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MODELING SHIELDING EFFECTIVENESS FOR COMPOSITE WALLS OF CONCRETE AND CARBON FILAMENTS

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Abstract—Concrete walls reinforced with rebars have poor shielding effectiveness for telecommunication frequencies (frequencies above 0.5 GHz). An effective method to increase the shielding effectiveness of the walls is to increase the complex permittivity of the concrete. This can be done by mixing in thin filaments of a material with high conductivity. One such material is carbon. In this paper the Maxwell Garnett mixing rule is used to model a concrete material with carbon filaments. The shielding effectiveness computed with the mixing rule is found to agree with previously published measurement results.

1. INTRODUCTION

The reinforced concrete walls used in construction have a poor shielding effectiveness, usually between 0–20 dB depending heavily on design and frequency [1]. For low frequencies the rebars can act as a frequency selective surface with a high pass characteristic where the spacing between the rebars and concrete permittivity sets the cutoff frequency.

A very effective method to increase the shielding effectiveness at high frequencies, in this case 200–500 MHz and above, is to mix the concrete with an additive with much higher conductivity than the concrete. Various materials can be used to increase the electromagnetic shielding: a review of suitable materials can be found in [2]. The additives are commonly divided into two categories, metal filling cement materials and carbon filling cement materials. Very good shielding performance is possible with both categories; therefore the choice of material must be based on other factors, e.g., mechanical,

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economical and how prone the material is to corrosion. Steel additives have the advantage of being magnetic and can therefore also be used for low frequency magnetic shielding but are prone to corrosion. A neat solution to the corrosion problem was presented in [3] where "stainless steel" fibers were used. However, a stainless alloy usually has a much lower permeability than normal steel.

Due to the corrosion stability, graphite is a very useful material and several experimental papers have been written describing mixtures of different types of carbon particles, see, e.g., [4–7]. From these papers it is apparent that additives can dramatically improve the shielding effectiveness of a wall, e.g., a 4 mm concrete layer has a shielding effectiveness of 0.4 dB at 1 GHz, and 20 dB when carbon filaments are added to the concrete. Furthermore the shape of the particles in the additives is important; narrow fibers result in higher shielding effectiveness than thicker fibers for a given volume fraction.

What are lacking in the previous work are material models for the concrete with additives. In this paper the Maxwell Garnett mixing rule is used to model the effective permittivity of the concrete carbon fibre composite materials. The effective permittivity can then be used to compute electromagnetic propagation as if the material was homogeneous. The Maxwell Garnett mixing rule has recently been used to analyze the density of steel fibres in cement [8], where it was shown that the permittivity of the material was a linear function of the material density. In [8] the polarizability of the steel fibers is solved numerically. However, a fiber can be approximated by a prolate ellipsoid for which the polarizability can be solved analytically. The latter approximation is used in this paper which results in an easily implemented analytical expression. The prolate ellipsoid approximation has previously been used to model carbon fibers in teflon as a method to increase the shielding effectiveness, see [9]. In [9] the composites were modeled using the Maxwell Garnett or the McLachlan formulation depending on the additive concentration. In this paper the focus is on composites with concentrations below the percolation threshold and only the Maxwell Garnet mixing rule is considered. The Maxwell Garnett mixing rule [10] is far from exact and should not be used without validation of the model against experimental data. In this paper we will compare our analytical results with published experimental results. The analytical results are shown to agree with measurements made in [6]. To be able to model the composite material we used the concrete models given by Robert [11] and Ogunsola et al. [12].

2. THE MAXWELL GARNETT MIXING RULE APPLIED TO CONCRETE AND CARBON FILAMENT COMPOSITES

The Maxwell Garnett mixing rule is based on the assumption that the inclusions that are mixed into the bulk material are evenly distributed and are small in terms of wavelength. The latter assumption is necessary since the mixing rule uses the polarizability of the particles, which is the particles' static response to an external electric field. The effective permittivity of the composite material will depend on the permittivity of the inclusions and bulk material, the shape of the particles, and the particles orientation in the bulk material.

Both concrete and carbon are isotropic materials, whose complex permittivity are here described by ε_{e} and ε_{i} respectively. However, if the inclusions, the fibers or filaments, are non-spherical in shape and align in some direction the composite material will become anisotropic. The inclusions that may be different in shape and orientation are divided into types denoted with the index k. For such a material the effective permittivity is tensor that can be written as

$$\bar{\bar{\varepsilon}}_{\mathit{eff}} = \sum_{i=x,y,z} \varepsilon_{\mathit{eff}\,i} \mathbf{u}_i \mathbf{u}_i, \tag{1}$$

where $\mathbf{u}_i$ is the unit vector codirectional with the i-axis. The permittivity in the i-direction is given by

$$\varepsilon_{\mathit{eff}\,i} = \frac{\varepsilon_{\mathrm{i}} \sum_k (f_k A_{ki}) + \varepsilon_{\mathrm{e}} \left(1 - \sum_k (f_k)\right)}{\sum_k (f_k A_{ki}) + \left(1 - \sum_k (f_k)\right)}, \tag{2}$$

where f_k is the volume fraction of inclusion type k, and A_{ki} is the absolute value of the fraction of the internal field in the inclusion of type k and the external i-directed electric field, i.e., $A_{ki} = \frac{|E_{\mathrm{i}ki}|}{|E_{\mathrm{e}i}|}$.

A common special case is when all the particles are identical in size and shape and they are mixed into the bulk material so their orientations are random. Then the randomness of the material can be modeled as if the particles are divided into three groups where each group is aligned with one of the cartesian-axis. The effective complex permittivity is then given by

$$\varepsilon_{\mathit{eff}\,i} = \left\{ f_k = \frac{f}{3} \right\} = \frac{\frac{f}{3} \varepsilon_{\mathrm{i}} \sum_k (A_{ki}) + \varepsilon_{\mathrm{e}} (1 - f)}{\frac{f}{3} \sum_k (A_{ki}) + (1 - f)}. \tag{3}$$

The fraction between the electric field in an inclusion and the external field, A_i, can be computed numerically using a static solver. However, for ellipsoid's it is possible to compute the fraction of the fields analytically. Ellipsoids are extremely useful since it possible to find good approximations of different inclusions by changing the semi-axis [10], e.g., a filament can be modeled as a prolate spheroid and a thin chip as an oblate spheroid. This is done by choosing values for the ellipsoids semi-axis so that the diameter and length (thickness) are the same as for the inclusions. For an ellipsoid with semi-axis a_x, a_y, and a_z the electric-field fraction in the i-direction is

$$A_i = \frac{\varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{e}} + N_i\left(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}\right)}, \tag{4}$$

where N_i is the depolarization factor. The depolarization factor in the x-direction for an ellipsoid with the semi-axis aligned with cartesian-axis is

$$N_x = \frac{a_x a_y a_z}{2} \int_0^\infty \frac{ds}{\left(s + a_x^2\right)\sqrt{\left(s + a_x^2\right)\left(s + a_y^2\right)\left(s + a_z^2\right)}}. \tag{5}$$

For prolate spheroids ($a_x > a_y = a_z$) the depolarization factors are,

$$N_x = \frac{1 - g^2}{2g^3}\left(\ln\frac{1+g}{1-g} - 2g\right) \tag{6}$$

and

$$N_y = N_z = \frac{1}{2}\left(1 - N_x\right), \tag{7}$$

where

$$g = \sqrt{1 - a_y^2/a_x^2}. \tag{8}$$

A material of randomly oriented ellipsoids will become an isotropic material. By inserting Equation (4) we get

$$\varepsilon_{\mathit{eff}} = \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{e}} \frac{\frac{f}{3}\sum\limits_{j=x,y,z} \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{e}} + N_j\left(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}\right)}}{1 - \frac{f}{3}\sum\limits_{j=x,y,z} \frac{N_j\left(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}\right)}{\varepsilon_{\mathrm{e}} + N_j\left(\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}\right)}}, \tag{9}$$

where the depolarization factors are for an ellipsoid aligned with one axis.

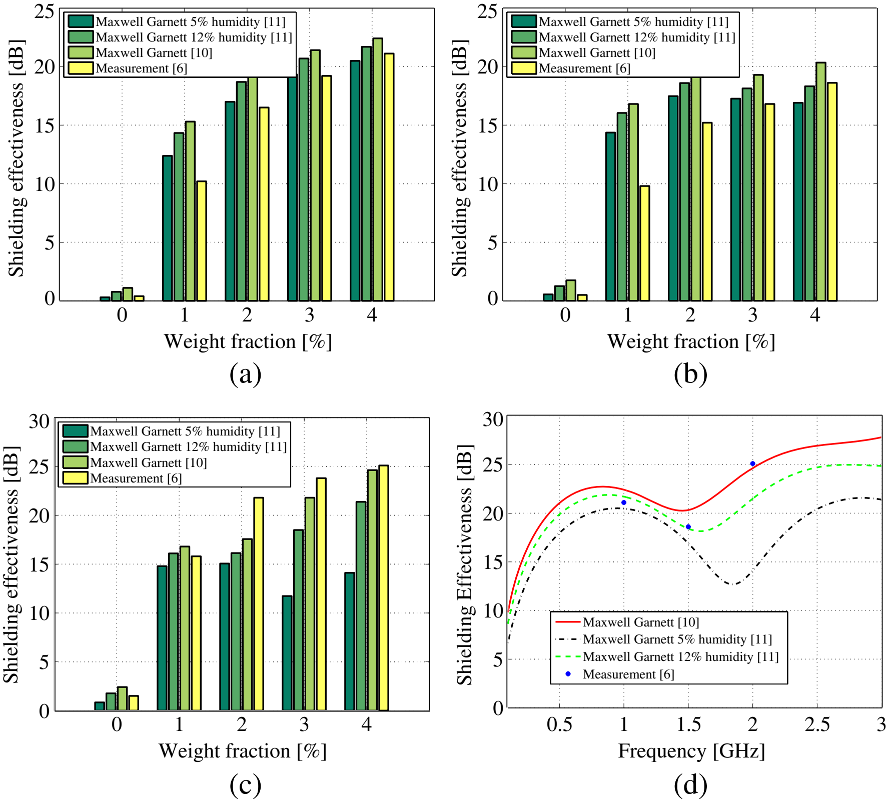
3. VERIFICATION AGAINST MEASUREMENTS

To validate the Maxwell Garnett mixing rule as a model for computing the effective permittivity for concrete and carbon fiber composites the results from the model will be compared with published results. To be able to compare experimental data with measurements it is very important that the size of the fibers is given. In [6] sufficient information is provided to reproduce the experiment. The fibers in the paper have a diameter of 12 μm, a length of 3.0 mm, and an electrical resistivity of 30 μΩm. However, the paper does not list the permittivity for the concrete. This is a problem since for a well conducting inclusion the effective permittivity of the material still depends on the bulk material. This can be seen in the limit where the permittivity of the inclusion goes to infinity in Equation (9).

$$\lim_{\varepsilon_{\mathrm{i}}\to\infty} \varepsilon_{\mathit{eff}} \to \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{e}} \frac{f}{3(1-f)} \sum_{j=x,y,z} \frac{1}{N_j} \tag{10}$$

In [6] an Elgal SET 19A shielding effectiveness tester was used for the measurements. This device has a coaxial cross section and we assume that the dominant mode is the TEM-mode. The dimensions of the tester are not openly available, but approximate measurements can be found in [6]. The first higher order mode has a cutoff frequency around 1.55 GHz. However, the symmetry of the device should suppress higher order mode excitation so that the TEM-mode is dominant for frequencies above 1.55 GHz. The TEM-mode in a coaxial cable is comparable to a plane wave in free space, and the group velocity is the same for both cases. The reflection and transmission coefficients will be the same for a plane wave with normal incidence to an infinite planar sheet with thickness d and a wave in a coaxial cable that is reflected by a media with thickness d, if the materials of the sheet and media are the same. This means that the experimental results are directly comparable to a very simple case that can be solved analytically.

There are five samples in [6] that can be directly comparable to the Maxwell Garnett mixing rule results. The weight fraction between the concrete and carbon fibers are varied between 0 to 4% in 1% steps, and the thickness varies from 3.6–4.1 mm. For these samples the shielding effectiveness was measured for 1.0, 1.5, and 2.0 GHz, and in Figure 1(a) the measured results are compared to three results computed using Equation (9). The three computed results differ in that they use different values for the permittivity of the concrete. The two first permittivity values $\varepsilon_{\mathrm{e}} = 6.1 - \mathrm{i}0.5$ and $\varepsilon_{\mathrm{e}} = 7.8 - \mathrm{i}1.7$ are from [12] for 5.5% and 12% moisture content, and the third value $\varepsilon_{\mathrm{e}} = 9.1 - \mathrm{i}2.5$ is from taken from [11]. At 1 GHz the first value agrees well with the experimental results when the weight fraction is $w = 0\%$. For small volume fractions the relationship to the weight fraction is approximately linear, $f \approx w/2.31$. As can be seen in Figure 1(a) the computed values for the composite materials agree fairly well with the experimental values, the best agreement is found for the lowest moisture content, and for the other two permittivity



(a) (b) (c) (d)

Figure 1. The shielding effectiveness is shown for the five experimental samples in Figures 1(a)–(c). The thicknesses of the samples are 3.6 mm, 3.8 mm, 3.9 mm, 4.1 mm, and 3.9 mm, arranged in order of increasing weight fraction 0–4%. In Figure 1(d) the shielding effectiveness is shown for the sample with a weight fraction of 4% as a function of frequency. (a) 1.0 GHz, (b) 1.5 GHz, (c) 2.0 GHz, (d) weight fraction 4%.

values the shielding effectiveness is higher than the experimental value.

In Figures 1(b) and 1(c) the results are shown for 1.5 GHz and 2.0 GHz, respectively. For these frequencies we have assumed that the concrete permittivity is the same as for 1.0 GHz. At these frequencies this assumption should not lead to a dramatic error in the results.

As can be seen in Figures 1(b) and 1(c) there are discrepancies between the shielding effectiveness for calculated and measured data. At 2.0 GHz the largest difference is between the shielding effectiveness computed using the 5.5% moisture level value and the permittivity values from [11]. The discrepancy between the two calculated values is solely due to the bulk materials being different. The cause for the rather large difference in shielding effectiveness is that the sheet of composite material is only $\sim 4\,\mathrm{mm}$ thick and there will be multiple reflections within this sheet. Depending on the frequency there will be constructive and destructive interference within the the sample and the shielding effectiveness will oscillate as a function of frequency. However for thicker samples, e.g., a wall, the attenuation within the sheet will remove this effect. In Figure 1(d) the shielding effectiveness is shown for the case when the weight fraction is 4%, and it is evident that there is a local minimum at 2 GHz for the 5.5% case. This leads us to suspect that the [11] and the 12% moisture permittivity is closer to the measured concrete sample than the 5.5% case, but also illustrates how much difference a change in the bulk material can make to the effective permittivity of the composite. This has practical importance since the shielding effectiveness will depend on humidity, and therefore will change with the seasons and weather.

However, there are other sources for error than the bulk material permittivity in this comparison. The Maxwell Garnett mixing rule is by no means exact, e.g., if a large percentage of the inclusion material is used there is a possibility that percolation occurs in the material, i.e., the fibers are mixed so that long chains are made which will increase the conductivity.

4. SHIELDING EFFECTIVENESS OF CARBON FILAMENT COMPOSITE CONCRETE WALLS

As seen in the previous section the Maxwell Garnett mixing rule can give an approximate description of the effective permittivity of a composite material of concrete and carbon fibers. In this section we will study the shielding effectiveness of concrete slabs with a thickness similar to a house wall. To simplify the analysis and the interpretation of the results we will consider the example where the plane wave has normal incidence to the wall. Since the wall will be fairly thick the effect of multiple reflections will be small due to attenuation. Furthermore, since the index of refraction is so much higher within the composite material than in the surrounding medium, the wave will approximately travel the same length in the wall regardless of its incident angle.

The carbon filaments chosen for the wall examples have the same dimensions and conductivity as the previous example, and the concrete data are taken from the paper by Robert [11]. The concrete is modeled

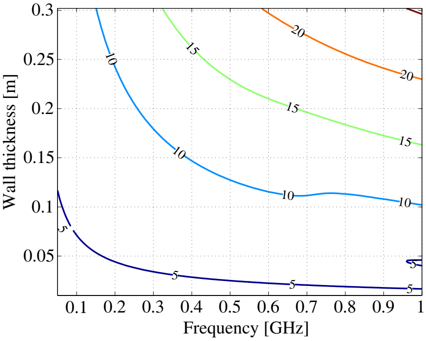


Figure 2. The shielding effectiveness for a concrete wall with a 0% weight fraction of carbon filaments.

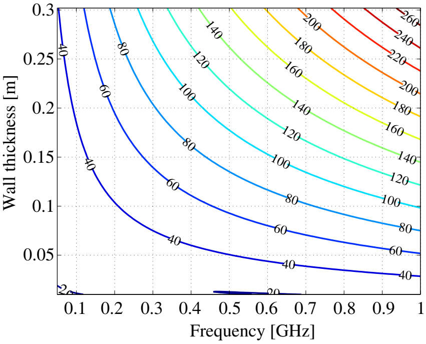


Figure 3. The shielding effectiveness for a concrete wall with a 3% weight fraction of carbon filaments.

by the Cole-Cole dispersion model

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_\mathrm{s} - \varepsilon_\infty}{1 + (\mathrm{i}\omega\tau)^{1-\alpha}} - \mathrm{i}\frac{\sigma_\mathrm{s}}{\omega\varepsilon_0}. \tag{11}$$

This model was chosen since it gave the best overall agreement with the measurement data in [11], see, e.g., Figure 1(d). However, the unknown parameters, $\varepsilon_\infty = 7.5$, $\varepsilon_\mathrm{s} = 21.23$, $\tau = 1.215 \cdot 10^{-9}\,\mathrm{s}$, $\alpha = 0.27$, $\sigma = 0.0135\,\mathrm{S/m}$, were not given in the paper and have therefore in this paper been curve fitted to the measurement data given in [11].

A 25 cm thick concrete wall with no additives has a shielding effectiveness of approximately 20 dB at mobile phone frequencies (900 MHz), see Figure 2. This is basically what is the expected shielding effectiveness for a concrete building at the GSM band.

If a 3% weight fraction of the carbon filaments are added to the concrete, the shielding effectiveness at 900 MHz is increased to more than 200 dB, a 180 dB increase from the plain concrete wall, see Figure 3. The increase of shielding effectiveness is due to the dramatic increase of the complex permittivity of the concrete with increasing weight fraction of additives, as can be seen in Figure 4. The reason for this dramatic increase in complex permittivity with such a small weight ratio of additives is the geometry of the carbon filaments. The optimal geometry for the filaments is when the aspect ratio between the filament length, $2a_x$, and thickness, $2a_y$, goes to infinity. If instead the carbon additives were spherical in shape and the weight fraction remained 3% there would not be a measurable improvement of the shielding effectiveness ($\sim 0.5\,\mathrm{dB}$ improvement). The downside with the prolate spheroids is that percolation will occur for smaller volume

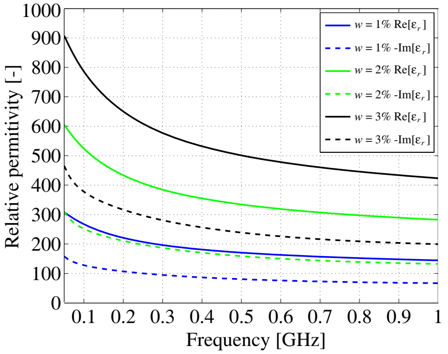


Figure 4. The complex relative permittivity of the composite material for three weight fractions ranging from 1 to 3%.

fraction than spheroid particles. When percolation occurs the Maxwell Garnet mixing rule is no longer valid and other mixing rules must be considered [9]. These effects are considered small when the weight fraction is below the percolation threshold. In [13] it was shown that the percolation threshold $f_c = \frac{9}{2}\frac{a_y}{a_x}$ is suitable for long fibers with an aspect ratio $\frac{a_x}{a_y} \gg 1$. This means that for this model to be valid in this example the weight fraction should be less than $w_c = 2.31 f_c \approx 4.2\%$.

From a manufacturing point of view, adding carbon filaments to a wall is an easy method to increase the shielding effectiveness of concrete walls; the extra labor for adding carbon filaments in the manufacturing process should not be too demanding. With this method the shielding effectiveness can easily be increased where it is needed in the construction, e.g., close to windows or ducts, by increasing the weight fraction of filaments. It is necessary to mention that this technique works best for higher frequencies, in these examples 500 MHz and higher. For lower frequencies the shielding efficiency has to be addressed by other means. Observe that we omit the question of the effect on the mechanical properties of the concrete due to the additives and how to successfully join different concrete sections to maintain a good shielding.

5. SUMMARY

In this paper it is shown that it is possible to compute analytically the complex permittivity of composites of concrete and carbon filaments using the Maxwell Garnett mixing rule for randomly oriented inclusions. The carbon filaments are modeled as prolate spheroids whose semi-axles are determined by the thickness and length of the filament. The analytical model was verified against published measurements and found to agree well with measurements for the studied cases but there were discrepancies. However, the measurements show that the Maxwell Garnett mixing rule will work well enough to approximate the shielding effectiveness of a concrete wall with additives. The difference in the shielding effectiveness between the measurement and analytical results are in the same order as the difference due to different moisture content. The analytical expression makes it easy to design the shielding effectiveness of concrete and carbon fiber composites. Furthermore, the shielding effectiveness of various walls were demonstrated and it was shown that the shielding effectiveness of a 25 cm thick wall can be increased by 180 dB by adding 3% weight percent of carbon filaments for the GSM 900 band.

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