

THE PERMITTIVITY FOR ANISOTROPIC DIELECTRICS WITH PERMANENT POLARIZATION

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Abstract—A new permittivity is defined for anisotropic dielectrics with permanent polarization, which allows obtaining simple connections between the quantities of electric field. As an application, using the defined quantity, we will demonstrate advantageous forms of the refraction theorems of the two-dimensional electric field lines at the separation surface of two anisotropic dielectrics with permanent polarization, anisotropic by orthogonal directions.

1. INTRODUCTION

We know that [1–3], for *dielectrics with permanent polarization*, the connection law, among electric flux density \overline{D} , electric field strength \overline{E} and polarization \overline{P} , is given by

$$\overline{D} = \varepsilon_0 \overline{E} + \overline{P}_\tau + \overline{P}_p, \quad (1)$$

where ε_0 is permittivity of the vacuum. The separation in temporary (\overline{P}_τ) and permanent (\overline{P}_p) components is unique only if \overline{P}_p is independent of \overline{E} , and \overline{P}_τ — which is depending on \overline{E} — is null at the same time with \overline{E} . From (1) follows that, for materials with permanent polarization ($\overline{P}_p \neq 0$), the relation between \overline{D} and \overline{E} (which for materials with $\overline{P}_p = 0$ represents the classic permittivity) is not univocally determined by material, because \overline{P}_p could have several values. In the case of the ferroelectric materials, for various electrical hysteresis cycles, the value of \overline{D} for $\overline{E} = 0$, i.e., remanent electric flux density

$$\overline{D}_r = \overline{D} |_{\overline{E}=0} = \overline{P}_p, \quad (2)$$

may have multiple values, depending on the considered electric hysteresis cycle (Figure 1).

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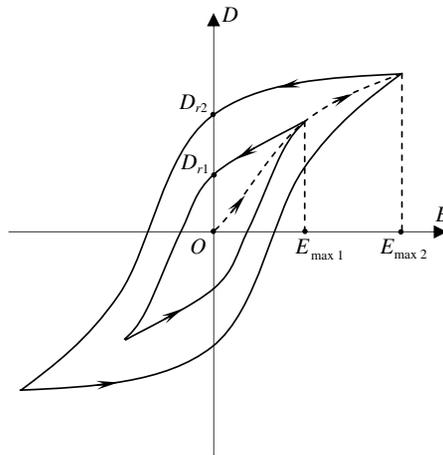


Figure 1. Remanent electric flux densities-specification.

In this context it is useful to define another permittivity (for dielectrics with permanent polarization), with which the equations have advantageous forms, and it is possible to identify some useful analogies with simpler case of the materials without permanent polarization.

2. ANOTHER PERMITTIVITY FOR ANISOTROPIC DIELECTRICS WITH PERMANENT POLARIZATION

The temporary polarization value of anisotropic materials depends on electric field, and the temporary polarization law is

$$\overline{P}_\tau = \varepsilon_0 \overline{\chi}_e \overline{E}, \tag{3}$$

where, for the nonlinear materials, the components of electric susceptibility $\overline{\chi}_e$ depend on electric field intensity.

Consequently, in case of the dielectrics with permanent polarization, nonlinear and anisotropic, (1) becomes

$$\overline{D} = \varepsilon_0 \left(\overline{1} + \overline{\chi}_e \right) \overline{E} + \overline{D}_r, \tag{4}$$

where the tensor's components are nonlinear functions depending on the components of \overline{E} .

If we introduce the calculation quantity

$$\overline{D}_p = \overline{D} - \overline{D}_r = \overline{D} - \overline{P}_p, \tag{5}$$

(4) becomes

$$\overline{D}_p = \varepsilon_0 \left(\overline{1} + \overline{\chi}_e \right) \overline{E}. \tag{6}$$

From (4), (5), (6), the relative ($\overline{\overline{\epsilon}}_{rp}$) and absolute ($\overline{\overline{\epsilon}}_p$) calculation permittivity tensors of anisotropic dielectrics with permanent polarization are defined with these equations:

$$\overline{\overline{\epsilon}}_{rp} = \overline{\overline{1}} + \overline{\overline{\chi}}_e; \quad \overline{\overline{\epsilon}}_p = \epsilon_0 \overline{\overline{\epsilon}}_{rp}. \tag{7}$$

By defining the vector \overline{D}_p (in (5)) and new absolute permittivity $\overline{\overline{\epsilon}}_p$ (in (7)), for *anisotropic dielectrics with permanent polarization* we obtain

$$\overline{D}_p = \overline{\overline{\epsilon}}_p \overline{E}. \tag{8}$$

With classical quantities [1–3], for anisotropic dielectrics with permanent polarization, we have $\overline{D} = \overline{\overline{\epsilon}} \overline{E} + \overline{P}_p$, so (8) is a more concentrated expression and simpler.

Formally, (8) is similar with the classical equation $\overline{D} = \overline{\overline{\epsilon}} \overline{E}$, but the latter written for the materials without permanent polarization.

For isotropic materials, even if they are with permanent polarization, (8) becomes $\overline{D}_p = \epsilon_p \overline{E}$ which shows that *the spectra lines of \overline{D}_p and \overline{E} are the same in this case.* We know that for materials with permanent polarization (even if they are isotropic) the spectra lines of \overline{D} and \overline{E} are different [1, 2, 4].

Following the polarization main directions, tensor $\overline{\overline{\chi}}_e$ has only three components [1–3]. If we note these directions (that in many cases are rectangular) with x, y, z indices, from (4) we have

$$D_v = \epsilon_0 (1 + \chi_{ev}) E_v + D_{rv}; \quad v = x, y, \text{ or } z, \tag{9}$$

and all the three components of tensor $\overline{\overline{\epsilon}}_{rp}$ are

$$\epsilon_{rpv} = \frac{D_v - D_{rv}}{\epsilon_0 E_v} = \frac{D_{pv}}{\epsilon_0 E_v}; \quad v = x, y, z. \tag{10}$$

Because (10) also contains the components of permanent polarization \overline{P}_p (which means the components of \overline{D}_r), with that new permittivity we have taken into account, in advantageously way, the nonlinearity of the depolarization curves of the dielectric with permanent polarization, any minor electric hysterezis cycles, i.e., for any $\overline{P}_p = \overline{D}_r$.

If the source of electric field is considered, a dielectric with permanent polarization for the operating point is obtained, $D < D_r$ (for components: $D_v < D_{rv}$) and $E < 0$ (for components: $E_v < 0$). The depolarization curve is the part from the second quadrant of the hysteresis cycle; the terminology is similar to that used in the magnetic field. Therefore, the components of tensor $\overline{\overline{\epsilon}}_{rp}$ are positive scalars.

It is interesting to specify that we should determine the nonlinear functions $\epsilon_{rpv}(E)$ following the procedure used by the author for the permeability of permanent magnets in [5], if we know all the three

electric hysteresis cycles following the polarization main axes. For these three main directions x, y, z the nonlinear function plots have similar forms, but they will be quantitatively different, as the depolarization curves following the three main directions of the anisotropic dielectric are different.

The numerical solution for the electric field problem in systems with permanent polarization is obtained with an iterative process, because the systems are, generally, nonlinear. The parameter used to control the convergence of the problem can be the relative permittivity defined with (10). For anisotropic materials, it is clear that the convergence of the calculation is made with the components of tensor $\bar{\bar{\epsilon}}_{rp}$ following the polarization main axes. Trough this defined calculation quantity we take, univocally and advantageously, into account the nonlinearity of the depolarization curves, no matter how the permanent polarization is (i.e., remanent electric flux density).

Obviously, if lacking the permanent polarization ($\bar{P}_p = 0$), \bar{D}_p is identical to \bar{D} , and the calculation permittivity $\bar{\bar{\epsilon}}_p$ is identical to classical $\bar{\epsilon}$, i.e., $\bar{\bar{\epsilon}}_{rp} \equiv \bar{\epsilon}_r$. If the temporary polarization is negligible, from (1), (6) and (8) follows that all the components of tensor $\bar{\bar{\epsilon}}_{rp}$ are approximated with 1.

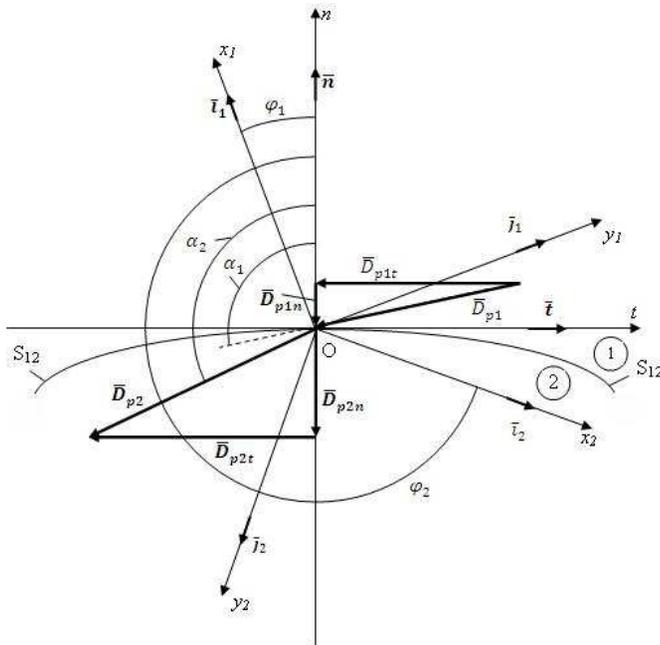


Figure 2. Refraction of \bar{D}_p .

3. APPLICATIONS FOR THE REFRACTION THEOREMS

Consider two different dielectric media 1 and 2 at rest, with permanent polarization, separated by smooth surface S_{12} , without free electric charge. The demonstration refers the electric field lines of \vec{E} and the calculation flux density \vec{D}_p (defined by (5)), for two-dimensional (2D) field, in dielectrics with permanent polarization, anisotropic by orthogonal directions. For medium 1, main directions of the polarization are noted with (x_1, y_1) and unit vectors (\vec{i}_1, \vec{j}_1) and for medium 2 are noted with (x_2, y_2) and unit vectors (\vec{i}_2, \vec{j}_2) (Figures 2 and 3).

In order to express the normal and tangent components of the electrical field at the separation surface S_{12} , in point 0 (where the refraction is analyzed) we attach the rectangular system $((n, t),$ with unit vectors \vec{n}, \vec{t}). The main axes of polarization in both media — therefore rectangular systems (x_1, y_1) and (x_2, y_2) — are different from each other and different from the system (n, t) .

In order to write the projections on axes of quantities \vec{D}_p and \vec{E} ,

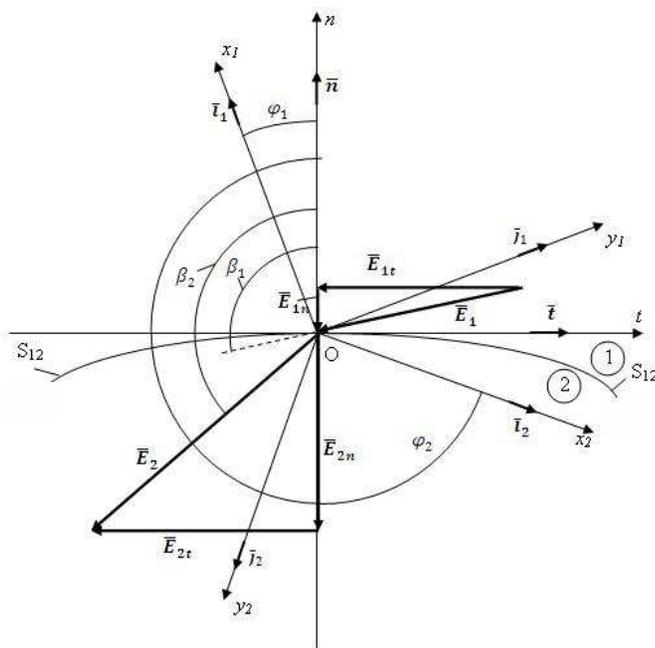


Figure 3. Refraction of \vec{E} .

where $\bar{\bar{\epsilon}}_{p\lambda} = \|\epsilon_{p\lambda x}\epsilon_{p\lambda y}\|$ are the tensors of calculation absolute permittivity in both dielectrics with permanent polarization. If we emphasize the components following the main directions (see also (10), where $\epsilon_0\epsilon_{rpv} = \epsilon_{pv}$), (14) becomes

$$D_{p\lambda\nu} = \epsilon_{p\lambda\nu}E_{\lambda\nu}, \quad \lambda = 1, 2 \text{ and } \nu = x, y \tag{15}$$

We remark that between \bar{D}_p and \bar{E} components, we can write relations similar to (15) only following the polarization main directions ($\nu = x$ or y), but not following n and t directions. We must specify that $D_{p\lambda\nu}$ and $E_{p\lambda\nu}$ are the projections of vectors $\bar{D}_{p\lambda}$ and \bar{E}_λ , following the polarization main directions, i.e., for the cases of 2D field showed in Figures 2 and 3 and the notices (11), we can write these equations (see also Figures 4 and 5):

$$\begin{aligned} \bar{D}_{p1} &= D_{p1x}\bar{i}_1 + D_{p1y}\bar{j}_1 \\ &= D_{p1} \cos(\alpha_1 - \varphi_1)\bar{i}_1 + D_{p1} \cos\left(\alpha_1 - \varphi_1 + \frac{\pi}{2}\right)\bar{j}_1 \\ &= D_{p1} [\cos(\alpha_1 - \varphi_1)\bar{i}_1 - \sin(\alpha_1 - \varphi_1)\bar{j}_1]; \\ \bar{D}_{p2} &= D_{p2x}\bar{i}_2 + D_{p2y}\bar{j}_2 \\ &= D_{p2} \cos(\alpha_2 + 2\pi - \varphi_2)\bar{i}_2 + D_{p2} \cos\left(\alpha_2 + 2\pi - \varphi_2 + \frac{\pi}{2}\right)\bar{j}_2 \\ &= D_{p2} [\cos(\varphi_2 - \alpha_2)\bar{i}_2 + \sin(\varphi_2 - \alpha_2)\bar{j}_2]; \\ \bar{E}_1 &= E_{1x}\bar{i}_1 + E_{1y}\bar{j}_1 \\ &= E_1 \cos(\beta_1 - \varphi_1)\bar{i}_1 + E_1 \cos\left(\beta_1 - \varphi_1 + \frac{\pi}{2}\right)\bar{j}_1 \\ &= E_1 [\cos(\beta_1 - \varphi_1)\bar{i}_1 - \sin(\beta_1 - \varphi_1)\bar{j}_1]; \\ \bar{E}_2 &= E_{2x}\bar{i}_2 + E_{2y}\bar{j}_2 \\ &= E_2 \cos(\beta_2 + 2\pi - \varphi_2)\bar{i}_2 + E_2 \cos\left(\beta_2 + 2\pi - \varphi_2 + \frac{\pi}{2}\right)\bar{j}_2 \\ &= E_2 [\cos(\varphi_2 - \beta_2)\bar{i}_2 + \sin(\varphi_2 - \beta_2)\bar{j}_2]. \end{aligned} \tag{16}$$

It is obvious that D_{p1} , D_{p2} , E_1 and E_2 are the modules of vectors, so they are positive scalars. The components $D_{p\lambda\nu}$ and $E_{\lambda\nu}$ ($\lambda = 1, 2$; $\nu = x, y$) are positive or negative scalars, as result of (16), Figures 4, and 5, i.e., for the case considered, we obtain the following relations:

$$\begin{aligned} D_{p1x} &= D_{p1} \cos(\alpha_1 - \varphi_1) > 0; & D_{p1y} &= -D_{p1} \sin(\alpha_1 - \varphi_1) < 0; \\ D_{p2x} &= D_{p2} \cos(\varphi_2 - \alpha_2) < 0; & D_{p2y} &= D_{p2} \sin(\varphi_2 - \alpha_2) > 0; \\ E_{1x} &= E_1 \cos(\beta_1 - \varphi_1) < 0; & E_{1y} &= -E_1 \sin(\beta_1 - \varphi_1) < 0; \\ E_{2x} &= E_2 \cos(\varphi_2 - \beta_2) < 0; & E_{2y} &= E_2 \sin(\varphi_2 - \beta_2) > 0. \end{aligned} \tag{17}$$

In Figures 2 and 3, only the components following the axes (n, t) are represented. In Figures 4 and 5 (at another scale and without

unit vectors), the components following the polarization main axes (x, y, z) are also represented. Figures 4 and 5 illustrate the components of vectors \overline{D}_{p1} and \overline{D}_{p2} , respectively \overline{E}_1 and \overline{E}_2 , which appear in relations.

If we write the vectors $\overline{D}_{p\lambda}$ and \overline{E}_λ depending on their components following the rectangular system (n, t) , we could write the equations:

$$\begin{aligned} \overline{D}_{p\lambda} &= D_{p\lambda n}\overline{n} + D_{p\lambda t}\overline{t}; \\ \overline{E}_\lambda &= E_{\lambda n}\overline{n} + E_{\lambda t}\overline{t}, \quad \lambda = 1, 2, \end{aligned} \tag{18}$$

where the components may be positive or negative, depending on the considered case.

3.1. The Refraction Theorem of Electric Field Strength Lines \overline{E}

The normal component of electric flux density \overline{D} in medium 1 can be written as sum of projections on normal direction of the two components following the polarization main directions (D_{1x} and D_{1y}):

$$D_{1n} = D_{1xn} + D_{1yn}. \tag{19}$$

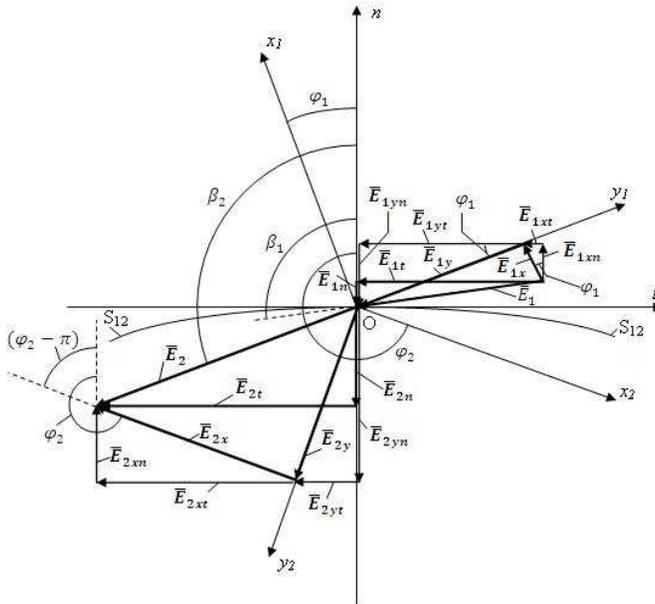


Figure 5. Components of \overline{E} .

Writing (5) for medium 1 ($\bar{D}_1 = \bar{D}_{p1} + \bar{P}_{p1}$), the two components are:

$$D_{1xn} = D_{p1xn} + P_{p1xn}, \quad D_{1yn} = D_{p1yn} + P_{p1yn}, \quad (20)$$

where D_{p1xn} and D_{p1yn} are the projections on normal axe (n) of the components D_{p1x} and D_{p1y} of the calculation electric flux density \bar{D}_{p1} following the polarization main directions (x_1, y_1) in medium 1; P_{p1xn} and P_{p1yn} similarly, but regarding the permanent polarization \bar{P}_{p1} of the medium 1. These are illustrated in Figure 4 and Equation (21) — for \bar{D}_{p1} — and in the Equation (22) for \bar{P}_{p1} .

$$\begin{aligned} \bar{D}_{p1} &= \bar{D}_{p1x} + \bar{D}_{p1y}, \\ \bar{D}_{p1x} &= D_{p1x}\bar{i}_1 = D_{p1xn}\bar{n} + D_{p1xt}\bar{t} = \bar{D}_{p1xn} + \bar{D}_{p1xt}, \end{aligned} \quad (21)$$

$$\begin{aligned} \bar{D}_{p1y} &= D_{p1y}\bar{j}_1 = D_{p1yn}\bar{n} + D_{p1yt}\bar{t} = \bar{D}_{p1yn} + \bar{D}_{p1yt}; \\ \bar{P}_{p1} &= \bar{P}_{p1x} + \bar{P}_{p1y}, \\ \bar{P}_{p1x} &= P_{p1x}\bar{i}_1 = P_{p1xn}\bar{n} + P_{p1xt}\bar{t}, \\ \bar{P}_{p1y} &= P_{p1y}\bar{j}_1 = P_{p1yn}\bar{n} + P_{p1yt}\bar{t}. \end{aligned} \quad (22)$$

The permanent polarizations are not represented in Figures 4 and 5, because their normal/tangent components are not preserved. Their values and directions are generally arbitrary.

From (19) and (20), we obtain

$$D_{1n} = (D_{p1xn} + D_{p1yn}) + (P_{p1xn} + P_{p1yn}), \quad (23)$$

where, for Figure 4, the components are:

$$D_{p1xn} = D_{p1x} \cos \varphi_1 > 0, \quad D_{p1yn} = D_{p1y} \sin \varphi_1 < 0.$$

Taking into account these equations and (15), (23) becomes

$$\begin{aligned} D_{1n} &= D_{p1x} \cos \varphi_1 + D_{p1y} \sin \varphi_1 + (P_{p1xn} + P_{p1yn}) \\ &= \varepsilon_{p1x} E_{1x} \cos \varphi_1 + \varepsilon_{p1y} E_{1y} \sin \varphi_1 + (P_{p1xn} + P_{p1yn}). \end{aligned} \quad (24)$$

For normal component of electric flux density in medium 2, we can write

$$D_{2n} = D_{2xn} + D_{2yn}. \quad (25)$$

Writing (5) for medium 2 ($\bar{D}_2 = \bar{D}_{p2} + \bar{P}_{p2}$), the two components are:

$$D_{2xn} = D_{p2xn} + P_{p2xn}, \quad D_{2yn} = D_{p2yn} + P_{p2yn}, \quad (26)$$

where D_{p2xn} and D_{p2yn} are the projections on normal axe (n) of the components D_{p2x} and D_{p2y} of the calculation electric flux density \bar{D}_{p2} following the polarization main directions (x_2, y_2) in medium 2; P_{p2xn} and P_{p2yn} similarly, but regarding the permanent polarization \bar{P}_{p2} of

medium 2. These are illustrated in Figure 4 and (27) — for \overline{D}_{p2} — and in (28) for \overline{P}_{p2} .

$$\begin{aligned}\overline{D}_{p2} &= \overline{D}_{p2x} + \overline{D}_{p2y}, \\ \overline{D}_{p2x} &= D_{p2x}\bar{i}_2 = D_{p2xn}\bar{n} + D_{p2xt}\bar{t} = \overline{D}_{p2xn} + \overline{D}_{p2xt},\end{aligned}\quad (27)$$

$$\begin{aligned}\overline{D}_{p2y} &= D_{p2y}\bar{j}_2 = D_{p2yn}\bar{n} + D_{p2yt}\bar{t} = \overline{D}_{p2yn} + \overline{D}_{p2yt}; \\ \overline{P}_{p2} &= \overline{P}_{p2x} + \overline{P}_{p2y}, \\ \overline{P}_{p2x} &= P_{p2x}\bar{i}_2 = P_{p2xn}\bar{n} + P_{p2xt}\bar{t},\end{aligned}\quad (28)$$

$$\overline{P}_{p2y} = P_{p2y}\bar{j}_2 = P_{p2yn}\bar{n} + P_{p2yt}\bar{t}.$$

From (25) and (26), we obtain

$$D_{2n} = (D_{p2xn} + D_{p2yn}) + (P_{p2xn} + P_{p2yn}), \quad (29)$$

where, for Figure 4, the components are:

$$\begin{aligned}D_{p2xn} &= |D_{p2x}| \cos(\varphi_2 - \pi) = -|D_{p2x}| \cos \varphi_2 > 0; \\ D_{p2yn} &= D_{p2y} \cos\left(\varphi_2 - \frac{\pi}{2}\right) = D_{p2y} \sin \varphi_2 < 0.\end{aligned}$$

Taking into account these (where $-|D_{p2x}| = D_{p2x} < 0$) and (15), (29) becomes

$$\begin{aligned}D_{2n} &= D_{p2x} \cos \varphi_2 + D_{p2y} \sin \varphi_2 + (P_{p2xn} + P_{p2yn}) \\ &= \varepsilon_{p2x} E_{2x} \cos \varphi_2 + \varepsilon_{p2y} E_{2y} \sin \varphi_2 + (P_{p2xn} + P_{p2yn}).\end{aligned}\quad (30)$$

By replacing (24) and (30) in (12), we obtain

$$\begin{aligned}(\varepsilon_{p1x} E_{1x} \cos \varphi_1 - \varepsilon_{p2x} E_{2x} \cos \varphi_2) + (\varepsilon_{p1y} E_{1y} \sin \varphi_1 - \varepsilon_{p2y} E_{2y} \sin \varphi_2) \\ + (P_{p1xn} - P_{p2xn}) + (P_{p1yn} - P_{p2yn}) = 0.\end{aligned}\quad (31)$$

If we emphasize the projections on normal direction of the vectors \overline{E}_λ ($\lambda = 1, 2$), from (31) we write

$$\begin{aligned}(\varepsilon_{p1x} E_{1xn} - \varepsilon_{p2x} E_{2xn}) + (\varepsilon_{p1y} E_{1yn} - \varepsilon_{p2y} E_{2yn}) + (P_{p1xn} - P_{p2xn}) \\ + (P_{p1yn} - P_{p2yn}) = 0,\end{aligned}\quad (32)$$

where:

$$\begin{aligned}E_{1xn} &= E_{1x} \cos \varphi_1 > 0; & E_{2xn} &= E_{2x} \cos \varphi_2 > 0; \\ E_{1yn} &= E_{1y} \sin \varphi_1 < 0; & E_{2yn} &= E_{2y} \sin \varphi_2 < 0.\end{aligned}\quad (33)$$

The components $D_{p\lambda\nu n}$, $E_{\lambda\nu n}$ and $P_{p\lambda\nu n}$ ($\lambda = 1, 2$; $\nu = x, y$) are positive or negative depending on the particular laying of vectors $\overline{D}_{p\lambda}$, \overline{E}_λ and $\overline{P}_{p\lambda}$ in comparison with the axes systems. We must remark that the sign (> 0 or < 0) of scalars $D_{p\lambda\nu n}$ and $E_{\lambda\nu n}$ could be easy to find for particular cases considered in Figures 4 and 5. Alike, we should

proceed also with $P_{p\lambda\nu n}$, the components of permanent polarization $\overline{P}_{p\lambda}$ of the two media. $P_{p\lambda\nu n}$ are no longer represented because of the reasons specified previously (please see the explanations included after relation (22)).

Consequently, the normal components of the electric field strength \overline{E} (components which are not conserved) follow (32); this relation will be named *the refraction theorem of the 2D electric field strength lines, in media with permanent polarization, anisotropic by orthogonal directions*.

3.2. The Refraction Theorem of Calculation Electric Flux Density Lines \overline{D}_p

The tangent component of electric field strength in medium 1 could be written as a sum of projections on tangent directions of the two components following the polarization main directions (E_{1x} and E_{1y}):

$$E_{1t} = E_{1xt} + E_{1yt}. \tag{34}$$

Regarding the meaning of E_{1xt} and E_{1yt} , the case of Figure 4, as well as (15), (34), becomes

$$E_{1t} = -E_{1x} \sin \varphi_1 + E_{1y} \cos \varphi_1 = -\frac{D_{p1x}}{\varepsilon_{p1x}} \sin \varphi_1 + \frac{D_{p1y}}{\varepsilon_{p1y}} \cos \varphi_1. \tag{35}$$

Alike, for the tangent component of \overline{E} in medium 2, we can write the following relation:

$$\begin{aligned} E_{2t} &= E_{2xt} + E_{2yt} = |E_{2x}| \cos \left(\varphi_2 - \pi + \frac{\pi}{2} \right) + E_{2y} \cos \varphi_2 \\ &= -E_{2x} \sin \varphi_2 + E_{2y} \cos \varphi_2 = -\frac{D_{p2x}}{\varepsilon_{p2x}} \sin \varphi_2 + \frac{D_{p2y}}{\varepsilon_{p2y}} \cos \varphi_2. \end{aligned} \tag{36}$$

By replacing (35) and (36) in (13), the following relation is obtained:

$$\left(-\frac{D_{p1x}}{\varepsilon_{p1x}} \sin \varphi_1 + \frac{D_{p2x}}{\varepsilon_{p2x}} \sin \varphi_2 \right) + \left(\frac{D_{p1y}}{\varepsilon_{p1y}} \cos \varphi_1 - \frac{D_{p2y}}{\varepsilon_{p2y}} \cos \varphi_2 \right) = 0. \tag{37}$$

If we put into evidence the projections on the tangent of the components following the magnetization main axes for $\overline{D}_{p\lambda}$ ($\lambda = 1, 2$), from (37) we obtain:

$$\left(\frac{D_{p1xt}}{\varepsilon_{p1x}} - \frac{D_{p2xt}}{\varepsilon_{p2x}} \right) + \left(\frac{D_{p1yt}}{\varepsilon_{p1y}} - \frac{D_{p2yt}}{\varepsilon_{p2y}} \right) = 0, \tag{38}$$

where:

$$\begin{aligned} D_{p1xt} &= -D_{p1x} \sin \varphi_1 < 0; & D_{p2xt} &= -D_{p2x} \sin \varphi_2 < 0; \\ D_{p1yt} &= D_{p1y} \cos \varphi_1 < 0; & D_{p2yt} &= D_{p2y} \cos \varphi_2 < 0. \end{aligned} \tag{39}$$

Components $D_{p\lambda\nu t}$ and $E_{\lambda\nu t}$ ($\lambda = 1, 2$; $\nu = x, y$) are positive or negative depending on the particular laying of vectors $\overline{D}_{p\lambda}$ and \overline{E}_λ in comparison with the axes systems.

Consequently, the tangent components of calculation electric flux density \overline{D}_p follow (38); this equation will be named *the theorem of 2D refraction of calculation electric flux density \overline{D}_p lines, in media with permanent polarization, anisotropic by orthogonal directions*. Since the materials are anisotropic, we can observe that the spectra lines of \overline{D}_p and \overline{E} are different. We should remark that the theorem (38) has a simpler form (see also [4], Equation (20)) than the refraction theorem of the electric flux density lines where we have considered the classical quantities (\overline{D} and $\overline{\varepsilon}$). So, in addition to the advantages shown in paragraph 2, the introduction of new quantities (\overline{D}_p and $\overline{\varepsilon}_p$) are helping us to express the refraction theorem in a simpler form.

If we write (5) for the normal components, in the two media, we obtain the relation

$$D_{p1n} = D_{p2n} - (P_{p1n} - P_{p2n}), \quad (40)$$

which reveals that, for different normal components of the permanent polarization in the two media ($P_{p1n} \neq P_{p2n}$), the normal components of vector \overline{D}_p are not preserved, even if the separation surface S_{12} has no free electric charge (under these conditions, the relation (12) was written).

4. PARTICULAR CASES OF THE REFRACTION THEOREMS

4.1. 2D Field in Isotropic Media with Permanent Polarization

For isotropic media, the calculation permittivity in the two materials is:

$$\varepsilon_{p1x} = \varepsilon_{p1y} = \varepsilon_{p1}; \quad \varepsilon_{p2x} = \varepsilon_{p2y} = \varepsilon_{p2}. \quad (41)$$

If we take into account (41), the theorem (32) for refraction of electric field strength lines becomes

$$\begin{aligned} &\varepsilon_{p1}(E_{1xn} + E_{1yn}) - \varepsilon_{p2}(E_{2xn} + E_{2yn}) \\ &+ [(P_{p1xn} + P_{p1yn}) - (P_{p2xn} + P_{p2yn})] = 0. \end{aligned} \quad (42)$$

Considering the significations of (32), (42) may be written in this way:

$$\varepsilon_{p1}E_{1n} = \varepsilon_{p2}E_{2n} - (P_{p1n} - P_{p2n}). \quad (43)$$

In case of isotropic dielectrics, (5) and (8), written for both media, become: $\overline{D}_{p\lambda} = \overline{D}_\lambda - \overline{P}_{p\lambda} = \varepsilon_{p\lambda} \overline{E}_\lambda$. In this case, between the normal components we can write $D_{p\lambda n} = \varepsilon_{p\lambda} E_{\lambda n} = D_{\lambda n} - P_{p\lambda n}$ or $D_{\lambda n} = \varepsilon_{p\lambda} E_{\lambda n} + P_{p\lambda n}$ ($\lambda = 1, 2$). Consequently, after a regrouping of the theorems, from (43) we track down (12), as we expect.

Alike, taking into account (40), the theorem (38) for refraction of calculation electric flux density lines becomes

$$\frac{1}{\varepsilon_{p1}}(D_{p1xt} + D_{p1yt}) - \frac{1}{\varepsilon_{p2}}(D_{p2xt} + D_{p2yt}) = 0. \tag{44}$$

Considering the significations of (38), (44) can be written in a more concise form:

$$\frac{D_{p1t}}{\varepsilon_{p1}} = \frac{D_{p2t}}{\varepsilon_{p2}}. \tag{45}$$

Equations (43) and (45) represent the theorems of refraction for \overline{E} and \overline{D}_p in 2D fields, for isotropic dielectrics with permanent polarization. We can remark that, for the tangent components of \overline{D}_p , *the theorem (45) of refraction in dielectrics with permanent polarization has a similar form (but another content) with the classical theorem of refraction in materials without permanent polarization.*

4.2. 2D Fields in Isotropic Media without Permanent Polarization

In this case, for $\overline{P}_p = 0$, from (5) we obtain $\overline{D}_p \equiv \overline{D}$. Also, from (8), for isotropic media we can write $\varepsilon_p = D_p/E = D/E$. So $\varepsilon_p = \varepsilon$, which means that the calculation permittivity is identical with the classical permittivity. Particularizing (43) and (45) for this case and taking into account the previous observations, we obtain

$$\frac{\varepsilon_{p1}}{\varepsilon_{p1}} = \frac{D_{p1t}}{D_{p2t}} = \frac{E_{2n}}{E_{1n}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{D_{1t}}{D_{2t}}, \tag{46}$$

that is the classical form of the refraction theorem for electric field lines (2D field, without permanent polarization, isotropic dielectrics). We should mention that in this case the lines spectra of the electric flux density \overline{D} and of the electric field strength \overline{E} are identical and refracting in the same way.

It is easy to remark that, from the general expression of the refraction theorem of \overline{D}_p and \overline{E} or the particular forms already mentioned, we can also obtain other particular forms. Such cases could be those in which one of the media has permanent polarization and the other one does not, when the permanent polarization vectors have particular orientations, etc.

5. CONCLUSIONS

The introduction of a new relative permittivity for dielectrics with permanent polarization (as described in Section 2 of this paper) is a useful operation, because the solution of field problem for nonlinear systems with permanent polarization can be obtained more easily. The relations obtained are more concise, so simpler. Also, it is possible to make useful analogies with the simpler cases of the materials without permanent polarization.

As an application, for anisotropic materials with orthogonal main directions of polarization and also with permanent polarization, the author has demonstrated new refraction theorems for 2D electric field, which are given by (32) (for electric field strength \bar{E}), and by (38) (for calculation electric flux density \bar{D}_p). Starting from these general forms of the theorems, some particular forms have been deduced: for 2D field in isotropic dielectrics, with permanent polarization, respectively for isotropic dielectrics without permanent polarization (par. 4), and another are possible.

Similar theorems were demonstrated by the same author (see [5–7]) for the magnetic field lines refraction in materials with permanent magnetization, i.e., permanent magnets.

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