

TO CONTROL THE PROPAGATION CHARACTERISTIC OF ONE-DIMENSIONAL PLASMA PHOTONIC CRYSTAL USING EXPONENTIALLY GRADED DIELECTRIC MATERIAL

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Abstract—The effect of exponentially graded material on the modal dispersion characteristics, group velocity and effective group index as well as phase index of refraction of a binary One-Dimensional Plasma Photonic Crystals (1D-PPCs) has been studied. The dispersion relation is derived by solving Maxwell's equations and using the transfer matrix method. The anomalous dispersion characteristics are observed for different values of selection parameters. The introduction of graded dielectric layers in 1D-PPCs provides additional parameters for controlling the propagation characteristics of 1D-PPCs. Also, the band gap is shown to become larger with the increase of plasma frequency and plasma width. Hence the structure having plasma and exponentially graded dielectric layer in unit cell is more useful for controlling and tuning of the plasma functioning devices than the structure having plasma and homogeneous dielectric layer in one unit cell.

1. INTRODUCTION

The dispersion characteristics of photonic crystals (PCs) depend on the strong coupling of forward and backward propagating electromagnetic (EM) waves generated by the periodic arrangement of indices within certain frequency range. Considerable amount of investigations have been dedicated towards the study of behavior of photons in 1D-PCs [1–4]. Discussions on a wide variety of such PCs have been presented to achieve suitable forbidden band gaps [5–7] by producing defects, disorders, etc. in it. Most of the explored 1D-PCs are

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now widely recognized and implemented in various issues related to optics and photonics. Recently, the study of interaction of EM waves with periodic plasma layers which is called Plasma Photonic Crystals (PPCs) has received great attention due to its various applications, such as plasma lens, plasma antenna, plasma stealth aircraft, frequency filters, etc. [8–11]. These PPCs can be controlled by external parameters and have the characteristics of PCs and plasma. The concept of PPCs was firstly proposed by Hojo and Mase [12]. The transmission properties of 1D-PPCs in terms of number of unit cells, thickness, and density of plasma layer is studied by Laxmi and Mahto [13]. These PPCs structure was found to work as a perfect reflector/mirror in a certain range of frequency [14, 15].

The propagation characteristics of PPCs are not only the function of plasma parameter but also the function of angle of incidence [16, 17]. Very recently, Prasad et al. [18, 19] have investigated the modal propagation characteristics of ternary 1D-PPCs structures and have shown the plasma frequency, plasma width and dielectric constant of dielectric media have influence on band gap, group index, group velocity and phase velocity.

In most of the works discussed above related to PPCs, the band gaps are controlled by plasma parameters, such as plasma density, plasma frequency, relative plasma width etc., but these parameters have their limitations viz., we cannot vary plasma density much. Hence to obtain some additional parameters for controlling the band gap in the desire range there are need of non-homogeneous material whose permittivity varying with space.

Moreover, the investigations pertaining to 1D-PCs containing graded materials are still less discussed in the literatures whereas some studies of propagation of light in optical waveguide having graded materials, whose physical properties can vary continuously in space, appear in the literature [20–22]. These studies show that the change of physical properties of graded materials makes them different in behavior from the homogeneous materials and conventional composite materials. The fabrications of graded materials are quite easy where the variation of permittivity and permeability along a direction can be achieved by imposing the temperature profile because permittivity and permeability of materials are related to temperature [23]. Also by using the diffusion method, one can fabricate compositionally graded material in which compositions vary along a direction [24]. The optical property of 1D-PCs containing graded materials is studied by Sang and Li [25]. They also studied the properties of defect modes in 1D-PCs containing a graded defect layer [26] and found that the introduction of a graded defect layer in 1D-PCs provides possible mechanism for tuning

the defect modes including the position, intensity and number of mode. Recently Thapa et al. [27] have studied the omni-directional 1D-PCs having exponential graded materials and found two omni-directional reflection (ODR) bands in this structure; one in the visible and other in the infrared region. The behavior of the ODR band in the infrared region is different from the usual Bragg ODR band in the visible region.

It is clear from above discussions that the graded material may also give additional degree of freedom to control the plasma photonic band gaps preserving the characteristics of conventional PCs and plasma. Therefore, in the present paper, the propagation of EM waves in 1D-PPCs having graded dielectric profile in the unit cell is considered which is not discussed by any researchers till now. Among the various graded index profiles such as Gaussian, exponential, complementary error function, etc., we choose only exponential graded profile because this type of profiles are explicitly solved in terms of Bessel functions. The paper is organized as follow: in Section 2 the dispersion relation of the proposed structure is given. The other necessary formulas used in this paper are also presented. Section 3 is devoted to result and discussion. A conclusion is drawn in Section 4.

2. THEORETICAL MODELING

The proposed binary 1D-PPCs consist of plasma and graded dielectric layer in one unit cell. In the graded dielectric layer the permittivity is exponentially varying in space. We assume that the space-variation of permittivity is perpendicular to the interface of layers. In this study we have considered nonmagnetic materials for both plasma and graded dielectric regions. The permittivity profile in a unit cell is written as

$$\varepsilon(\omega, x) = \begin{cases} 1 - \frac{\omega_{pe}^2}{\omega^2}; & (n-1)\Lambda + b < x < n\Lambda \\ \alpha e^{\beta x}; & (n-1)\Lambda < x < (n-1)\Lambda + b \end{cases} \quad (1)$$

with condition that $\varepsilon(\omega, x) = \varepsilon(\omega, x + \Lambda)$, where $\Lambda = a + b$ with a and b are widths of plasma and graded dielectric layer as shown in Fig. 1. Here $\omega_{pe} = \sqrt{e^2 n_p / \varepsilon_0 m}$ is the electron plasma frequency, n_p is density of plasma, $\beta = \frac{1}{b} \ln \frac{\varepsilon_b}{\alpha}$ with α and ε_b are the initial and final permittivity of graded dielectric layer at boundaries $x = 0$ and $x = b$ respectively.

We know that the one dimensional Maxwell's equation for $E_n(x)$ at normal incidence in n th unit cell is written as

$$\frac{d^2}{dx^2} E_n(x) + \frac{\omega^2}{c^2} \varepsilon(\omega, x) E_n(x) = 0, \quad (2)$$

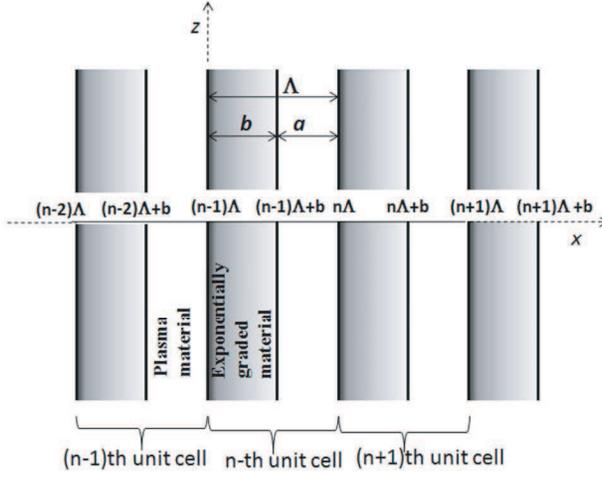


Figure 1. Schematic representation of the unit cell of binary 1D-PPCs having exponentially graded material.

Hence the one dimensional Maxwell's equation for both layers in n th unit cell is separately written as

$$\frac{d^2}{dx^2} E_n(x) + \frac{\omega^2}{c^2} \left(1 - \frac{\omega_{pe}^2}{\omega^2} \right) E_n(x) = 0, \quad (n-1)\Lambda + b < x < n\Lambda \quad (3)$$

$$\frac{d^2}{dx^2} E_n(x) + \frac{\omega^2}{c^2} \left(\alpha e^{\beta x} \right) E_n(x) = 0, \quad (n-1)\Lambda < x < (n-1)\Lambda + b$$

By solving the above Maxwell's wave Equation (3), we can write the electric field in n th unit cell for $\omega > \omega_{pe}$ as

$$E_n(x) = \begin{cases} a_n e^{ik_p(x-n\Lambda)} + b_n e^{-ik_p(x-n\Lambda)} & (n-1)\Lambda + b < x < n\Lambda \\ c_n J_0 \left(\frac{2\omega\sqrt{\alpha} e^{\beta(x-n\Lambda)}}{c\beta} \right) + d_n Y_0 \left(\frac{2\omega\sqrt{\alpha} e^{\beta(x-n\Lambda)}}{c\beta} \right) & (n-1)\Lambda < x < (n-1)\Lambda + b \end{cases} \quad (4)$$

and for $\omega < \omega_{pe}$ as

$$E_n(x) = \begin{cases} a_n e^{\kappa(x-n\Lambda)} + b_n e^{-\kappa(x-n\Lambda)} & (n-1)\Lambda + b < x < n\Lambda \\ c_n J_0 \left(\frac{2\omega\sqrt{\alpha} e^{\beta(x-n\Lambda)}}{c\beta} \right) + d_n Y_0 \left(\frac{2\omega\sqrt{\alpha} e^{\beta(x-n\Lambda)}}{c\beta} \right) & (n-1)\Lambda < x < (n-1)\Lambda + b \end{cases} \quad (5)$$

where $k_p = \frac{\omega}{c} \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}$, $\kappa = \frac{\omega}{c} \sqrt{\frac{\omega_{pe}^2}{\omega^2} - 1}$. Here J_0 and Y_0 are the zeroth order Bessel function of first kind and second kind respectively.

Imposing the continuity of electric field $E_n(x)$ and its derivatives $\frac{\partial E_n(x)}{\partial x}$ at interfaces $x = (n-1)\Lambda$, $x = (n-1)\Lambda + b$ and arranging

coefficients a_{n-1} , b_{n-1} , a_n and b_n by transfer matrix method [28], we obtained the following matrix relation:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad (6)$$

where all the matrix elements A, B, C and D are given in Appendix A. The matrix in Equation (6) is unit cell transfer matrix which relates the complex wave amplitudes of incident wave and reflected wave in $(n - 1)$ th unit cell to respective amplitudes in the n th unit cell.

According to Floquet Theorem, a wave propagating in a periodic medium is of form $E(x) = E(x) e^{i(\omega t - Kx)}$, where $E(x)$ is periodic with period Λ , that is, $E(x + \Lambda) = E(x)$. Hence the dispersion relation [28] for proposed structure can be written as

$$K = \left(\frac{1}{\Lambda}\right) \cos^{-1} \left[\frac{1}{2} (A + D) \right]; \quad (7)$$

Here K is Bloch wave number. Some other parameters such as; group velocity, effective phase index ($n_{eff}(p)$) and effective group index (n_g) are also calculated for the proposed structure. The expression of group velocity can be obtained by taking derivative of $K(\omega)$ with respect to ω which is written as:

$$V_g = \left(\frac{dK(\omega)}{d\omega} \right)^{-1} \quad (8)$$

Expression for effective phase index ($n_{eff}(p)$), which is effective index associated with the effective phase velocity is given by

$$n_{eff}(p) = \frac{cK(\omega)}{\omega} \quad (9)$$

The effective group index (n_g) [4] can be expressed as:

$$n_g = \frac{c}{V_g} \quad (10)$$

3. RESULTS AND DISCUSSION

We are now in position to present the results of numerical calculations for Equations (7) to (10).

The exponentially graded material of width b with permittivity $\varepsilon_b = \alpha e^{\beta x}$ is taken. For the comparison purpose with the result reported in [12], two sets of values for α and ε , namely, $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$ have been chosen in such a way that the average volume permittivity remain fixed at 5. The plasma of width a having frequency dependent permittivity $\varepsilon = 1 - (\omega_{pe}^2/\omega^2)$ is

taken. Here $a = d \times b$ and d is a constant related to the width of plasma layer (for $d = 1$, $a = b$). Also the normalized plasma frequency ($P = \omega_{pe} \times b/c$) 1, 2, and 4 and the normalized frequency $\omega \times b/c$, have been considered. There are five selection parameters P , b , d , α and ε_b considered in the numerical calculation which may affect the propagation characteristics of proposed structure.

The curve between normalized frequencies and Bloch wavevector K (called dispersion curve) is shown in Fig. 2 to Fig. 4. These curves show an anomalous dispersion relation for different values of selection parameters. Here the effects of these parameters on the band gap have been estimated. Fig. 2 shows the dispersion relation of EM waves for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$. These dispersion curves are plotted for two different variations in the permittivity of graded dielectric layers keeping average volume permittivity as constant. If the slope of the permittivity of graded dielectric layer changes sharply then the width of the gap and phase velocity increase considerably. This is due to reason that sharp varying permittivity of graded dielectric layer increases the Bragg's reflection. The comparison of present dispersion curves with those obtained in [12] show that the width of the band gap in the present case is nearly constant in the given frequency range. This is the novelty

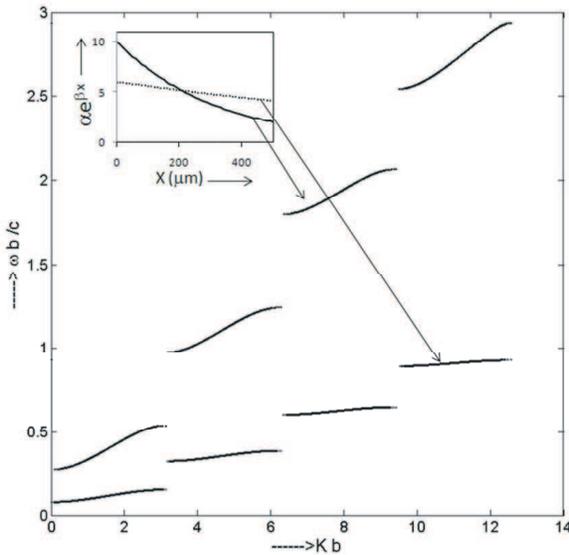


Figure 2. Dispersion curves for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$.

in the present dispersion curves. Fig. 3 shows the dispersion relation of EM waves for $P = 4$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\epsilon_b = 2.04$ and $\alpha = 6$, $\epsilon_b = 4.13$. This figure is plotted here to see the effect of plasma frequency or plasma density. By comparing Fig. 3 with Fig. 2 it is found that the increase in plasma frequency or plasma density flattened the dispersion curve and increases the band gap considerably. The similar results are also obtained in [12]. Fig. 4 shows the effect of plasma width on the dispersion curves for $P = 2$, $b = 500 \mu\text{m}$, $\alpha = 6$, $\epsilon_b = 4.13$ with $d = 0.1$ and $d = 1$. It is observed that as the width of plasma layer increases from $d = 0.1$ to $d = 1$ the band gap get compressed and phase velocity decreases up to $\omega \times b/c \cong 0.15$ after this the phase velocity increases very rapidly and become larger than those obtained for $d = 0.1$. This is an interesting result in the dispersion of the proposed PPCs structure which is not obtained in [12].

The curves of effective phase index $n_{eff}(p)$ with respect to the normalized frequency for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\epsilon_b = 2.04$ and $\alpha = 6$, $\epsilon_b = 4.13$ are shown in Fig. 5. This figure shows that the effective phase index shifted toward higher frequency at the cost of its magnitude for the slow variation in the permittivity of graded dielectric layers. The variation of normalized group velocity with respect to the normalized frequency for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\epsilon_b = 2.04$ and $\alpha = 6$, $\epsilon_b = 4.13$ is also studied and shown in Fig. 6. It is clear that group velocity

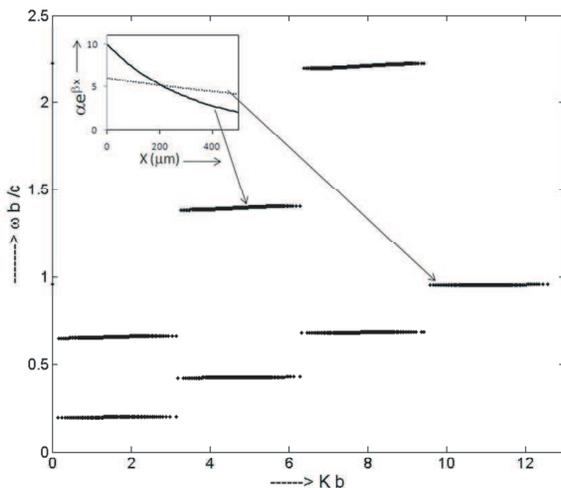


Figure 3. Dispersion curves for $P = 4$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\epsilon_b = 2.04$ and $\alpha = 6$, $\epsilon_b = 4.13$.

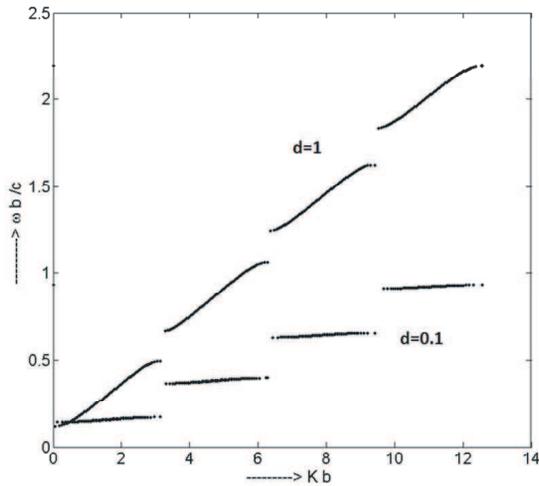


Figure 4. Dispersion curves for $P = 2$, $b = 500 \mu\text{m}$, $\alpha = 6$, $\varepsilon_b = 4.13$ with $d = 0.1$ and $d = 1$.

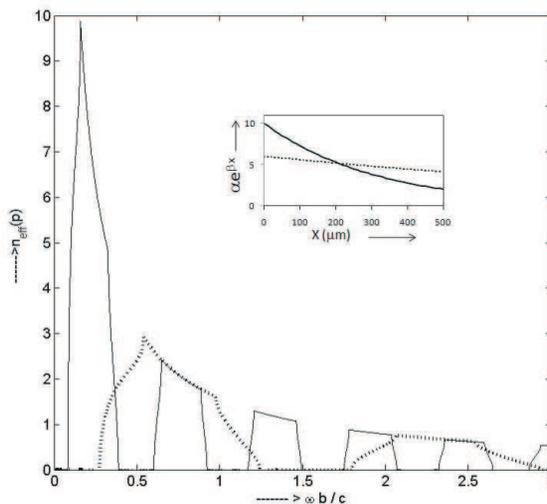


Figure 5. The variation of the effective phase index $n_{eff}(p)$ with respect to normalized frequency for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$.

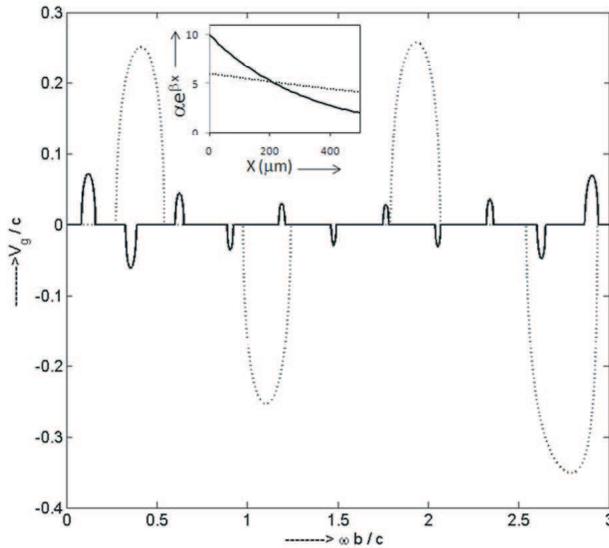


Figure 6. The variation of normalized group velocity with respect to normalized frequency for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$.

becomes negative in certain frequency range for both considered cases because of anomalous dispersion relations. The left-right and up-down symmetries for negative as well as for positive group velocities in both cases are observed. This figure shows that if the variation in the permittivity of graded dielectric layer is sharp then the numbers of peaks for positive and negative group velocities are larger than those for slowly varying permittivity in given frequency range. It is also observed that the sharp varying graded dielectric layer permittivity shift the group velocities towards lower frequency range and decrease the magnitude of peaks of negative and positive group velocities.

Finally, the curves of effective group index n_g with respect to the normalized frequency for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$ are obtained and shown in Fig. 7. These curves have been studied for two different permittivity profiles in the exponential graded layer. The magnitude of group index increases considerably, near the band gap edges, for sharp varying permittivity profile of graded dielectric layer. The negative effective group of refraction, near band edges, for slow varying permittivity is at $\omega \times b/c \cong 1$ and 2.5 and for sharp varying permittivity it is at $\omega \times b/c \cong 0.3, 0.85, 1.48, 2.1$ and 2.6 . This negative effective group of refraction may be caused by wave interference from Bragg reflection.

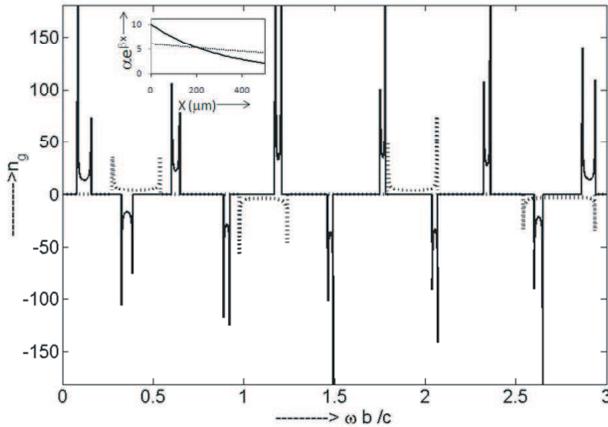


Figure 7. The variation of the effective group index n_g with respect to normalized frequency for $P = 1$, $b = 500 \mu\text{m}$, $d = 1$ with $\alpha = 10$, $\varepsilon_b = 2.04$ and $\alpha = 6$, $\varepsilon_b = 4.13$.

4. CONCLUSION

The effect of exponentially varying dielectric permittivity on the propagation characteristic of 1D-PPCs has been studied. The two exponentially varying dielectric profiles (sharp and slow) are chosen in such a way that the volume average permittivity remains constant. As expected, all the known characteristics of PPCs are found. It is observed that as the slope of the permittivity profile in the graded dielectric layer changes slowly, the band gaps get compressed, and number of band gaps increases considerably. Therefore, by changing the slope of the permittivity profile in the graded dielectric layer, the band gap can be controlled or tuned instead of changing the plasma density or plasma layer width.

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APPENDIX A. UNIT CELL TRANSFER MATRIX ELEMENTS

Unit cell transfer matrix elements A , B , C and D for $\omega > \omega_{pe}$ are given below:

$$A = \frac{\frac{1}{2}e^{-ik_p a} \begin{pmatrix} -J_0(\frac{2t_1}{\beta})k_p Y_1(\frac{2t_2}{\beta})t_2 - iJ_0(\frac{2t_1}{\beta})k_p^2 Y_0(\frac{2t_2}{\beta}) \\ -it_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 + t_1 Y_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})k_p \\ + Y_0(\frac{2t_1}{\beta})k_p J_1(\frac{2t_2}{\beta})t_2 + iY_0(\frac{2t_1}{\beta})k_p^2 J_0(\frac{2t_2}{\beta}) \\ + it_1 Y_1(1, \frac{2t_1}{\beta})J_1(1, \frac{2t_2}{\beta})t_2 - t_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})k_p \end{pmatrix}}{\left(k_p \left(-J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) + Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta})\right) t_2\right)} \quad (A1)$$

$$B = \frac{\frac{1}{2}e^{ik_p a} \begin{pmatrix} -J_0(\frac{2t_1}{\beta})k_p Y_1(\frac{2t_2}{\beta})t_2 + iJ_0(\frac{2t_1}{\beta})k_p^2 Y_0(\frac{2t_2}{\beta}) \\ -it_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 - t_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})k_p \\ + Y_0(\frac{2t_1}{\beta})k_p J_1(\frac{2t_2}{\beta})t_2 - iY_0(\frac{2t_1}{\beta})k_p^2 J_0(\frac{2t_2}{\beta}) \\ + it_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 + t_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})k_p \end{pmatrix}}{\left(k_p \left(-J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) + Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta})\right) t_2\right)} \quad (A2)$$

$$C = \frac{-\frac{1}{2}e^{-ik_p a} \begin{pmatrix} -iJ_0(\frac{2t_1}{\beta})k_p Y_1(\frac{2t_2}{\beta})t_2 + J_0(\frac{2t_1}{\beta})k_p^2 Y_0(\frac{2t_2}{\beta}) \\ -t_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 - it_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})k_p \\ + iY_0(\frac{2t_1}{\beta})k_p J_1(\frac{2t_2}{\beta})t_2 - Y_0(\frac{2t_1}{\beta})k_p^2 J_0(\frac{2t_2}{\beta}) \\ + t_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 + it_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})k_p \end{pmatrix}}{\left(k_p \left(-J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) + Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta})\right) t_2\right)} \quad (A3)$$

$$D = \frac{-\frac{1}{2}e^{ik_p a} \begin{pmatrix} -iJ_0(\frac{2t_1}{\beta})k_p Y_1(\frac{2t_2}{\beta})t_2 - J_0(\frac{2t_1}{\beta})k_p^2 Y_0(\frac{2t_2}{\beta}) \\ -t_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 + it_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})k_p \\ + iY_0(\frac{2t_1}{\beta})k_p J_1(\frac{2t_2}{\beta})t_2 + Y_0(\frac{2t_1}{\beta})k_p^2 J_0(\frac{2t_2}{\beta}) \\ + t_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 - it_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})k_p \end{pmatrix}}{\left(k_p \left(-J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) + Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta})\right) t_2\right)} \quad (A4)$$

where $t_1 = \frac{\omega}{c} \sqrt{\epsilon_0 e^{-\beta} \Lambda}$; $t_2 = \frac{\omega}{c} \sqrt{\epsilon_0 e^{-\beta} a}$, $k_p = \frac{\omega}{c} \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}$.

Similarly, unit cell transfer matrix elements A , B , C and D for

$\omega < \omega_{pe}$ can be written as:

$$A = \frac{-\frac{1}{2}e^{-\kappa a} \begin{pmatrix} -J_0(\frac{2t_1}{\beta})\kappa Y_1(\frac{2t_2}{\beta})t_2 - J_0(\frac{2t_1}{\beta})\kappa^2 Y_0(\frac{2t_2}{\beta}) \\ + t_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 + t_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})\kappa \\ + Y_0(\frac{2t_1}{\beta})\kappa J_1(\frac{2t_2}{\beta})t_2 + Y_0(\frac{2t_1}{\beta})\kappa^2 J_0(\frac{2t_2}{\beta}) \\ - t_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 - t_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})\kappa \end{pmatrix}}{\left(\kappa \left(-J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) - Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta}) \right) t_2 \right)} \quad (\text{A5})$$

$$B = \frac{-\frac{1}{2}e^{\kappa a} \begin{pmatrix} -J_0(\frac{2t_1}{\beta})\kappa Y_1(\frac{2t_2}{\beta})t_2 + J_0(\frac{2t_1}{\beta})\kappa^2 Y_0(\frac{2t_2}{\beta}) \\ + t_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 - t_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})\kappa \\ + Y_0(\frac{2t_1}{\beta})\kappa_p J_1(\frac{2t_2}{\beta})t_2 - Y_0(\frac{2t_1}{\beta})\kappa^2 J_0(\frac{2t_2}{\beta}) \\ - t_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 + t_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})\kappa \end{pmatrix}}{\left(\kappa \left(-J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) - Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta}) \right) t_2 \right)} \quad (\text{A6})$$

$$C = \frac{\frac{1}{2}ie^{-\kappa a} \begin{pmatrix} J_0(\frac{2t_1}{\beta})\kappa Y_1(\frac{2t_2}{\beta})t_2 + J_0(\frac{2t_1}{\beta})\kappa^2 Y_0(\frac{2t_2}{\beta}) \\ + t_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 + t_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})\kappa \\ - Y_0(\frac{2t_1}{\beta})\kappa J_1(\frac{2t_2}{\beta})t_2 - Y_0(\frac{2t_1}{\beta})\kappa J_0(\frac{2t_2}{\beta}) \\ - t_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 - t_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})\kappa \end{pmatrix}}{\left(\kappa \left(J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) - Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta}) \right) t_2 \right)} \quad (\text{A7})$$

$$D = \frac{\frac{1}{2}ie^{\kappa a} \begin{pmatrix} J_0(\frac{2t_1}{\beta})\kappa Y_1(\frac{2t_2}{\beta})t_2 - J_0(\frac{2t_1}{\beta})\kappa^2 Y_0(\frac{2t_2}{\beta}) \\ + t_1 J_1(\frac{2t_1}{\beta})Y_1(\frac{2t_2}{\beta})t_2 - t_1 J_1(\frac{2t_1}{\beta})Y_0(\frac{2t_2}{\beta})\kappa - \\ Y_0(\frac{2t_1}{\beta})\kappa J_1(\frac{2t_2}{\beta})t_2 + Y_0(\frac{2t_1}{\beta})\kappa^2 J_0(\frac{2t_2}{\beta}) \\ - t_1 Y_1(\frac{2t_1}{\beta})J_1(\frac{2t_2}{\beta})t_2 + t_1 Y_1(\frac{2t_1}{\beta})J_0(\frac{2t_2}{\beta})\kappa \end{pmatrix}}{\left(\kappa \left(J_0(\frac{2t_2}{\beta})Y_1(\frac{2t_2}{\beta}) - Y_0(\frac{2t_2}{\beta})J_1(\frac{2t_2}{\beta}) \right) t_2 \right)} \quad (\text{A8})$$

where $t_1 = \frac{\omega}{c} \sqrt{\varepsilon_0 e^{-\beta \Lambda}}$, $t_2 = \frac{\omega}{c} \sqrt{\varepsilon_0 e^{-\beta a}}$, $k_p = \frac{\omega}{c} \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}$ and $\kappa = \frac{\omega}{c} \sqrt{\frac{\omega_{pe}^2}{\omega^2} - 1}$.

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