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LINEAR DIFFUSION INTO A FARADAY CAGE

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Abstract—In this paper, linear lightning diffusion into a Faraday cage is studied. The high-altitude Electromagnetic Pulse (HEMP) and nearby lightning are used as examples for a uniform field drive and the direct-strike lightning adjacent to the enclosure is used as a worst-case configuration of a line source excitation. The time-derivative of the magnetic field ($HDOT$) inside the enclosure for a uniform field drive with a decaying exponential waveform is analyzed and numerically determined. The physically relevant time-derivative of the magnetic field and voltage characterizations of an optimum coupling loop inside the enclosure for a decaying exponential waveform in a worst-case line source coupling configuration are numerically determined. First, the impulse and the unit step response peaks are shown to bound the decaying exponential peaks. Next, a simple fit function for a decaying exponential peak $HDOT$ or a voltage bound for a single-turn loop inside the Faraday cage is constructed from peak responses of the unit step and impulse limiting cases. Excitations used are from (1) a uniform field drive of HEMP or nearby lightning and (2) a line source of direct-strike lightning. Comparisons of $HDOT$ and voltage bounds of the fit function and actual numerical evaluations are given in Table 3.

1. INTRODUCTION

An impulse is typically used as an idealized waveform for approximating a HEMP when treating the magnetic diffusion into a metallic enclosure [1]. A unit step on the other hand is typically used to calculate the maximum voltage induced on an optimum coupling loop inside a metallic enclosure and on the opposite side of the enclosure wall by lightning [2]. The rationale of these treatments is clear. A thick enclosure wall allows the use of an impulse; a thin wall needs to

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Table 1. $\alpha\tau_d$ values for different enclosure wall thicknesses and for different lightning decay constants.

Δ (wall thickness in inches)	$\tau_d\,(=\Delta^2\mu\rho)$ for 6061 Aluminum Alloy	$\alpha\tau_d$	
		$\alpha = 3466$ (1% lightning)	$\alpha = 13864$ (50% lightning)
1/2	5.27 ms	18.27	73
1/4	1.32 ms	4.567	18.27
1/8	329 µs	1.142	4.567
1/16	82 µs	0.2855	1.142
1/32	21 µs	0.0714	0.2855

use a unit step. The questions that arise in deciding which approach to take are, "What is the quantitative criterion for determining which one is more accurate? What is the relevant parameter? What errors are incurred if the criterion is violated?"

The parameter most relevant to the diffusion penetration is the diffusion time ($\tau_d = \nabla^2\mu\sigma$) of the enclosure wall where μ, σ and Δ are the permeability, conductivity and thickness of the wall material. Table 1 lists various $\alpha\tau_d$ where α is the decay constant of the lightning waveform. All of these thicknesses are used in various aerospace applications. The table shows that the diffusion time is both large and small compared to the fall time of the lightning waveform, depending on the thickness, so that neither the impulse approximation, nor the step-function approximation is universally valid. Use of very thin conducting foils for shielding Electronics is very common; in this case, the unit-step model is quite adequate.

The decaying exponential waveform characterizes the principal energy contribution of naturally occurring physical phenomena, e.g., lightning. The decaying exponential response closely describes the transient behavior and is the response of interest in many physical problems. The decay constant α is inversely proportional to the fall time of the waveform. Therefore, $\alpha\tau_d$ is proportional to the ratio of diffusion time to the fall time and is the transition parameter. Limiting cases are simple and easier to calculate and serve as useful models. When $\alpha \to 0$ and the fall time goes to infinity, the decaying exponential becomes a unit step that contains low frequencies. This is considered a thin limit because low frequencies penetrate the enclosure wall. When $\alpha \to \infty$ and the fall time goes to zero, the decaying exponential becomes an impulse that contains high frequencies. In the thick limit, we consider how high frequencies penetrate the wall.

2. APPROACH

The Laplace transform of the enclosure interior field for a decaying exponential waveform in a normalized time (t/τ_d) is the product of $\frac{1}{\alpha\tau_d+s'}$ and the impulse response transform [3]. In the time domain, we can denote the decaying exponential enclosure interior field as H_{in}^e, where superscript "e" represents decaying exponential and subscript "in" represents interior. As $\alpha\tau_d \to 0$, H_{in}^e becomes H_{in}^s, which is the unit step enclosure interior field. The subscript "s" represents unit step. This is the thin limit and the penetrant interior magnetic field has a wide pulse. On the other hand, as $\alpha\tau_d \to \infty$, H_{in}^e becomes $H_{\text{in}}^i/(\alpha\tau_d)$. H_{in}^i is the impulse enclosure interior field and subscript "i" represents impulse. Note that $1/(\alpha\tau_d)$ is the moment of the time domain moment of the impulse. This is the thick limit and the penetrant interior magnetic field has a comparative narrow pulse. This consideration applies to the time-derivative magnetic field or the induced voltage on a loop inside the enclosure.

We vary $\alpha\tau_d$ through the transition range from thin to thick. Figure 1 illustrates the peak interior $HDOT$ for nearby lightning enclosure interior fields. Strictly speaking, the unit step response is only valid for $\alpha\tau_d = 0$ and the impulse response is only valid for $\alpha\tau_d \to \infty$; however, each of the models can be used to approximate the problem under study. The peak $HDOT$ determines the peak induced voltage and therefore we will emphasize $HDOT$ in our discussion. For voltage calculations in an externally uniform field drive like HEMP and nearby lightning, the spatial variation of the nearby lightning H and $HDOT$ inside the enclosure is assumed to be constant. For direct strikes, the spatial variation of the $HDOT$ waveform is, in general,

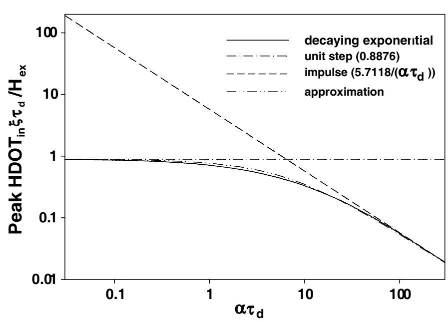

Figure 1. This figure shows the peak decaying exponential response compared to the peak unit step and the peak impulse responses for nearby lightning.

unknown and therefore the direct calculation of a voltage bound is also included.

The *HDOT* and the loop voltage are of the greatest significance in shielding and are the main parameters of our discussion.

3. MAIN RESULT

First, we note that both unit step and impulse responses give bounds for the decaying exponential response: Convolution integrals and elementary Calculus are used to prove two inequalities.

$$HDOT^s_{\text{in}}(t) - HDOT^e_{\text{in}}(t) = \int_0^t \left[1 - e^{-\alpha t'}\right] HDOT^i_{\text{in}}\left(t - t'\right) dt'$$

$$= \left[1 - e^{-\alpha t_0}\right] \int_0^t HDOT^i_{\text{in}}\left(t'\right) dt' > \left[1 - e^{-\alpha t_0}\right] HDOT^s_{\text{in}}(t) > 0. \quad (1)$$

$HDOT^i_{\text{in}}$ can only be negative after $HDOT^s_{\text{in}}(t)$ already attains the peak and therefore the integrand in (1) is strictly positive for the range of interest. The key step is the application of the mean-value theorem for integrals where $0 < t_0 < t$ [4]. Also,

$$HDOT^e_{\text{in}}(t) =$$

$$\int_0^t e^{-\alpha(t-t')} HDOT^i_{\text{in}}(t')dt' < HDOT^i_{\text{in}}(t_p) \int_0^t e^{-\alpha(t-t')} dt' < \frac{1}{\alpha} HDOT^i_{\text{in}}(t_p), \quad (2)$$

where t_p is the time corresponding to peak $HDOT^i_{\text{in}}$. From (1) and (2),

$$\text{peak}\, HDOT^e_{\text{in}} < \text{peak}\, HDOT^s_{\text{in}}; \;\; \text{peak}\, HDOT^e_{\text{in}} < \frac{1}{\alpha}\, \text{peak}\, HDOT^i_{\text{in}}. \quad (3)$$

Note that, if normalized time t/τ_d is used as an independent variable, the only modification to (3) is the replacement of α by $\alpha\tau_d$.

Next, we compared the unit step, impulse, and decaying exponential responses and found that approximate *HDOT* peaks for decaying exponentials can be obtained by combining the unit step coupling and the impulse coupling (treating them as independent, for the former is dominated by low frequencies and the latter is dominated by high frequencies). A parallel "combination" of the unit step coupling and the impulse coupling yields an approximate formula for peak *HDOT*:

$$\frac{1}{HDOT^e_{\text{in}}} = \frac{1}{HDOT^s_{\text{in}}} + \frac{1}{HDOT^i_{\text{in}}}, \quad (4)$$

where superscript "*e*" is for decaying exponential, superscript "*s*" is for unit step and superscript "*i*" is for impulse and subscript "*in*" is for interior field.

The inequalities (3) and the approximate fit function (4) are applicable to other physical quantities such as voltage bounds.

3.1. An Example of Fit Function for Uniform Field Drive

Figure 1 shows the peak decaying exponential response compared to the peak unit step and the peak impulse responses for nearby lightning. Note that 0.8876 is the peak response of coupling from the unit step, 5.7118 is the slope of the peak response of the coupling from the unit impulse and $1/(\alpha\tau_d)$ is the impulse moment. These numerical values are obtained from solving the limiting cases. Note the unit step and impulse intersect at $\alpha\tau_d = 6.4351$ where each waveform overestimates the peak derivative compared to the decaying exponential†. $\xi = 6.088$ is used for obtaining peak responses. The scale factor $\xi\tau_d$ is the product between the enclosure geometric factor (defined as $\xi = \frac{\mu_0}{\mu}\frac{V}{S\Delta}$) [1] and diffusion time. As an example, we consider a cylindrical enclosure with diameter of $2a = 2\,\text{ft}$ (0.61 m), length of $b = 6\,\text{ft}$ (1.83 m) and $\Delta = 20\,\text{mils}$ (0.5 mm), $\xi = 257.3$. Scaled peak responses are not very sensitive to the ξ value. In [3], small variations for different ξ's are discussed.

The approximation in Figure 1 makes use of (4) in combining the unit step contribution with the impulse contribution.

$$\text{Peak } HDOT^e_{\text{in}}(\xi\tau_d/H_{ex}) \approx \frac{1}{\frac{1}{0.8876} + \frac{\alpha\tau_d}{5.7118}}. \tag{5}$$

At the intersection value of $\alpha\tau_d$ (6.4351), the error is approximately a factor of 2 in either the impulse or step responses. Large errors can incur if the unit step is applied to the thick wall ($\alpha\tau_d$ is large) or if the impulse response is applied to thin wall ($\alpha\tau_d$ is small). HEMP has a decay constant $\alpha = 4 \times 10^6$ [5]. The intersection point corresponds to $\tau_d = 1.61\,\mu\text{s}$, or approximately 9 mil (0.23 mm) aluminum foil. For the impulse response to be accurate, the enclosure wall has to be at least 20 mils (0.5 mm) in thickness. Note that $\xi\tau_d = \Delta\frac{\mu_0\sigma V}{S}$ is proportional to Δ and therefore the peak $HDOT$ for a unit step is inversely proportional to Δ.

Consider now a HEMP (Electric Field Peak = 50 kV/m and Magnetic Field Peak = 133 A/m) incident on the cylindrical enclosure discussed before ($2a = 0.34\,\text{m}$ and $b = 1.83\,\text{m}$). We assume a wall thickness of $\Delta = 20\,\text{mils}$ (0.5 mm), a magnetic field perpendicular to the axis of the cylinder, and an optimum coupling loop oriented to capture a maximum penetrant magnetic flux. The induced voltage

† The corresponding intersection for peak H response is $\alpha\tau_d = 0.1458$. An adequate formula for describing $HDOT$ may not be adequate for describing H and vice versa.

is the time derivative of the magnetic flux through the loop. Let us use an impulse model as shown in Figure 1. The 5 mm (20 mil)-aluminum alloy wall has $\tau_d = 8.4\,\mu\text{s}$, $\alpha\tau_d = 33.6$ and the geometric factor $\xi = 257.3$,

$$V = \mu_0 \frac{dH}{dt} 2ab = \mu_0 HDOT_{\text{in}} 2ab = \frac{\mu_0 H_{ex}}{\xi\tau_d} \frac{5.7118}{\alpha\tau_d} 2ab \approx 14.5\,\text{mV}.$$

Equation (2) for calculating the voltage of a decaying exponential waveform gives 12.2 mV.

Similarly, let us assume that the peak magnetic field from the nearby lightning is 320 A/m. Using the same cylindrical enclosure for the HEMP problem, the induced voltage for the maximum coupling loop as defined before is 35 mV for the impulse model and 29.3 mV for the more accurate decaying exponential.

3.2. An Example For Line Source

The direct lightning model assumed is one for which the lightning current is adjacent to the enclosure but electrically insulated from the enclosure (Figure 2) [2]. The lightning channel may have high potential and the assumed lightning line source is not easily realizable for a small separation between the lightning carrying cable and the enclosure. However, the worst-case coupling can be approached when lightning strikes a well-insulated cable that is isolated from the enclosure but their separation is sufficient to withstand the high potential. The relevancy of the model should be based on the potentially physical configuration that might be susceptible to this particular threat.

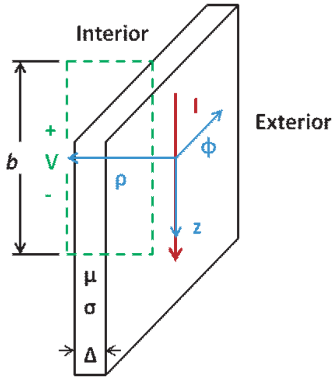

Figure 2. Direct lightning strike to an insulated cable parallel to the enclosure wall and a maximum coupling loop.

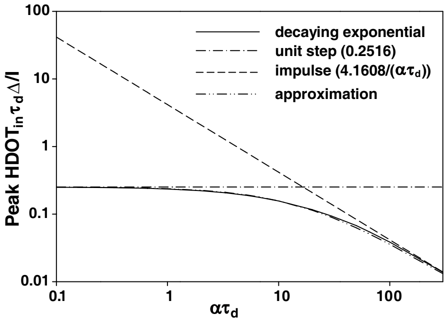

Figure 3. $HDOT$ peak ($\rho = \Delta$) for the decaying exponential and an approximation are compared to the unit step and impulse responses ($\mu = \mu_0$).

Peak $HDOT$ for a direct strike next to the enclosure is given in Figure 3. This figure is used to determine the maximum voltage induced on an optimally coupled loop. The unit step coupling peak is 0.2516 and the slope of the unit impulse coupling peak is 4.1608. The approximation in Figure 3 makes use of (4) for and $\rho = \Delta$ and $\mu = \mu_0$ as

$$\text{Peak } HDOT_{\text{in}}^{e}(\tau_d \Delta / I) \approx \frac{1}{\frac{1}{0.2516} + \frac{\alpha\tau_d}{4.1608}}. \tag{6}$$

Here "I" is the direct strike peak current. Peak $HDOT$ for a unit step is inversely proportional to Δ.

Note that the intersection of the unit step and the impulse peaks occurs at $\alpha\tau_d = 16.5374$ (Figure 3). For 1-percentile lightning, the decay constant is determined to be $\alpha = 3466$. A 1/2-inch aluminum wall thickness has $\alpha\tau_d = 18.27$ (Table 1). At this value of $\alpha\tau_d$, 1.83 m (6 ft) is b as defined in Figure 2.

$$\begin{aligned} V &< \int_{\Delta}^{\infty} \mu_0 HDOT_{\text{in}}|_{\rho=\Delta} \frac{\Delta^2 b d\rho}{\rho^2} = \mu_0 HDOT_{\text{in}}|_{\rho=\Delta} b\Delta \\ &= \frac{\mu_0 2 \times 10^5}{0.00527} 0.2516 \times 1.83 \approx 21.95\,\text{V} \end{aligned} \tag{7}$$

for the unit step case. The spatial dependence of $\frac{1}{\rho^2}$ is assumed for this unit step example. Equation (6) gives 10.4 V for the decaying exponential.

$HDOT$ values in the enclosure indicate what induced voltage on a given loop might be. However, because the spatial dependence of the $HDOT$ for a general excitation is unknown, the induced loop

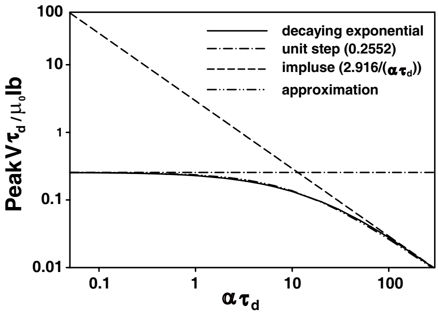

Figure 4. Peak voltage bound for direct strikes.

voltage waveform (that is the integration of the *HDOT* waveform on the loop area) cannot be accurately calculated. The voltage bound on an optimally coupled loop is a useful alternative for describing the enclosure interior direct strike lightning coupling (Figure 4). Note the close agreement between the unit step response of 0.2516 for *HDOT* (Figure 3) and the unit step response 0f 0.2552 for the voltage bound (Figure 4). This is because the *HDOT* for the unit step has an approximate spatial variation of ($\approx \frac{1}{\rho^2}$). The voltage bound in this case is inversely proportional to Δ^2.

Note that the intersection of the peak unit step voltage and the peak impulse voltage occurs at $\alpha\tau_d = 11.4263$. Nevertheless, the unit step induced voltage for the direct strike problem (Figure 2) just discussed can be calculated by (Figure 4)

$$V = 0.2552\frac{\mu_0}{\tau_d}Ib \approx \frac{0.2552 \times \mu_0}{0.00527}2 \times 10^5 \times 1.83 \approx 22.27\,\mathrm{V}$$

The peak impulse voltage (Figure 4) is somewhat smaller than the voltage obtained from peak *HDOT* (Figure 3) because *HDOT* from the impulse drops off much faster than $\frac{1}{\rho^2}$. The approximation in Figure 4 is a formula similar to (6):

$$V\tau_d/\mu_0 Ib \approx \frac{1}{\frac{1}{0.2552} + \frac{\alpha\tau_d}{2.916}}. \tag{8}$$

The more accurate decaying exponential voltage bound (8) gives $V \approx 0.0982\frac{\mu_0 Ib}{\tau_d} \approx 8.57\,\mathrm{V}$.

HDOT for $\rho = \Delta$ that gives the maximum induced voltage of an optimally coupled loop is a universal response and can be scaled for any enclosure wall thickness Δ: The impulse solution is scaled by $(\tau_d)^2\Delta$ and thus is inversely proportional to Δ^5. Similarly, the unit step

HDOT solution is inversely proportional to Δ^3. The corresponding induced voltage is scaled Δ^{-4} by for the impulse and Δ^{-2} for the unit step, which agrees with voltage bound (8). As an example, the induced voltage (4) for a 1/8-inch (3.175-mm) aluminum enclosure ($\alpha\tau_d = 1.142$) with the same geometry and 1/8-in wall thickness is $V \approx 0.232\frac{\mu_0 Ib}{\tau_d} \approx 324\,\mathrm{V}$.

We must emphasize at this point that there is no reliable way to know which one of the two models (the unit step or impulse) to use because the intersection point in Figures 1, 3 and 4 cannot be determined a priori. For example, how do we know $\alpha\tau_d$ of 1.142 is too small for using the impulse model? If the impulse model is used for calculating the peak voltage bound, the resulting estimated voltage is approximately a factor of 11 too high.

The procedure for deriving the approximate expression (4) for the enclosure interior peak *HDOT* as a function of $\alpha\tau_d$ can be used to address this situation. When both the unit step and impulse *HDOT* are known, an accurate fit function is available for use. Furthermore, the technique of using a fit function is applicable to other diffusion problems. For instance, the insulated conductor that is struck by lightning can be only a small distance away from the enclosure. In this case, solving for the unit step and impulse responses is considerably simpler than the decaying exponential response. The fit function for decaying exponentials can thus be constructed with the simpler unit step and impulse responses.

4. FIT FUNCTIONS FOR PEAK RESPONSES

Let us summarize the fit functions (5), (6) and (8) as

$$g^e = \frac{1}{\frac{\alpha\tau_d}{l} + \frac{1}{m}}, \quad g^i = \frac{l}{\alpha\tau_d}, \quad g^s = m. \tag{9}$$

where the subscript "e" for the decaying exponential, "i" for impulse and "s" for unit step and the parameters given in Table 2. "b" is the length of the loop in the direction of the lightning current as defined in Figure 2.

The 1st, 2nd and 6th row of data in Table 2 correspond to fit functions (5), (6) and (8), respectively. In Table 2, the numerical values l for are obtained from peaks of impulse responses, and those for m are from peaks of unit step responses, respectively.

Figure 5 as summarized in the 3rd row of data ($\rho = 10\Delta$) in Table 2 is relevant to the coupling loop located away from the enclosure wall. The intersection of the unit step and the impulse peaks occurs at $\alpha\tau_d = 7.15$, which is not very different from the nearby lightning value shown in Figure 1 but very different from the direct strike value for $\rho = \Delta$ (Figure 3). The closeness of these values in these two cases is closely related to the pulse widths of their unit step and impulse *HDOT* responses.

Table 2. Parameters for approximate formula (9) for peak *HDOT* and voltage bounds.

Environment	Physical Quantity	Field point	Permeability	Approximation Parameters		
				g	l	m
Nearby Lightning	*HDOT*	Everywhere inside enclosure	$\mu \le 10\mu_0$	$\frac{HDOT_{in}\,\xi\tau_d}{H_{ex}}$	5.7118	0.8876
Direct Lightning	*HDOT*	$\rho = \Delta$	$\mu = \mu_0$	$\frac{HDOT_{in}\,\tau_d\Delta}{I}$	4.1608	0.2516
Direct Lightning	*HDOT*	$\rho = 10\,\Delta$	$\mu = \mu_0$	$\frac{HDOT_{in}\rho^2\tau_d}{I\Delta}$	2.0997	0.2938
Direct Lightning	*HDOT*	$\rho = \Delta$	$\mu = 10\mu_0$	$\frac{HDOT_{in}\,\tau_d\Delta}{I}$	4.7065	0.2281
Direct Lightning	*HDOT*	$\rho = 10\,\Delta$	$\mu = 10\mu_0$	$\frac{HDOT_{in}\rho^2\tau_d}{I\Delta}$	11.8978	1.1272
Direct Lightning	Voltage Bound	Single-Turn Loop	$\mu = \mu_0$	$\frac{V\tau_d}{\mu_0 Ib}$	2.916	0.2552
Direct Lightning	Voltage Bound	Single-Turn Loop	$\mu = 10\mu_0$	$\frac{V\tau_d}{\mu_0 Ib}$	6.949	0.5167

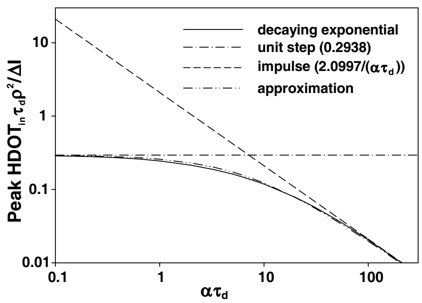

Figure 5. *HDOT* peak at $\rho = 10\Delta$ for decaying exponential and an approximation are compared to those of the unit step and the impulse excitations ($\mu = \mu_0$).

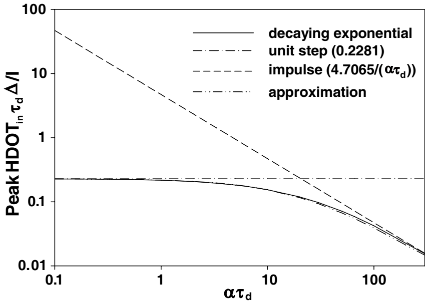

Figure 6. *HDOT* peak ($\rho = \Delta$) for the decaying exponential and an approximation are compared to the unit step and the impulse responses ($\mu = 10\mu_0$).

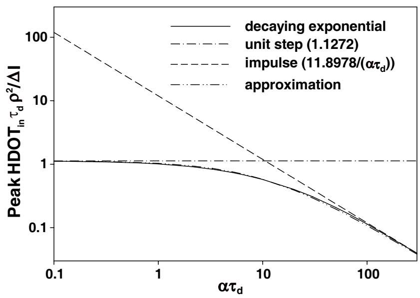

Figure 7. *HDOT* peak at $\rho = 10\Delta$ for decaying exponential and an approximation are compared to those of the unit step and the impulse excitations ($\mu = 10\mu_0$).

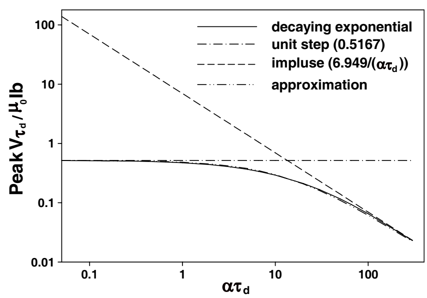

Figure 8. Peak voltage bounds for direct strikes ($\mu = 10\mu_0$).

Figures 6 and 7 or Row 4 and 5 of data in Table 2 give the corresponding comparison for $\mu = 10\mu_0$ and are applicable to magnetic steel with low permeability. Figure 8 gives the peak voltage bounds for $\mu = 10\mu_0$.

5. NUMERICAL RESULTS

The detailed analysis that leads to the numerical results reported in this section is given in [3]. For a magnetic uniform field drive, residue expansions of Bedrosian and Lee [1, 6] of the transient magnetic field transfer function into a metallic enclosure are numerically evaluated to give the enclosure interior magnetic field and its time derivative caused by a decaying exponential excitation.

For a direct strike scenario, the previous enclosure interior solutions for a worst-case line source configuration shown in Figure 2 [2] are extended to obtain numerical results for the magnetic field, its time derivative and the voltage bound for an optimally coupled single-turn loop inside the enclosure for a decaying exponential waveform. Table 3 provides the actual numerical value followed by the value from the fit function for various *HDOT* and voltage bounds.

The loop voltage of the HEMP example given previously was estimated to be 12.2 mV using the fit function (2). Notice the discrepancy between the actual numerical value and the approximation for $\alpha\tau_d = 30$ is 2% higher for the actual numerical value, resulting in a more accurate loop voltage 12.4 mV.

The loop voltage of the lightning problem for $1/2$-in enclosure is estimated from (7) to be 8.57 V. The discrepancy between the numerical value and the approximation for a voltage bound with $\alpha\tau_d = 20$ is *1.6%* higher for the numerical value, resulting in a loop voltage of 8.69 V.

Fit functions of peak H for nearby and direct lightning have also been constructed [3]; however, they are not as accurate as (6) for *HDOT*.

Table 3. Comparison of *HDOT* and voltage actual peaks with approximate formula (9). The actual peak is followed by the approximation indicated by (A) in each data entry.

$\alpha\tau_d$ value	Nearby *HDOT* Peak	Direct *HDOT* Peak $\rho = \Delta$ $\mu = \mu_0$	Direct *HDOT* Peak $\rho = 10\Delta$ $\mu = \mu_0$	Direct *HDOT* Peak $\rho = \Delta$, $\mu = 10\mu_0$	Direct *HDOT* Peak $\rho = 10\Delta$ $\mu = 10\mu_0$	Direct Voltage Bound $\mu = \mu_0$	Direct Voltage Bound $\mu = 10\mu_0$
0.05	0.8730 0.8808 (A)	0.2507 0.2508 (A)	0.2904 0.2918 (A)	0.2275 0.2275 (A)	1.1208 1.1219 (A)	0.2537 0.2541 (A)	0.5142 0.5148 (A)
0.066	0.8688 0.8786 (A)	0.2504 0.2506 (A)	0.2894 0.2911 (A)	0.2273 0.2274 (A)	1.1182 1.1202 (A)	0.2532 0.2537 (A)	0.5134 0.5142 (A)
0.1	0.8602 0.874 (A)	0.2498 0.2501 (A)	0.2871 0.2897 (A)	0.2269 0.2270 (A)	1.1138 1.1166 (A)	0.2522 0.253 (A)	0.5117 0.5129 (A)
0.2	0.8371 0.8608 (A)	0.2480 0.2486 (A)	0.2811 0.2858 (A)	0.2257 0.2259 (A)	1.1005 1.1062 (A)	0.2492 0.2508 (A)	0.5069 0.5091 (A)
0.33	0.8103 0.8443 (A)	0.2458 0.2467 (A)	0.2739 0.2808 (A)	0.2242 0.2245 (A)	1.0839 1.0930 (A)	0.2455 0.248 (A)	0.5009 0.5053 (A)
0.5	0.7805 0.8236 (A)	0.2430 0.2442 (A)	0.2654 0.2746 (A)	0.2221 0.2227 (A)	1.0636 1.0762 (A)	0.2410 0.2445 (A)	0.4933 0.4982 (A)
0.66	0.7562 0.805 (A)	0.2403 0.2419 (A)	0.2581 0.269 (A)	0.2203 0.2210 (A)	1.0458 1.0609 (A)	0.2371 0.2413 (A)	0.4865 0.4925 (A)
1	0.7111 0.7682 (A)	0.2351 0.2373 (A)	0.2446 0.2577 (A)	0.2166 0.2176 (A)	1.0097 1.0297 (A)	0.2292 0.2347 (A)	0.4730 0.4809 (A)
2	0.6155 0.6771 (A)	0.2214 0.2245 (A)	0.2144 0.2296 (A)	0.2066 0.2079 (A)	0.9220 0.9476 (A)	0.2099 0.2172 (A)	0.4389 0.4498 (A)
3.3	0.5325 0.5867 (A)	0.2064 0.2097 (A)	0.1871 0.201 (A)	0.1951 0.1966 (A)	0.8338 0.8587 (A)	0.1905 0.198 (A)	0.4033 0.4149 (A)
5	0.4575 0.4995 (A)	0.1902 0.1932 (A)	0.1620 0.1729 (A)	0.1824 0.1836 (A)	0.7448 0.7649 (A)	0.1710 0.1775 (A)	0.3664 0.3767 (A)
6.6	0.4061 0.4382 (A)	0.1776 0.1798 (A)	0.1444 0.1527 (A)	0.1722 0.1728 (A)	0.6794 0.6935 (A)	0.1565 0.1618 (A)	0.3382 0.3466 (A)
10	0.3304 0.3475 (A)	0.1563 0.1568 (A)	0.1183 0.1225 (A)	0.1542 0.1536 (A)	0.5751 0.5788 (A)	0.1334 0.1361 (A)	0.2922 0.2963 (A)
20	0.2157 0.2161 (A)	0.1167 0.1139 (A)	0.0780 0.0773 (A)	0.1190 0.1158 (A)	0.4001 0.3894 (A)	0.0941 0.0928 (A)	0.2112 0.2078 (A)
30	0.1603 0.1568 (A)	0.0934 0.0894 (A)	0.0583 0.0565 (A)	0.0972 0.0930 (A)	0.3073 0.2934 (A)	0.0729 0.0704 (A)	0.1660 0.1599 (A)

6. CONCLUSIONS

Faraday cages constructed as metallic enclosures must be evaluated for their attributes of shielding effectiveness against external electrical insults. In the case of transient insults such as HEMP and lightning, the enclosure interior $HDOT$ and voltage induced in a loop inside the enclosure are the most direct attributes for characterizing shielding.

Fit functions of peak $HDOT$ for a uniform magnetic field drive (such as nearby lightning) and for a line source (such as lightning attached to a metallic cable that is insulated from the enclosure) have been shown to give an accurate approximation to the actual numerical calculation. Fit functions of voltage bounds for a single-turn optimum coupling loop for a line source (such as from direct strike lightning) have been found to be accurate. These functions can be constructed from the unit step peak and impulse peak, which are considerably easier to obtain than the peak from a decaying exponential waveform.

These simple fit functions provide yardsticks for shielding effectiveness characterizations of metallic enclosures of varying thicknesses.

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