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Modeling of Wave Propagation in General Dispersive Materials with Efficient ADE-WLP-FDTD Method

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Abstract—Within the framework of the finite-difference time-domain (FDTD) and the weighted Laguerre polynomials (WLPs), we derive an effective update equation of the electromagnetic in the dispersive media by introducing the factorization-splitting (FS) schemes and auxiliary differential equation (ADE). As two examples, we employ a 2-D parallel plate waveguide loaded with two dispersive medium columns and a thin grapheme sheet to calculate the plane wave propagation by using the FS-ADE-WLP-FDTD method. Compared with the ADE-FDTD and the ADE-WLP-FDTD methods, the results from our proposed method show its accuracy and efficiency for dispersive media simulation.

1. INTRODUCTION

The finite-difference time-domain (FDTD) method has been widely used for electromagnetic modeling due to its easy implementation [1]. However, because of Courant-Friedrich-Levy (CFL) stability constraint, the conventional FDTD is not very suitable for electromagnetic problems which involve fine grid division. To eliminate the limitation, some techniques, e.g., alternating-direction implicit (ADI) [2–4] and locally one-dimensional (LOD) [5–7] methods, were proposed. Although these techniques can get more accurate simulation results and higher computational efficiency than the conventional FDTD, a large time step inevitably results in a large numerical dispersion error. Also, an unconditionally stable FDTD method using Laguerre polynomials has been proposed [8]. This marching-on-in-order scheme shows better efficiency than the conventional FDTD method when analyzing multi-scale structure.

Based on auxiliary differential equation (ADE), an unconditionally stable WLP-FDTD was proposed to simulate electromagnetic wave propagation in general dispersive materials [9]. The method introduces an ADE technique which establishes the relationship between the electric displacement vector and electric field intensity with a differential equation rather than a convolution integral. However, it leads to a huge sparse matrix equation, which is very challenging to solve. To solve the huge sparse matrix equation, an efficient algorithm is regularly used to implement the WLP-FDTD method [10], in which the huge sparse matrix equation is solved into a sub-steps procedure with a factorized-splitting scheme.

In this paper, a hybrid algorithm, known as factorization-splitting ADE-WLP- FDTD, is presented to improve its simulation performance. Based on the FS and ADE technique, our proposed algorithm only solves two tri-diagonal matrices and computes one explicit equation in 2-D problem. In comparison with the conventional implementation, less CPU runtime is spent. The accuracy and efficiency of the proposed method is verified by simulating electromagnetic wave propagation in a variety of dispersive media.

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2. MATHEMATICAL FORMULATION

With lossless and dispersive media, the Maxwell's equations read

$$\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} = \nabla \times \mathbf{H}(\mathbf{r},t) - \mathbf{J}(\mathbf{r},t) \tag{1}$$

$$\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} = -\frac{1}{\mu_0}\nabla \times \mathbf{E}(\mathbf{r},t) \tag{2}$$

where μ_0 is the magnetic permeability of free space. The electric displacement vector $\mathbf{D}$ is related to the electric field intensity $\mathbf{E}$ through the relative dielectric constant ε_r of the local tissue by

$$\mathbf{D}(\omega) = \varepsilon_0\varepsilon_r(\omega)\mathbf{E}(\omega) \tag{3}$$

where ε_0 is the electric permittivity in free space. In the frequency domain, ε_r can be written as [9, 11]

$$\varepsilon_r(\omega) = \varepsilon_\infty\left(1 + \sum_n^{N_d} \frac{a_n}{b_n + j\omega c_n - d_n\omega^2}\right) \tag{4}$$

where ε_∞ is the infinite dielectric constant, ω the angular frequency, and a_n, b_n, c_n and d_n are known constants determined by the properties of the electric fields $\mathbf{E}(\omega)$. Substituting Eq. (4) into Eq. (3), we get

$$\mathbf{D}(\omega) = \varepsilon_0\varepsilon_\infty\left[\mathbf{E}(\omega) + \sum_n^{N_d}\mathbf{S}_n(\omega)\right] \tag{5}$$

with

$$\mathbf{S}_n(\omega) = \frac{a_n}{b_n + j\omega c_n - d_n\omega^2}\mathbf{E}(\omega) \tag{6}$$

In terms of the transition relationship $j\omega \to \partial/\partial t$, Eqs. (5) and (6) can be casted into

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0\varepsilon_\infty\left(\mathbf{E}(\mathbf{r},t) + \sum_{n=1}^{N_d}\mathbf{S}_n(\mathbf{r},t)\right) \tag{7}$$

$$b_n\mathbf{S}_n(\mathbf{r},t) + c_n\frac{\partial \mathbf{S}_n(\mathbf{r},t)}{\partial t} + d_n\frac{\partial^2 \mathbf{S}_n(\mathbf{r},t)}{\partial t^2} = a_n\mathbf{E}(\mathbf{r},t) \tag{8}$$

Substituting Eq. (7) into Eq. (1) results in

$$\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} + \sum_{n=1}^{N_d}\frac{\partial \mathbf{S}_n(\mathbf{r},t)}{\partial t} = \frac{1}{\varepsilon_0\varepsilon_\infty}\nabla \times \mathbf{H}(\mathbf{r},t) - \frac{1}{\varepsilon_0\varepsilon_\infty}\mathbf{J}(\mathbf{r},t) \tag{9}$$

Using the weighted Laguerre basis functions $\varphi_q(st)$, the field components can be expanded as [8]

$$\{\mathbf{E},\mathbf{H},\mathbf{S}(\mathbf{r},t)\} = \sum_{q=0}^{\infty}\{\mathbf{E}^q,\mathbf{H}^q,\mathbf{S}^q(\mathbf{r})\}\,\varphi_q(st) \tag{10}$$

where s, q are time-scale factor and the order of Laguerre functions, respectively. For an arbitrary field component $\mathbf{U}(\mathbf{r},\, t)$, for example, $\mathbf{E}$, $\mathbf{H}$, $\mathbf{S}(\mathbf{r},\, t)$, etc., the first and second derivatives of $\mathbf{U}(\mathbf{r},\, t)$ obey the following equations [8, 12], respectively,

$$\frac{\partial \mathbf{U}(\mathbf{r},t)}{\partial t} = s\sum_{q=0}^{\infty}\left[0.5\mathbf{U}^q(\mathbf{r}) + \sum_{k=0,q>0}^{q-1}\mathbf{U}^k(\mathbf{r})\right]\varphi_q(st) \tag{11}$$

$$\frac{\partial^2 \mathbf{U}(\mathbf{r},t)}{\partial t^2} = s^2\sum_{q=0}^{\infty}\left[\frac{\mathbf{U}^q(\mathbf{r})}{4} + \sum_{k=0,q>0}^{q-1}(q-k)\mathbf{U}^k(\mathbf{r})\right]\varphi_q(st) \tag{12}$$

Inserting Eqs. (10)–(12) into Eqs. (2), (8) and (9), multiplying both sides by $\varphi_p(st)$, and integrating over $st \in [0, \infty)$, we have

$$\mathbf{E}^q(\mathbf{r}) + \sum_{n=1}^{N_d} \mathbf{S}_n^q(\mathbf{r}) = \frac{2}{s\varepsilon_0\varepsilon_\infty}\nabla \times \mathbf{H}^q(\mathbf{r}) - \frac{2}{s\varepsilon_0\varepsilon_\infty}\mathbf{J}^q(\mathbf{r}) - 2\sum_{k=0,q>0}^{q-1}\mathbf{E}^k(\mathbf{r}) - 2\sum_{n=1}^{N_d}\sum_{k=0,q>0}^{q-1}\mathbf{S}_n^k(\mathbf{r}) \tag{13}$$

$$\mathbf{S}_n^q(\mathbf{r}) = \frac{1}{A_n}\left\{a_n\mathbf{E}^q(\mathbf{r}) - \sum_{k=0,q>0}^{q-1}\left[c_n s + d_n s^2(q-k)\right]\mathbf{S}_n^k(\mathbf{r})\right\} \tag{14}$$

$$\mathbf{H}^q(\mathbf{r}) = -\frac{2}{s\mu_0}\nabla \times \mathbf{E}^q(\mathbf{r}) - 2\sum_{k=0,q>0}^{q-1}\mathbf{H}^k(\mathbf{r}) \tag{15}$$

where $\mathbf{J}^q(\mathbf{r}) = \int_0^{T_f}\mathbf{J}(\mathbf{r},t)\varphi_p(st)d(st)$, $A_n = b_n + 0.5sc_n + 0.25s^2d_n$, and T_f is a finite time interval. Substituting Eq. (14) into Eq. (13), we may then write, instead of Eq. (13),

$$\begin{aligned}\left(1+\sum_{n=1}^{N_d}\frac{a_n}{A_n}\right)\mathbf{E}^q(\mathbf{r}) &= -2\sum_{k=0,q>0}^{q-1}\mathbf{E}^k(\mathbf{r}) + \sum_{n=1}^{N_d}\left(\frac{sa_n}{A_n}-2\right)\sum_{k=0,q>0}^{q-1}\mathbf{S}_n^k(\mathbf{r})\\ &\quad + \frac{2}{s\varepsilon_0\varepsilon_\infty}\nabla\times\mathbf{H}^q(\mathbf{r}) - \frac{2}{s\varepsilon_0\varepsilon_\infty}\mathbf{J}^q(\mathbf{r}) + \frac{s^2d}{A_n}\sum_{n=1}^{N_d}\sum_{k=0,q>0}^{q-1}(q-k)\mathbf{S}_n^k(\mathbf{r})\end{aligned} \tag{16}$$

Hence, Eqs. (15) and (16) can be written as a matrix equation form [8]. After obtaining the auxiliary differential variable $\mathbf{S}$ from Eq. (14), the electric fields are obtained by solving the matrix equation.

For the sake of simplicity, in the following sections we will employ a 2-D TE_z case and single pole dispersive media ($N_d = 1$) to describe the procedures for deriving the FS-ADE-WLP-FDTD algorithm, then the z-component of $\mathbf{H}^q(\mathbf{r})$ in (15) reads

$$H_z^q(\mathbf{r}) = \sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\sigma b D_\alpha E_\beta^q(\mathbf{r}) + V_H^{q-1}(\mathbf{r}) \tag{17}$$

where $b = 2/(\mu_0 s)$, $V_H^{q-1}(\mathbf{r}) = -2\sum_{k=0,q>0}^{q-1}H_z^k(\mathbf{r})$. $D_\alpha = \partial/\partial\alpha$ (α, $\beta = x$, y), is the first-order partial differential operator, and $\alpha = x$, $\sigma = -1$, $\alpha = y$, $\sigma = 1$. The α-components of $\mathbf{E}^q(\mathbf{r})$ and $\mathbf{S}_1^q(\mathbf{r})$ in Eqs. (14) and (16) are given by

$$E_\alpha^q(\mathbf{r}) = A_\alpha D_\beta H_z^q(\mathbf{r}) + J_{E\alpha}^q(\mathbf{r}) + V_{E\alpha}^{q-1}(\mathbf{r}) + V_{S\alpha}^{q-1}(\mathbf{r}) \tag{18}$$

$$S_{1\alpha}^q(\mathbf{r}) = 1/A_{1\alpha}\left\{a_{1\alpha}E_\alpha^q(\mathbf{r}) - \sum\nolimits_{k=0,q>0}^{q-1}\left[c_{1\alpha}s + d_{1\alpha}s^2(q-k)\right]S_{1\alpha}^k(\mathbf{r})\right\} \tag{19}$$

where A_α, $J_{E\alpha}^q$, $V_{E\alpha}^{q-1}$ and $V_{S\alpha}^{q-1}$ are given by

$$A_\alpha = A_{1\alpha}/[0.5\varepsilon_0\varepsilon_{\alpha,\infty}s(a_{1\alpha}+A_{1\alpha})] \tag{20}$$

$$J_{E\alpha}^q = -A_{1\alpha}J_\alpha^q(\mathbf{r})/[0.5\varepsilon_0\varepsilon_{\alpha,\infty}s(a_{1\alpha}+A_{1\alpha})] \tag{21}$$

with J_α^q describing the incident electric current excitation source along α axes.

$$V_{E\alpha}^{q-1}(\mathbf{r}) = -2A_{1\alpha}/(a_{1\alpha}+A_{1\alpha})\sum_{k=0,q>0}^{q-1}E_\alpha^k(\mathbf{r}) \tag{22}$$

$$V_{S\alpha}^{q-1}(\mathbf{r}) = (c_{1\alpha}s - 2A_{1\alpha})/(a_{1\alpha}+A_{1\alpha})\sum_{k=0,q>0}^{q-1}S_{1\alpha}^k(\mathbf{r}) + d_{1\alpha}s^2/(a_{1\alpha}+A_{1\alpha})\sum_{k=0,q>0}^{q-1}(q-k)S_{1\alpha}^k(\mathbf{r}) \tag{23}$$

Similar to the derivational procedure in [10], Eqs. (17)–(19) can be written as a matrix form

$$\mathbf{W}_E^q = \mathbf{D}_H\mathbf{W}_H^q + \mathbf{J}_E^q + \mathbf{V}_E^{q-1} + \mathbf{V}_S^{q-1} \tag{24}$$

$$\mathbf{W}_H^q = \mathbf{D}_E W_E^q + \mathbf{V}_H^{q-1} \tag{25}$$

where $\mathbf{W}_E^q = [E_x^q \;\; E_y^q]^T$, $\mathbf{W}_H^q = [H_z^q]$, $\mathbf{J}_E^q = [J_{Ex}^q \;\; J_{Ey}^q]^T$, $\mathbf{D}_H = [A_x D_y - A_y D_x]^T$, $\mathbf{D}_\mathrm{E} = [bD_y - bD_x]$, $\mathbf{V}_E^{q-1} = \left[V_{Ex}^{q-1} \;\; V_{Ey}^{q-1}\right]^T$, $\mathbf{V}_S^{q-1} = \left[V_{Sx}^{q-1} \;\; V_{Sy}^{q-1}\right]^T$. Combining Eqs. (24) and (25) leads to

$$\begin{bmatrix} \mathbf{W}_E^q \\ \mathbf{W}_H^q \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{D}_H \\ \mathbf{D}_E & 0 \end{bmatrix} \begin{bmatrix} \mathbf{W}_E^q \\ \mathbf{W}_H^q \end{bmatrix} + \begin{bmatrix} \mathbf{J}_E^q \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{V}_E^{q-1} \\ \mathbf{V}_H^{q-1} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_S^{q-1} \\ 0 \end{bmatrix} \tag{26}$$

Let $\mathbf{W}_{EH}^q = \left[\mathbf{W}_E^q \;\; \mathbf{W}_H^q\right]^T$, $\mathbf{J}_{EH}^q = \left[\mathbf{J}_E^q \;\; 0\right]$, $\mathbf{V}_{EH}^{q-1} = \left[\mathbf{V}_E^{q-1} \;\; \mathbf{V}_H^{q-1}\right]$ and $\mathbf{V}_{SH}^{q-1} = \left[\mathbf{V}_S^{q-1} \;\; 0\right]^T$, then Eq. (26) becomes

$$(\mathbf{I} - \mathbf{A} - \mathbf{B})\, \mathbf{W}_{EH}^q = \mathbf{V}_{EH}^{q-1} + \mathbf{V}_{SH}^{q-1} + \mathbf{J}_{EH}^q \tag{27}$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{D}_{Ha} \\ \mathbf{D}_{Ea} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -A_y D_x \\ 0 & -bD_x & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & \mathbf{D}_{Hb} \\ \mathbf{D}_{Eb} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_x D_y \\ 0 & 0 & 0 \\ bD_y & 0 & 0 \end{bmatrix}$$

Adding a perturbation term $\mathbf{AB}(\mathbf{W}_{EH}^q - \mathbf{V}_{EH}^{q-1})$ to Eq. (27), we can obtain the factorized form

$$(\mathbf{I} - \mathbf{A})\,(\mathbf{I} - \mathbf{B})\, \mathbf{W}_{EH}^q = \mathbf{AB}\mathbf{V}_{EH}^{q-1} + \mathbf{V}_{EH}^{q-1} + \mathbf{V}_{SH}^{q-1} + \mathbf{J}_{EH}^q \tag{28}$$

Equation (28) can be computed into two sub-steps as following,

$$(\mathbf{I} - \mathbf{A})\, \mathbf{W}_{EH}^{*q} = (\mathbf{I} + \mathbf{B})\, \mathbf{V}_{EH}^{q-1} + \mathbf{V}_{SH}^{q-1} + \mathbf{J}_{EH}^q \tag{29}$$

$$(\mathbf{I} - \mathbf{B})\, \mathbf{W}_{EH}^q = \mathbf{W}_{EH}^{*q} - \mathbf{B}\mathbf{V}_{EH}^{q-1} \tag{30}$$

where $\mathbf{W}_{EH}^{*q} = \left[\mathbf{W}_E^{*q} \;\; \mathbf{W}_H^{*q}\right]^T = \left[\mathbf{E}_x^{*q} \;\; \mathbf{E}_y^{*q} \;\; \mathbf{H}_z^{*q}\right]^T$. Using Eqs. (29) and (30) to solve Eq. (28) with some manipulations, we get

$$(\mathbf{I} - \mathbf{D}_{Ha}\mathbf{D}_{Ea})\, \mathbf{W}_E^{*q} = (\mathbf{D}_{Ha} + \mathbf{D}_{Hb})\, \mathbf{V}_H^{q-1} + (\mathbf{I} + \mathbf{D}_{Ha}\mathbf{D}_{Eb})\, \mathbf{V}_E^{q-1} + \mathbf{V}_S^{q-1} + \mathbf{J}_E^q \tag{31}$$

$$(\mathbf{I} - \mathbf{D}_{Hb}\mathbf{D}_{Eb})\, \mathbf{W}_E^q = (\mathbf{I} + \mathbf{D}_{Hb}\mathbf{D}_{Ea})\, \mathbf{W}_E^{*q} \tag{32}$$

$$\mathbf{W}_H^q = \mathbf{D}_{Eb}\mathbf{W}_E^q + \mathbf{D}_{Ea}\mathbf{W}_E^{*q} + \mathbf{V}_H^{q-1} \tag{33}$$

Expanding Eqs. (31)–(33) leads to

$$E_x^{*q} = A_x D_y V_H^{q-1} + V_{Ex}^{q-1} + V_{Sx}^{q-1} + J_{Ex}^q \tag{34}$$

$$E_y^q = E_y^{*q} \tag{35}$$

$$(I - bA_y D_{2x})\, E_y^{*q} = -A_y D_x V_H^{q-1} + V_{Ey}^{q-1} - bA_y D_x D_y V_{Ex}^{q-1} + V_{Sy}^{q-1} + J_{Ey}^q \tag{36}$$

$$(I - bA_x D_{2y})\, E_x^q = E_x^{*q} - bA_x D_y D_x E_y^{*q} \tag{37}$$

$$H_z^q = bD_y E_x^q - bD_x E_y^{*q} + V_H^{q-1} \tag{38}$$

where $D_{2\alpha}$ ($\alpha = x, y$) is the second-order partial differential operator. Substituting Eqs. (34) and (35) into Eqs. (36)–(38), we have

$$(I - bA_y D_{2x})\, E_y^q = -A_y D_x V_H^{q-1} + V_{Ey}^{q-1} - bA_y D_x D_y V_{Ex}^{q-1} + V_{Sy}^{q-1} + J_{Ey}^q \tag{39}$$

$$(I - bA_x D_{2y})\, E_x^q = A_x D_y V_H^{q-1} + V_{Ex}^{q-1} + V_{Sx}^{q-1} + J_{Ex}^q - bA_x D_y D_x E_y^q \tag{40}$$

$$H_z^q = bD_y E_x^q - bD_x E_y^q + V_H^{q-1} \tag{41}$$

Equations (39)–(41) are the update equations for efficient 2-D ADE-WLP-FDTD method. According to the central-difference scheme introduced by Yee, we discretize space Equations (39)–(41) and obtain the following form:

$$\left[1+\frac{bA_y\,|_{i,j}}{\Delta\bar{x}\,|_{i,j}}\left(\frac{1}{\Delta x\,|_{i,j}}+\frac{1}{\Delta x\,|_{i-1,j}}\right)\right]E_y^q\,|_{i,j}-\frac{bA_y\,|_{i+1,j}}{\Delta x\,|_{i,j}\;\Delta\bar{x}\,|_{i,j}}E_y^q\,|_{i+1,j}-\frac{bA_y\,|_{i-1,j}}{\Delta x\,|_{i-1,j}\;\Delta\bar{x}\,|_{i,j}}E_y^q\,|_{i-1,j}$$
$$=\frac{A_y\,|_{i,j}}{\Delta\bar{x}\,|_{i,j}}\left(V_H^{q-1}\,|_{i,j}-V_H^{q-1}\,|_{i-1,j}\right)+J_{Ey}^q\,|_{i,j}+V_{Ey}^{q-1}\,|_{i,j}+V_{Sy}^{q-1}\,|_{i,j}$$
$$-\frac{A_y\,|_{i,j}\;b}{\Delta y\,|_{i,j}\;\Delta\bar{x}\,|_{i,j}}\left(V_{Ex}^{q-1}\,|_{i,j+1}-V_{Ex}^{q-1}\,|_{i,j}-V_{Ex}^{q-1}\,|_{i-1,j+1}+V_{Ex}^{q-1}\,|_{i-1,j}\right) \tag{42}$$

$$\left[1+\frac{bA_x\,|_{i,j}}{\Delta\bar{y}\,|_{i,j}}\left(\frac{1}{\Delta y\,|_{i,j-1}}+\frac{1}{\Delta y\,|_{i,j}}\right)\right]E_x^q\,|_{i,j}-\frac{bA_x\,|_{i,j+1}}{\Delta y\,|_{i,j}\;\Delta\bar{y}\,|_{i,j}}E_x^q\,|_{i,j+1}-\frac{bA_x\,|_{i,j-1}}{\Delta y\,|_{i,j-1}\;\Delta\bar{y}\,|_{i,j}}E_x^q\,|_{i,j-1}$$
$$=-\frac{A_x\,|_{i,j}}{\Delta\bar{y}\,|_{i,j}}\left(V_H^k\,|_{i,j}-V_H^k\,|_{i,j-1}\right)+V_{Ex}^{q-1}\,|_{i,j}+J_{Ex}^q\,|_{i,j}+V_{Sx}^k\,|_{i,j}$$
$$-\frac{A_x\,|_{i,j}\;b}{\Delta x\,|_{i,j}\;\Delta\bar{y}\,|_{i,j}}\left(E_y^q\,|_{i+1,j}-E_y^q\,|_{i,j}-E_y^q\,|_{i,+1j-1}+E_y^q\,|_{i,j-1}\right) \tag{43}$$

$$H_z^q\,|_{i,j}=\frac{b}{\Delta y\,|_{i,j}}\left(E_x^q\,|_{i,j+1}-E_x^q\,|_{i,j}\right)-\frac{b}{\Delta x\,|_{i,j}}\left(E_y^q\,|_{i+1,j}-E_y^q\,|_{i,j}\right)-2\sum_{k=0,q>0}^{q-1}H_z^k\,|_{i,j} \tag{44}$$

Comparing Eqs. (39) and (40) with [10], one can find that some parameters determined by dispersive media, A_α, $\alpha = x$, y for example, are included.

3. NUMERICAL RESULTS

In order to validate the effectiveness of the FS-ADE-WLP-FDTD method, as the first example, we employ the wave transmission in a 2-D parallel plate waveguide with two dispersive medium columns, as depicted in Fig. 1. The staircase approximation is introduced to model dispersive medium columns. To improve the simulation precision, a fine grid division with cell size of 0.3 mm × 0.3 mm is applied to the staircase region. The graded mesh is applied to rest computational regions, and the maximal cell is 10 mm × 10 mm [9]. For simplicity, Mur's 1st-order absorbing boundary conditions are used to truncate the computational area [8].

The first dispersive medium column is Debye model, in which the relative complex permittivity is given by

$$\varepsilon_r(\omega)=\varepsilon_\infty+\frac{\varepsilon_{\rm S}-\varepsilon_\infty}{1+j\omega\tau} \tag{45}$$

where $\varepsilon_s = 4.301$, $\varepsilon_\infty = 4.096$ and $\tau = 2.294 \times 10^{-9}$. The second dispersive medium column is Lorentz model, in which the relative complex permittivity is given by

$$\varepsilon_r(\omega)=\varepsilon_\infty+(\varepsilon_{\rm S}-\varepsilon_\infty)\frac{G_1\omega_1^2}{\omega_1^2+2j\delta_1\omega-\omega^2} \tag{46}$$

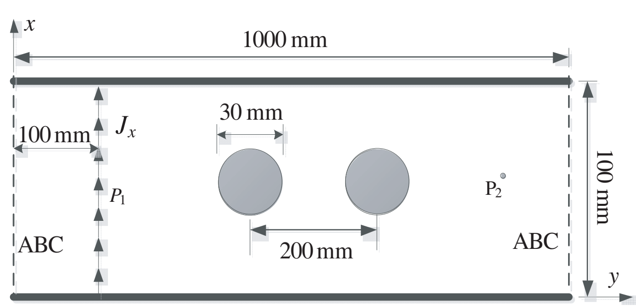

Figure 1. 2-D parallel plate waveguide with two dispersive media columns.

where $\varepsilon_s = 3$, $\varepsilon_\infty = 1.5$, $\omega_1 = 2 \times 10^9$ rad/s, $G_1 = 0.4$ and $\delta_1 = 0.1\omega_1$. A sinusoidally modulated Gaussian pulse is used as a x-incident electric current profile

$$J_x(t) = \exp\left[-\left(\frac{t-T_c}{T_d}\right)^2\right] \sin 2\pi f_c(t-T_c) \tag{47}$$

where $T_d = 1/(2f_c)$, $T_c = 3T_d$ and $f_c = 1$ GHz. And we choose the time duration $T_f = 11.71$ ns, time scaling factor $s = 1.1902 \times 10^{10}$ and order-marching step number $N_L = 142$.

Figure 2 shows the calculated results given by the FS-ADE-WLP-FDTD, ADE-WLP-FDTD and ADE-FDTD. From their profiles, one can find that the FS-ADE-WLP-FDTD is accurate.

Table 1 shows the required computational resource and computing time for the numerical simulations. Compared with the ADE-WLP-FDTD and the ADE-FDTD, the FS-ADE-WLP-FDTD shows much improvement in computation efficiency. All calculations have been performed on an AMD Phenom II $\times$ 6 2.80 GHz machine with 8 GB RAM.

In the second example, the transmission coefficient of wave propagation in graphene sheets is calculated, as shown in Fig. 3. Here, we also choose x-polarization as the electric current excitation, and $T_c = 3T_d$, $f_c = 5000$ GHz, the time duration $T_f = 1.5 \times 10^{-12}$ s, time scaling factor $s = 3.7699 \times 10^{14}$ and order-marching step number $N = 150$. Due to the structure with a thin layer in the computational domain, a fine grid division with the cell size of 1 nm $\times$ 1500 nm is applied to the graphene layer. The graded mesh is applied to the rest computational regions, and the maximal cell is 1500 nm $\times$ 1500 nm. In this example, the dispersive model of grapheme can be written as

$$\varepsilon_r(\omega) = \left(1 + \frac{\sigma_0/\varepsilon_0}{j\omega - \tau\omega^2}\right) \tag{48}$$

with

$$\sigma_0 = \frac{e^2 \tau k_B T}{\pi \hbar^2 \Delta}\left(\frac{\mu_c}{k_B T} + 2\ln\left(e^{-\frac{\mu_c}{k_B T}} + 1\right)\right)$$

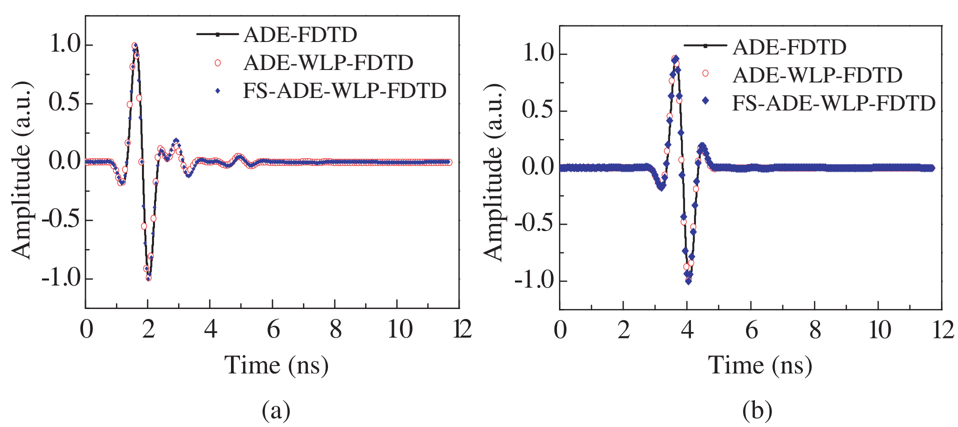

Figure 2. Transient electric fields of the x component (a) at P_1 and (b) P_2.

Table 1. Comparison of the computational efforts for the 2-D waveguide.

Method	Δt (ps)	Meshing size	Marching-on steps	Memory (MB)	CPU time(s)
ADE-FDTD	0.5	320 $\times$ 120	23420	5.2	710
ADE-WLP-FDTD	30	320 $\times$ 120	142	103	242
FS-ADE-WLP-FDTD	30	320 $\times$ 120	142	97	60

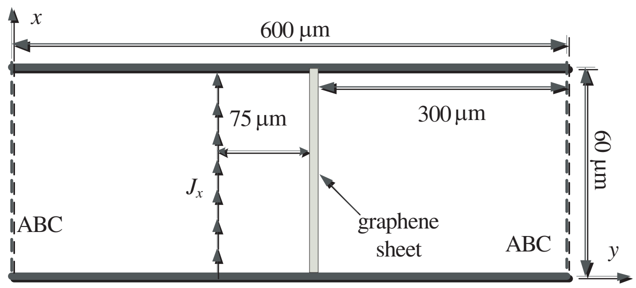

Figure 3. Diagram of computational domain for WLP-FDTD analysis of graphene sheet.

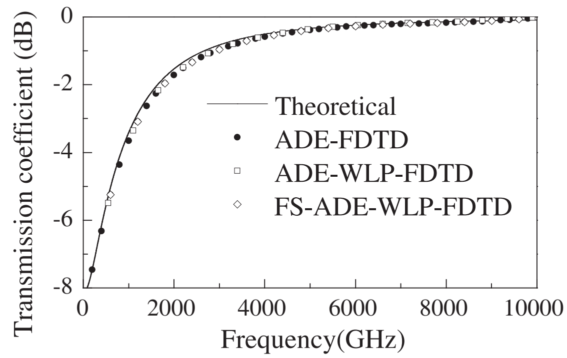

Figure 4. Transmission coefficient calculated with the FS-ADE-WLP-FDTD, ADE-FDTDADE-WLP-FDTD and the theoretical solution.

where Δ, $e, \hbar = h/2\pi$, k_{B}, T, τ and μ_c are the thickness of graphene sheets, electron charge, reduced Plank's constant, Boltzmann constant, temperature, scattering time and chemical potential, respectively [13]. Fig. 4 plots the numerical results of FS-ADE-WLP-FDTD, ADE-WLP-FDTD, ADE-FDTD and theory by setting $\Delta = 10\,\mathrm{nm}$, $\mu_c = 0.5\,\mathrm{eV}$, $T = 300\,\mathrm{K}$ and $\tau = 0.5 \times 10^{-12}\,\mathrm{s}$. Compared with the theoretical solution, the accuracy of the FS-ADE-WLP-FDTD method is verified.

Table 2 shows the comparison of the computing times among the three numerical methods. In Table 2, the FS-ADE-WLP-FDTD method also shows much more improvement in computation efficiency than the ADE-WLP-FDTD and ADE-FDTD methods.

Table 2. Comparison of the computational efforts for the graphene sheet.

Method	Δt (fs)	Meshing size	Marching-on steps	Memory (MB)	CPU time(s)
ADE-FDTD	1.67×10^{-3}	462×40	9×10^5	29	1722
ADE-WLP-FDTD	2.5	462×40	150	52	52
FS-ADE-WLP-FDTD	2.5	462×40	150	50	10

4. CONCLUSION

An ADE-WLP-FDTD method based on factorization splitting technique for general dispersive media is presented in this paper. Compared with the ADE-FDTD and ADE-WLP-FDTD, the FS-ADE-WLP-FDTD method can reduce the calculation burden. Two examples verify the accuracy and efficiency of the FS-ADE-WLP-FDTD method.

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