Space-Frequency Approach to Design of Displacement Tolerant Transcutaneous Energy Transfer System

Arseny A. Danilov, Eduard A. Mindubaev*, and Sergey V. Selishchev

Abstract—One of the main concerns for transcutaneous energy transfer via inductive coupling is misalignments of coils, especially in the case of mechanical circulatory support systems, when coils placed on a chest wall or an abdomen. We proposed a space-frequency approach to this problem. It is possible to find values of so called splitting frequency by expression which incorporate the value of coupling coefficient. Given that coupling coefficient depends on the system geometry, it allows one to determine the optimal operating frequency for the specified relative position of the coils. Numerical calculations of transcutaneous energy transfer parameters show the capability of the proposed method. It was found that the operation at splitting frequency provided more stable output with respect to changes in a system geometry. The output power of the proposed system changes for not more than 5% for a distance in a range of 5–25 mm. At the same time, the output power of the system which operates at fixed resonant frequency changes for about 40%. Similar results were obtained for lateral displacements in a range of 0–20 mm.

1. INTRODUCTION

Transcutaneous energy transfer (TET) via inductive coupling is a promising technique for the energy supply of implantable medical devices (IMD) [1–3]. But, in spite of great interest since early 60s [4], there is a limited number of clinically used IMD with TET. Several complex problems are still to be solved. One of the main concerns is misalignments of coils, especially in the case of mechanical circulatory support systems, when coils are placed on a chest wall or an abdomen [5]. Unavoidable displacements caused by movement of the patient and by postoperative swelling lead to substantial alteration in transferring power.

The problem of coils displacements was actively investigated during several decades [6–8]. It can be said that there are two main ways to overcome this problem. First is based on coils form factor selection [9]. It is also possible to achieve displacement tolerance by frequency control with some kind of a feedback loop [10]. However, this solution is not problem free. Main issue here is operating frequency hopping [11].

We propose a more general approach, in which both the geometry and frequency are taken into account. The main feature here is a consideration of frequency splitting effect to relate the coils position and operating frequency to each other. To examine the potential of our approach, we performed numerical calculations of energy transfer parameters by a TET system with primary series and secondary series (PSSS) compensated topology. It has been shown that the proposed space-frequency approach can provide stable output power in a wide range of lateral displacements.
2. BASIC THEORY OF INDUCTIVE COUPLING

Key parameters to describe an inductive link are coupling coefficient $k$ and mutual inductance $M$. The coupling coefficient can be described as the fraction of the flux generated by the primary coil which is captured by the secondary coil (and vice versa) and can be calculated as follows [11]:

$$k = \frac{M}{\sqrt{L_T L_R}}. \quad (1)$$

Here $L_T$ is the self-inductance of the primary (transmitting) coil and $L_R$ the self-inductance of the secondary (receiving) coil. Mutual inductance is a proportionality factor between the rate, at which current in one circuit changes, and electromotive force around the other circuit due to this current flow. So, for a pair of a coil it can be written

$$\varepsilon_T = L_T \frac{dI_T}{dt} + M \frac{dI_R}{dt} \quad (2)$$

$$\varepsilon_R = L_R \frac{dI_R}{dt} + M \frac{dI_T}{dt} \quad (3)$$

Here $I_T$ is the current flowing in transmitting part of the system and $I_R$ the current flowing in receiving part of the system.

For farther analysis TET system with so called PSSS topology (inductor and capacitor connected in series in both primary (transmitting) and secondary (receiving) parts of the system) was chosen (Figure 1). One of characteristic properties of this topology is zero reactive power flow in resonant circuits [11]. It has significant advantage in TET systems due to reduced power losses in the receiving part and, therefore, reduced heating.

Total impedance seen at the power source of the inductive energy transfer system shown in Figure 1 is derived as:

$$Z_{total} = R_S + \frac{1}{j\omega C_T} + j\omega L_T + \frac{1}{j\omega C_R} + j\omega L_R + R_L \quad (4)$$

Here, $\omega$ is the angular operating frequency of the system, $R_S$ the internal resistance of power source, $R_L$ the load resistance, and $C_T$ and $C_R$ are the transmitting and receiving coils capacitances, respectively.

For the power source voltage $V_S$ and the transmitter current $I_T$ it can be written:

$$I_T = \frac{V_S}{Z_{total}} \quad (5)$$

The receiver current $I_R$ can be derived from Kirchhoff’s laws and is equal to:

$$I_R = \frac{j\omega MI_T}{\frac{1}{j\omega C_R} + j\omega L_R + R_L} \quad (6)$$

![Figure 1](image-url)  

**Figure 1.** (a) Energy transfer system schematic and (b) its representation with T-equivalent transformer circuit.
Load voltage can be calculated as:

\[ V_L = I_R R_L \]  

(7)

By substituting Eq. (5) into Eq. (6) and then substituting the resulting equation for \( I_R \) into Eq. (7), one can obtain:

\[ V_L = j\omega MV_S R_L \left( \frac{1}{j\omega C_R} + j\omega L_R + R_L \right) Z_{Total} \]

(8)

Power transferred to the load is derived as:

\[ P_L = \frac{|V_L^2|}{2R_L} \]

(9)

Substituting Eq. (8) into Eq. (9), we obtain:

\[ P_L = \frac{\omega^2 MC_R V_S}{(\omega^2 L_RC_R - j\omega C_R R_L - 1) Z_{Total}} \left( R_L \right) \]

(10)

Total power consumption of TET system is calculated as:

\[ P_{Total} = \frac{1}{2} \left| \frac{V_S^2}{Z_{Total}} \right| \]

(11)

Total energy transfer efficiency is equal to:

\[ \eta = \frac{P_L}{P_{Total}} \]

(12)

Finally, by substituting Eqs. (10) and (11) into Eq. (12), one can obtain for overall efficiency:

\[ \eta = \frac{\omega^2 MC_R}{(\omega^2 L_RC_R - j\omega C_R R_L - 1) Z_{Total}} \left( R_L \right) \]

(13)

3. FREQUENCY SPLITTING EFFECT

The frequency splitting effect can occur in wireless energy transfer systems via inductive coupling. If the coupling coefficient is greater than a certain value (so called splitting coupling, see below expression (14)), resonance frequency splits into two modes [12]. Figure 2 shows calculated values of power transferred to the load in dependence of coupling coefficient and operating frequency. It can be seen that for \( k \) higher than some value two frequencies which correspond to the peak values of the transferred power exist. These two frequencies are called odd and even splitting frequencies and vary with changes of coupling coefficient.

<table>
<thead>
<tr>
<th>Table 1. Component values of modeled system.</th>
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<tbody>
<tr>
<td>Component</td>
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<tr>
<td>Power supply, ( V_S )</td>
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<tr>
<td>Transmitting part resistance, ( R_S )</td>
</tr>
<tr>
<td>Transmitting coil inductance, ( L_T )</td>
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<tr>
<td>Receiving coil inductance, ( L_R )</td>
</tr>
<tr>
<td>Transmitting part capacitor, ( C_T )</td>
</tr>
<tr>
<td>Receiving part capacitor, ( C_R )</td>
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<tr>
<td>Load resistance, ( R_L )</td>
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</table>
Value of splitting coupling can be calculated as [12]:

$$k_{splitting} = \sqrt{\frac{(R_{S}^2 + R_{L}^2)}{2}} \frac{C}{L} \quad (14)$$

If coupling coefficient between the coils is lower than splitting coupling, resonant frequency of the TET system is equal to:

$$f_{res} = \frac{1}{2\pi\sqrt{LC}} \quad (15)$$

Coupling coefficient between the coils can be defined as a function of transmitting and receiving coils geometrical characteristics (coil diameter, coil form-factor, distance between the coils etc.). Therefore, dependencies from the geometrical characteristics can be obtained for the system output parameters. Any of these characteristics can be singled out from the coupling coefficient. For example, Figure 3 shows power transferred to the load in dependence of lateral displacement and operating frequency. The figure shows that specific range of displacements and corresponding range of operating frequencies can be allocated for each value of output power.

It is worth noting, that the frequency sweep of output power is not fully identical to the frequency sweep of energy transfer efficiency. For example, for the system component values listed in Table 1 and a coupling coefficient of 0.6 energy transfer stays highly efficient (about 85%) for a range of frequencies from odd to even splitting frequencies (Figure 4(b)). However, power transferred to the load has two peaks corresponding to odd and even splitting frequencies (Figure 4(a)). Therefore, the energy transfer efficiency cannot be used as a reliable parameter to optimize energy transfer stability.

Thus, it can be said that optimization of the TET via inductive coupling is a more difficult task than just maintaining of the minimal distance between the two perfectly aligned coils and operating at the resonant frequency. Influence of coupling coefficient between the coils and the fact that power transferred to the load can be low, even when energy transfer efficiency is high, must be considered.
Figure 4. (a) Power transferred to the load and (b) total energy transfer efficiency versus operating frequency. It can be seen that the frequency sweep of power transferred to the load is not fully identical to the frequency sweep of energy transfer efficiency. System parameters are given in Table 1. Coupling coefficient for both (a) and (b) is equal to 0.6. Power transferred to the load is calculated via Equation (10). Energy transfer efficiency is calculated via Equation (13).

4. OPTIMIZATION APPROACH

As shown in Section 3, power transferred to the load depends on system operating frequency, as well as coupling coefficient between the coils. We propose a method for the TET system tuning based on the utilization of a frequency splitting. The measured value of output power should be fed back to the control unit. If this value is lower than a specified level, tuning procedure begins. First of all, it is necessary to determine the coils position. Relative position of the coils can be determined directly (by means of optical radiation, ultrasound, magnetic positioning systems etc.) or can be derived indirectly by so called M-mapping [13]. After that, demanded value of operating frequency can be calculated by using the obtained information about the position of the coils. Operating frequency should be automatically adapted accordingly to changes in coupling coefficient to increase energy transfer stability of the TET system. Key steps of the adaptation process are described in detail further.

4.1. Mutual Inductance Calculation

Mutual inductance between two arbitrary oriented coils can be calculated by Neumann formula:

\[ M = \frac{\mu_0}{4\pi} \oint_{l_T} \oint_{l_R} \frac{dl_T dl_R}{r_{bc}} \]  

(16)

where \( r_{bc} \) — distance between \( dl_T \) and \( dl_R \) elements of the coils, \( \mu_0 \) — permeability of free space. However, this equation cannot be solved analytically in general form. Numerical methods can be used to calculate it.

Mutual inductance of the two coils with arbitrary number of turns can be calculated as:

\[ M = \sum_{t=1}^{T} \sum_{n=1}^{N} M_{tn} \]  

(17)

where \( T \) is the number of turns of the first coil, \( N \) the number of turns of the second coil and \( M_{tn} \) the mutual inductance between two respective turns.

4.2. Coupling Coefficient Calculation

Mutual inductance is needed to be calculated numerically. The self-inductances of the transmitting and receiving coils can be measured in advance and does not change much within defined frequencies range. Initialization of the inductances of the transmitting and receiving coils allows for faster calculation of coupling coefficient. Therefore, coupling coefficient between the coils can be calculated via Equation (1).
4.3. Splitting Frequency Calculation

For wireless inductive energy transfer system, odd and even splitting frequencies could be evaluated as [14]:

\[
f_{\text{odd}} = \frac{f_{\text{res}}}{\sqrt{1 + k}} \quad (18)
\]

\[
f_{\text{even}} = \frac{f_{\text{res}}}{\sqrt{1 - k}} \quad (19)
\]

It should be noted that these equations did not locate values of odd and even frequencies perfectly. The exact calculation of the splitting coupling and odd and even splitting frequencies is complex, because it relates to the tenth-order equation [15]. An asymptotic analysis might be used to estimate values of the parameters. On the other hand, it can be seen from Figure 5 that expressions (18) and (19) have reasonable accuracy. Calculated values of odd and even frequencies are close enough to the peaks.

Operating on the odd frequency is preferable, because it changes more smoothly with alteration of a system geometry (Figure 6).

**Figure 5.** The dependence of the output power from coupling coefficient and operating frequency, calculated by means of expression (10). The black dashed-line shows operating frequency dependency from coupling coefficient calculated via Equations (19) (left line) and (20) (right line). Calculated values of odd and even frequencies are close to the peaks, corresponding to maximum output power. System parameters are given in Table 1.

4.4. Operating Frequency Tuning

TET system with operating frequency tuned to the odd splitting frequency was evaluated to examine the capability of the proposed approach. It was compared to the system that operates at the resonant frequency. Main parameters \((V_S, R_S, R_L, C_T, C_R)\) are taken from Table 1. On the other hand, coils geometry was specified (see Table 2) instead of taking preset values of self-inductance \((L_T, L_R)\). Geometry specific for coils used in TET systems with ventricular assist devices (VADs) was chosen. Transmitting and receiving coils are identical. Maximum feasible coil diameter in VAD application is limited to 70 mm [16].

Comparison between the operation at fixed resonant frequency and adaptation to the odd splitting frequency is shown in Figure 7. Usually, distance between the coils is limited to about 10–20 mm. Figure 7(c) shows that for a system operating at resonant frequency output power changes as 0.41–1.33 of the chosen nominal output power value (power transferred to the load at a distance equal to 15 mm and zero lateral displacement) for ±10 mm changes of a distance between the coils. While, for the system with operating frequency tuned to the splitting frequency output power changes as 0.81–1.04 of the nominal output power value for the conditions equal to described above. Thus, significant increase of
Figure 6. (■) Odd (Equation (19)) and (▲) even (Equation (20)) splitting frequencies calculated in dependence of (a) distance between the coils (lateral displacement is constant and equal to 0 mm) and (b) lateral displacement between the coils (distance between the coils is constant and equal to 15 mm). Odd splitting frequency changes more smoothly with changes of displacements and has narrower band. TET system parameters are given in Table 1. Transmitting and receiving coils geometrical parameters are equal and described in Table 2.

Table 2. Coils parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Outer diameter</td>
<td>70 mm</td>
</tr>
<tr>
<td>Wire cross-section diameter</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Number of turns</td>
<td>10</td>
</tr>
<tr>
<td>Distance between turns</td>
<td>1 mm</td>
</tr>
</tbody>
</table>

stability is shown in the case of frequency adaptation in comparison with the fixed frequency operation. It comes at the cost of lower power level, but it can be said that excess of power for the system with fixed operating frequency cannot be utilized due to great variation with changes in the distance between the coils. Similar observations can be made in the case of lateral displacements (see Figures 7(b) and 7(d)).

It is worth noting that for chosen TET system parameters quality factors are relatively small. Transmitting part quality factor is 11.5, and receiving part quality factor is 2.3. Increase of the quality factors will lead to decrease of critical coupling and, therefore, extending of stable energy transfer range.

4.5. Applicable Scope

Although stable output power for the changing value of displacements is shown, the obtained results are based on the assumption that mutual inductance was calculated with perfect precision. However, in practical application, coils displacements will be calculated with rated accuracy. Therefore, the error tolerance calculation is provided for the proposed approach.

Numerical modeling was completed to estimate the required accuracy of the coils positions determination. Figure 8 shows a comparison of output power calculated in absence of displacement estimation error and output power calculated for displacements calculated with error values of ±1 mm and ±2 mm. For most cases of TET usage, VAD maximum displacements very rarely exceed 10–20 mm. Therefore, error of 1–2 mm corresponds to the relative error in displacements determination about 10%, and this value can be used as the upper limit of the error for a positioning unit. Also, from Figures 8(a) and 8(b) we can observe that errors in coils position estimation can lead to underload as well as overload of a system. So, it should be taken into account during the system design procedure.
Figure 7. Power transferred to the load for system operating frequency tuned to the (▲) splitting frequency and for a system operating at (■) resonant frequency in dependence of (a) distance between the coils and (b) lateral displacement between the coils. Relative changes in power transferred to the load in respect to the value for a distance equal to 15 mm and zero lateral displacement for system operating frequency tuned to the (▲) splitting frequency and for a system operating at (■) resonant frequency in dependence of distance between the coils (c) and lateral displacement between the coils (d). Significant increase of stability is shown in the case of frequency adaptation in comparison with the fixed frequency operation. TET system parameters are given in Table 1. Transmitting and receiving coils geometrical parameters are equal and described in Table 2.

Figure 8. Power transferred to the load for system operating frequency tuned to splitting frequency in dependence of (a) distance between the coils and (b) lateral displacement between the coils. Displacement determination error: (■) none, (▲) ±1 mm, (●) ±2 mm. Error of 1–2 mm corresponds to the relative error in displacements determination about 10% and this value can be used as the upper limit of the error for a positioning unit. From Figures 8(a) and 8(b) we can observe, that errors in coils position estimation could lead to underload as well as overload of a system. TET system parameters are given in Table 1. Transmitting and receiving coils geometrical parameters are equal and described in Table 2.
5. CONCLUSION

One of the main concerns for TET systems is misalignments of coils, especially in the case of mechanical circulatory support systems, when coils are placed on a chest wall or an abdomen. By using theory which describes this effect, it is possible to find the relation between geometry of a system and operating frequency which provide stable output power.

Our approach is based on the utilization of a frequency splitting. By measuring the value of transmitted power and determining the position of the coils, demanded value of operating frequency (namely, odd frequency) can be calculated. After that, operating frequency should be automatically adapted accordingly to changes in coupling coefficient to increase energy transfer stability of the TET system.

Numerical calculations of TET parameters show the capability of the proposed method. It was found that the operation at splitting frequency provided more stable output with respect to changes in a system geometry. The output power of a proposed system changes not more than 5% for a distance in a range of 5–25 mm. At the same time, the output power of a system which operates at fixed resonant frequency changes for about 40%. Similar results were obtained for a lateral displacement in a range of 0–20 mm.

The system design must take into account a possible error in the estimation of the coils relative position. The error tolerance calculation is provided for the proposed approach, and it was shown that errors could lead to decrease as well as increase of output power. To deal with it, some kind of a feedback loop for the slight tuning of the operating frequency in a narrow band near the odd frequency can be useful.

It should be noted that we use the simple approximated expression for calculation of the splitting frequency. Also, the more exact expression could improve capability of our approach.

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REFERENCES


