

## Optical Bistability in a Grating with Slits Filled Nonlinear Media

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**Abstract**—An approximate self-consistent solution of the problem of plane electromagnetic wave diffraction on a thick grating of metallic bars with slits between the bars filled a Kerr-type nonlinear dielectric is solved. The bistable operating regime of wave transmission through the grating is studied.

### 1. INTRODUCTION

Diffraction gratings made of metallic or dielectric bars are widely used in various quasi-optic and electronic devices, particularly in wavemeters, interferometers, polarization converters, phase shifters, microwave vacuum electronic devices, etc. Therefore, the problem of electromagnetic wave diffraction on such gratings is a classic problem in the theory of diffraction [1].

In recent years the interest in solution of the problem of electromagnetic waves diffraction on gratings of bars has been rekindled due to advances in technologies of production of the periodic structures whose characteristic dimensions are comparable to the wavelength of optical and infrared radiations [2]. An important point is that, unlike to the microwave band, in the optical and infrared bands, on the one hand, there is a lot of transparent materials with significant nonlinear characteristics and, on the other hand, powerful light sources (lasers) are available which can produce radiation with sufficiently high intensity enough for the nonlinearity to become apparent.

An undoubted importance for practical applications are nonlinear resonant structures that exhibit the effects of optical bistability or multistability in transmission, reflection or light polarization conversion. Bistable systems, which have two stable states at the same control parameters (e.g., for the same intensity of the incident light), allow designing miniature optical switches, efficient power limiters, optoelectronic systems and optical transistors (transphasors) which are nonlinear optical devices that use one light beam to modulate another, in a manner analogous to an electronic transistor, and that operate through the transference of a phase shift from one beam to the other.

A classical example of the bistable device is a Fabry-Perot interferometer, filled with a Kerr-type nonlinear medium [3]. In this case, the resonator provides feedback, which is essential to obtain a multivalued intensity at the structure's output. Examples of recently proposed new miniature nonlinear resonant elements are photonic crystal microcavities [4] and quantum well structures [5]. With the assistance of surface plasmon polaritons to the effects of confining and enhancing the local optical field intensity, optical bistability has also been shown in different metal nanostructures such as surface plasmon polaritonic crystals [6], one-dimensional and two-dimensional subwavelength gratings [7–10], metamaterials [11–15], etc.. In their design the main attention is focused on achieving an enhancement of nonlinear effects along with reducing the material volume and intensity of the operating light. As nonlinear materials some semiconductors, like indium antimonide (InSb), gallium arsenide (GaAs) and indium arsenide (InAs) are typically used in such systems. These materials are characterized by a relatively strong nonlinearity and have acceptable switching time (relaxation time) of the nonlinear response.

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The goal of this paper is to study the features of bistable optical response (especially phase transformations) of a grating of metallic bars with slits filled a Kerr-type nonlinear dielectric. Our investigations are provided in the near-infrared wavelength range, in particular, at the wavelength of 1240 nm which is close to the second Telecom Window of the fiber-optic communication lines.

## 2. PROBLEM STATEMENT AND SOLUTION

Let us assume an electromagnetic plane wave

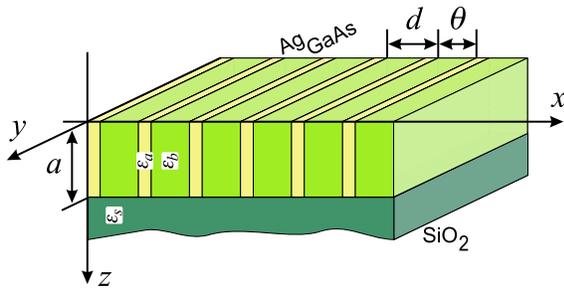
$$\vec{E}_i(z) = \vec{P} \exp[i(kz - \omega t)], \quad (1)$$

normally incidents on a periodic grating depicted in Fig. 1. In (1)  $\vec{P}$  is a vector which defines the magnitude and polarization state of the incident field, and  $k = 2\pi/\lambda$  the free-space wavenumber. We neglect the effects of high-order harmonic generations, so we suppose that both transmitted and reflected fields have the time dependence in the form  $\exp(-i\omega t)$ .

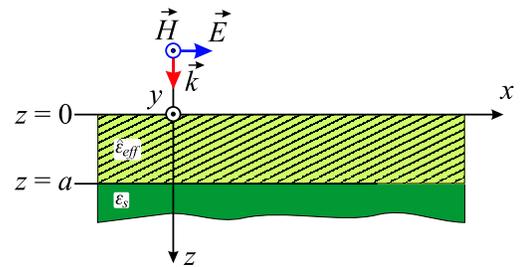
The grating is made of silver bars with rectangular cross section which are periodically arranged on a flat substrate of silica. The slits between the bars are filled with gallium arsenide. So the final structure is the layered geometry, and consists of alternating layers of two materials with different linear and nonlinear optical properties. The grating period and thickness are  $d$  and  $a$ , respectively. The width of slits is  $\theta$ .

At the wavelength  $\lambda = 1240$  nm, the relative permittivity of silver is almost independent of the electromagnetic field strength and is about  $\varepsilon_a = -81.5 + i5.1$  [16]. On the contrary, gallium arsenide which fills the slits between the silver bars is a nonlinear material. In the first approximation, its permittivity is linearly dependent on the intensity of the electric field, i.e., it is a Kerr-type nonlinear medium with permittivity  $\varepsilon_b = \varepsilon_{b1} + \varepsilon_{b2}|E|^2$ , where  $\varepsilon_{b1} = 11.0$ ,  $\varepsilon_{b2} = 1.3 \times 10^{-3} \text{ cm}^2/\text{kW}$  [3]. The substrate is made of a transparent and linear material (silica) which permittivity is  $\varepsilon_s = 2.1$ . We suppose that all structure's components are nonmagnetic, i.e.,  $\mu = 1$ .

Further we assume that the grating is made from ultra thin layers of metal and nonlinear dielectric. Thus the structure period  $d$  is much smaller than the electromagnetic wavelength in the grating materials ( $d \ll \lambda$ ). Under this condition, the propagation of light can be described by effective values of the optical constants that are obtained by performing a suitable volume average of the local optical response of the material. In fact, performing such an average can be rather subtle for situations involving the nonlinear optical response, because it is the nonlinear polarization that must be averaged, and the nonlinear polarization depends on the spatially inhomogeneous electric field amplitude in the composite material. We will, however, make the assumption that the spatial extent of each region is sufficiently large that we can describe its response using macroscopic concepts [17–22] rather than using microscopic ones. So, these conditions limit the grating period by several tens of nanometers. Thus, to calculate a nonlinear response of the studied structure we can follow the approach treating the grating like a slab of effective



**Figure 1.** A periodic grating of pairs of alternating metallic (silver) and dielectric (gallium arsenide) bars placed on a silica substrate.



**Figure 2.** The normal incidence of the plane electromagnetic wave on the homogeneous anisotropic layer which is equivalent to the grating.

continuous anisotropic medium [23] with effective tensor of nonlinear dielectric permittivity

$$\hat{\varepsilon}_{eff} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}. \quad (2)$$

This problem becomes to be reduced to the problem of finding the relation between the local and macroscopic field in a composite material containing nonlinear inclusions. By this means, the investigation of the wave interaction with an inhomogeneous periodic structure is reduced to the solution of the boundary-value problem of conjugations of an equivalent homogeneous anisotropic layer with surrounding spaces (Fig. 2).

Electromagnetic field within the grating must satisfy the condition of continuity of the tangential component of the electric field and the normal component of the electric induction at the boundaries between the layers of silver and gallium arsenide. Using these boundary conditions, it is easy to obtain expressions for the components of effective permittivity tensor  $\hat{\varepsilon}_{eff}$  in the case when the vector  $\vec{E}$  is parallel to bars ( $E$ -polarization)

$$\varepsilon_{yy} = \varepsilon_{zz} = \eta\varepsilon_b + (1 - \eta)\varepsilon_a, \quad (3)$$

and in the case when the vector  $\vec{E}$  is perpendicular to bars ( $H$ -polarization)

$$\varepsilon_{xx} = \frac{\varepsilon_a\varepsilon_b}{\eta\varepsilon_a + (1 - \eta)\varepsilon_b}, \quad (4)$$

where  $\eta = \theta/d$  is the dielectric infilling factor.

When the incident field intensity is small (i.e., in the linear regime), the effective permittivity of the equivalent layer is a constant. In this case the reflected ( $z < 0$ ), inner ( $0 < z < a$ ) and transmitted ( $z > a$ ) fields can be written as follows

$$E_{ref}(z) = P \frac{\gamma - \sqrt{\varepsilon_e} - \sqrt{\varepsilon_s} + (\delta + \sqrt{\varepsilon_e} - \sqrt{\varepsilon_s}) \exp(2ik_e a)}{\alpha + \beta \exp(2ik_e a)} \exp(-ikz), \quad (5)$$

$$E_{in}(z) = 2P \frac{\gamma \exp(ik_e z) + \delta \exp[ik_e(2a - z)]}{\alpha + \beta \exp(2ik_e a)}, \quad (6)$$

$$E_{tr}(z) = 4P \frac{\exp[i(k_e - k_s)a]}{\alpha + \beta \exp(2ik_e a)} \exp(ik_s z), \quad (7)$$

where  $k_e = k\sqrt{\varepsilon_e}$ ,  $k_s = k\sqrt{\varepsilon_s}$ ,  $\alpha = 1 + \sqrt{\varepsilon_e} + \sqrt{\varepsilon_s} + \sqrt{\varepsilon_s/\varepsilon_e}$ ,  $\beta = 1 - \sqrt{\varepsilon_e} + \sqrt{\varepsilon_s} - \sqrt{\varepsilon_s/\varepsilon_e}$ ,  $\gamma = 1 + \sqrt{\varepsilon_s/\varepsilon_e}$ ,  $\delta = 1 - \sqrt{\varepsilon_s/\varepsilon_e}$ . The difference between  $H$ -polarization and  $E$ -polarization involves the use in Eqs. (5)–(7) of two different effective permittivities  $\varepsilon_e = \varepsilon_{xx}$  and  $\varepsilon_e = \varepsilon_{yy}$ , respectively.

When the incident field intensity is high (i.e., in the nonlinear regime), the permittivity of gallium arsenide in the grating slits depends on the electric field intensity, and therefore the grating becomes to be inhomogeneous along the  $z$ -axis. In our self-consistent approach we neglect the effect of this inhomogeneity on the optical properties of the studied structure and seek a solution of the nonlinear diffraction problem by averaging the squared magnitude of the electric field inside the grating along its thickness and then use this averaged value to determine the actual permittivity within the slits

$$\varepsilon_b = \varepsilon_{b1} + \varepsilon_{b2} \overline{|E_{in}(z)|^2}, \quad (8)$$

and then the effective permittivity  $\varepsilon_e$  of the equivalent layer. In Eq. (8) the averaged value of the squared amplitude of the electric field is calculated using (6) as the next

$$F = \overline{|E_{in}(z)|^2} = \frac{1}{a} \int_0^a |E_{in}(z)|^2 dz = \frac{4|P|^2}{a} \int_0^a \left| \frac{\gamma \exp(ik_e z) + \delta \exp[ik_e(2a - z)]}{\alpha + \beta \exp(2ik_e a)} \right|^2 dz. \quad (9)$$

Thus a nonlinear equation related to the averaged value of the squared amplitude of the electric field can be formulated in the form [13–15]

$$F = \tilde{P} \Phi_F(\lambda, \varepsilon_{b1} + \varepsilon_{b2}(F)) = \tilde{P} \Phi_F(\lambda, \varepsilon_e(F)), \quad (10)$$

where  $\tilde{P}$  is a dimensionless coefficient which depicts how many times the incident field magnitude  $P$  is greater than  $1 \text{ V cm}^{-1}$ . The input field magnitude  $P$  is a parameter of this nonlinear equation. So, at

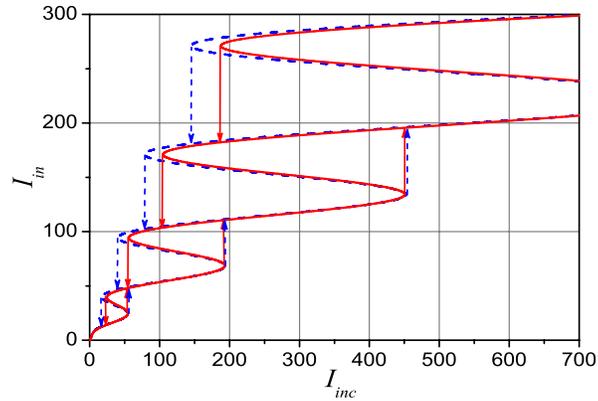
a fixed wavelength  $\lambda$ , the solution of this equation gives us the averaged value of the squared amplitude of the electric field  $F$  which depends on the magnitude of the incident field  $P$ . Since the effective permittivity  $\varepsilon_e$  depends on the averaged value of the squared amplitude of the electric field  $F$ , Eq. (10) is a nonlinear equation with respect to  $F$  in which the incident field magnitude  $P$  is a parameter. For each value of the magnitude of the incident field Eq. (10) can have one solution or more than one solutions, and accordingly, the whole diffraction problem has a set of solutions.

Since these solutions are depended on the incident field intensity, it is convenient to provide an analysis of the diffraction problem solution in terms of field intensities. Thus the intensity of the electromagnetic wave, which electric field strength is expressed in  $\text{V cm}^{-1}$  in a nonmagnetic medium with a refractive index  $n$ , is further determined by the formula  $I = n|E|^2/240\pi \text{ W cm}^{-2}$ .

The set of solutions of the nonlinear Equation (10) can be found at least in two ways. The first one is to find the roots of the equation for any given value of the magnitude (intensity) of the incident wave. The second less obvious way consists in assigning  $F$  in some interval of its possible values and then finding the field intensity of the incident wave, which leads to the appearance of the appropriate inner field. The second method is more robust since it does not need searching for multiple roots of the equation in the case when their number is not known a priori, and we use such a treatment in our numerical calculations.

### 3. RESULTS AND DISCUSSION

The characteristic curves of dependence of the inner field intensity on the incident field intensity ( $I_{inc} = |P|^2/240\pi \text{ kW cm}^{-2}$ ) are depicted in Fig. 3 in the case of the  $H$ -polarized incident wave. At low magnitude of the incident wave, electromagnetic field intensity inside the structure is a single-valued function  $I_{in}(I_{inc})$  of the intensity of the incident wave. As magnitude of the incident wave rises, the function  $I_{in}(I_{inc})$  becomes to be two-valued and then multi-valued one.



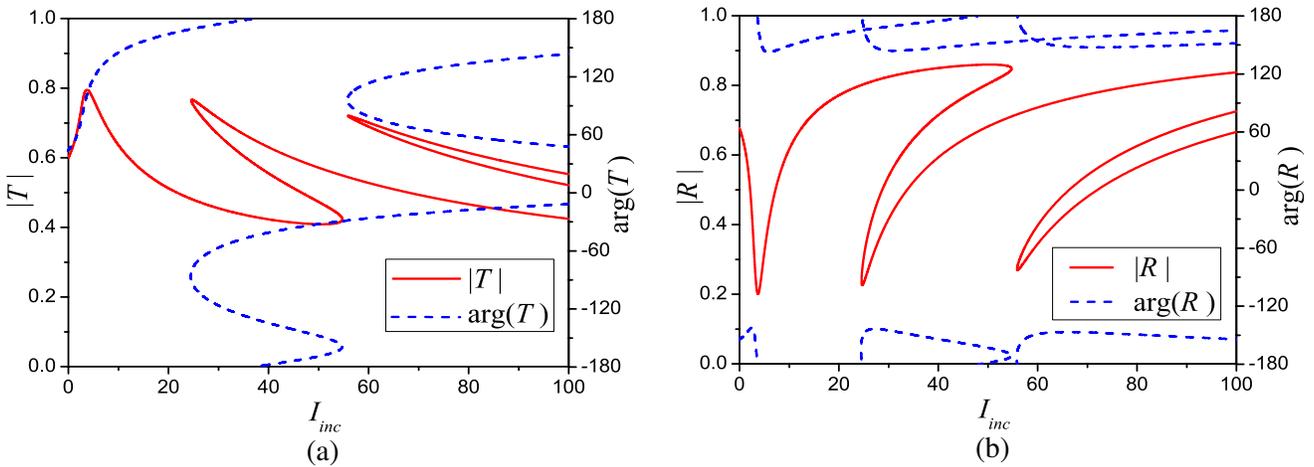
**Figure 3.** Dependences of the inner field intensity on the incident field intensity of  $H$ -polarized wave for the grating in free space (solid red line) and the grating placed on a silica substrate (dash blue line);  $\eta = 0.8$ ,  $a/\lambda = 0.5$ .

The inner field intensity experiences an abrupt transitions between different distinct states as the incident field magnitude increases/decreases. In Fig. 3 these transitions are marked with vertical arrows. It is important that the field intensity inside the structure is changed in a different way as the magnitude of the incident wave increases and decreases, forming typical for nonlinear systems  $S$ -type hysteresis loops.

Once the averaged intensity of the inner field (i.e., the parameter  $F$  which is a solution of Eq. (10)) is found, the transmission ( $T$ ) and reflection ( $R$ ) coefficients can be determined in the form

$$T = E_{tr}(a)/E_{inc}(a) = E_{tr}(a) \exp(-ika)/P, \quad R = E_{ref}(0)/P, \quad (11)$$

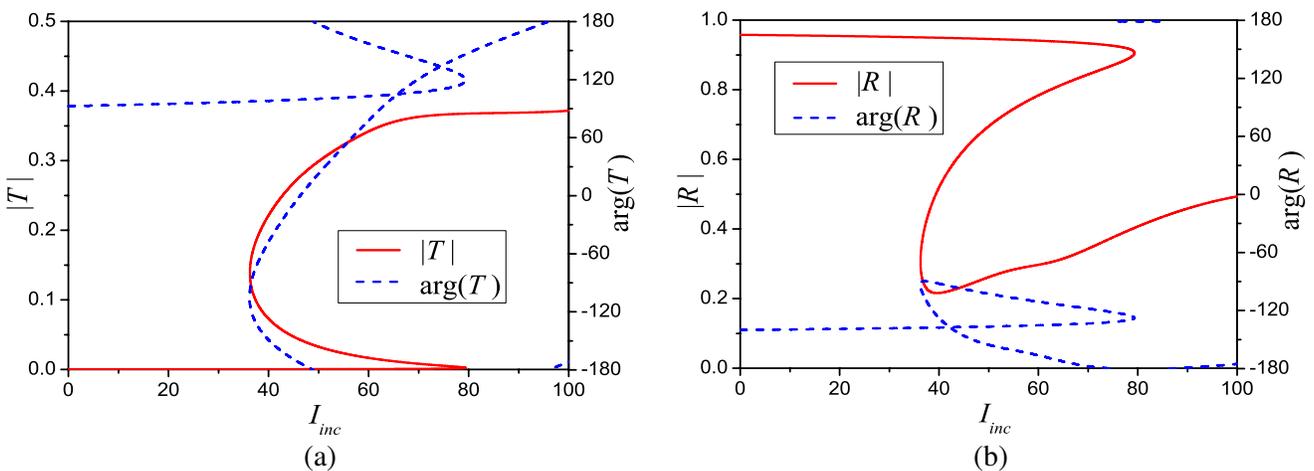
apart for  $E$ -polarized and  $H$ -polarized waves.



**Figure 4.** Magnitude (solid red line) and phase (dash blue line) of the (a) transmission and (b) reflection coefficients versus the incident field intensity of  $H$ -polarized wave for the grating placed on a silica substrate;  $\eta = 0.8$ ,  $a/\lambda = 0.5$ .

The dependences of the magnitude of the transmission and reflection coefficients on the incident field intensity for  $H$ -polarized incident wave at a wavelength of 1240 nm are presented in Fig. 4. In the case of such polarization, the electric field vector of the incident wave is perpendicular to the grating bars. Therefore, the fundamental waveguide mode of the planar waveguide with metallic sidewalls is effectively excited inside the grating slits for any their width. As a result of the fundamental waveguide mode reflection from open ends of planar waveguides in planes  $z = 0$  and  $z = a$  the standing wave field is formed within each slit. At a particular wavelength this standing wave field becomes to be resonant. The resonant wavelength depends on the permittivity of slits filling, which, in turn, is determined by the inner field intensity. Thus the dependences of the magnitude of the transmission and reflection coefficients on the incident field intensity appear as alternating maxima and minima which are results of the constructive and destructive interference of waves inside the structure, and characteristic of this wave interference changes as  $I_{in}$  increases or decreases.

The phase changing of the reflection and transmission coefficients against the background of their magnitudes as a function of the incident field intensity of  $H$ -polarized wave is also depicted in Fig. 4. One can see that the phase also experiences discontinuous switching as  $I_{inc}$  increases and decreases.



**Figure 5.** Magnitude (solid red line) and phase (dash blue line) of the (a) transmission and (b) reflection coefficients versus the incident field intensity of  $E$ -polarized wave for the grating placed on a silica substrate;  $\eta = 0.8$ ,  $a/\lambda = 0.5$ .

In the case of  $E$ -polarization of the incident waves (i.e., when the electric field vector  $\vec{E}$  is parallel to the grating bars) at a low intensity of the incident field the wave is almost completely reflected from the grating (see Fig. 5). The phase of the reflection coefficient is close to  $-180^\circ$ . The grating can be approached as an equivalent layer of the plasma-like medium which dispersion curve is considered to be in the frequency band below the plasma frequency. As the incident field intensity increases the optical properties of this medium vary and the system becomes to be partially transparent.

It is obvious when the incident wave possesses some intermediate polarization, which is a superposition of  $H$ -polarized and  $E$ -polarized waves, some abrupt changes in the polarization state of the reflected and transmitted fields (namely, the stepwise changing in the polarization azimuth and ellipticity) can appear with the incident field intensity increasing due to effects of bistability and multistability in the reflection and transmission coefficients.

#### 4. CONCLUSION

The studied structure made in the form of grating of metal bars with slits filled with a Kerr-type nonlinear dielectric manifests bistable and multistable behaviors of the reflection and transmission coefficients in the near infrared band. From the specific parameters used in our numerical calculations, it is reasonable to conclude that bistable response and stepwise polarization switching can already be achieved at the incident power densities of  $10\text{--}100\text{ kW cm}^{-2}$  with available materials in the considered structure configuration. The studied structure can be applied in the infrared band for designing miniature switches, limiters, logic gates, etc..

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