general since it is not applicable to the whole system with charged particles.

Extending the approach to charges, one has to add into Eq. (A2) a component responsible for interaction of charges with EM fields (see [5]). It leads to the following expression for the Lagrangian density of a whole system of *"charged particles and EM field"* as it was suggested in [9]

$$L = -\frac{1}{c}A^{\mu}j^{\mu} - \frac{1}{16\pi}F_{\gamma\alpha}F^{\gamma\alpha}. \tag{A3}$$

The first term in *rhs* of Eq. (A3) might be referred as an *interaction part* since it takes into consideration the presence of charged particles, being responsible for interaction. By analogy, the second term can be regarded as *field part*, which, in general, deals with both bound and free EM field components. By substituting Eq. (A3) into Eq. (A1), we again arrive at Eq. (8).

APPENDIX B. PROOF OF THE EQUALITY $\partial_\mu (A^\mu J^0)_{PR}=0$

In order to prove this equality, we write the term $(A^\mu j^0)$ in the factorized form, considering an isolated system of $N > 1$ charged particles (in the units $c = 1$) and assuming the Lorenz gauge ($\partial_\mu A^\mu = 0$):

$$\left(A^\mu j^0\right)_{pr} = \sum_k \underset{(k)}{A}^\mu \underset{(k)}{j}^0, \tag{B1}$$

where $k = 1, \ldots, N$ is the particle index.

Hence

$$\partial_\mu \sum_k \underset{(k)}{A}^\mu \underset{(k)}{j}^0 = \left(\sum_k \partial_\mu \underset{(k)}{A}^\mu\right) \underset{(k)}{j}^0 + \sum_k \underset{(k)}{A}^\mu \left(\partial_\mu \underset{(k)}{j}^0\right) = \sum_k \underset{(k)}{A}^\mu \partial_\mu \underset{(k)}{\rho}, \tag{B2}$$

where ρ is the charge density. Taking into account that the four-potential of each particle can be written in the form $\underset{(k)}{A}^0 = \underset{(k)}{\varphi}$, $\underset{(k)}{A}^i = \underset{(k)}{\varphi}\, \underset{(k)}{v}^i$ (φ being the scalar potential and v^i is the velocity component, $i = 1 \ldots 3$), we further obtain

$$\sum_k \underset{(k)}{A}^\mu \partial_\mu \underset{(k)}{\rho} = \sum_k \underset{(k)}{\varphi} \frac{\partial \underset{(k)}{\rho}}{\partial t} + \sum_k \underset{(k)}{\varphi}\, \underset{(k)}{v}^i \frac{\partial \underset{(k)}{\rho}}{\partial x^i} = \sum_k \underset{(k)}{\varphi} \frac{d \underset{(k)}{\rho}}{dt} = 0 \tag{B3}$$

due to conservation of charge for each particle (i.e., $\frac{d\rho}{dt} = 0$ for each k). The latter equality proves that

$$\partial_\mu \left(A^\mu j^0\right)_{pr} = 0.$$

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