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Electromagnetic Waves Radiation by a Vibrators System with Variable Surface Impedance

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Abstract—The problem of electromagnetic waves radiation by a vibrators system with variable distributed surface impedance along their axes located in free space is solved by the generalized method of induced electromotive forces (EMF). The distinctive peculiarity of this method is the use of the functional distributions, obtained as a result of the analytical solution of the integral equation for the current by the asymptotic averaging method before, as the basic approximations for the currents along the impedance vibrators. The multi-parameter characteristics of three-element and multi-element antennas with variable impedance vibrators are calculated.

1. INTRODUCTION

Systems of perfectly conducting vibrators are widely used both as antennas with directional axial radiation and as a multi-element antenna array in the meter and decimeter wave bands [1–10]. An additional parameter, which allows forming a required amplitude-phase distribution of vibrators currents, and thus, modifying and optimizing the electromagnetic characteristics of the system as a whole, can be distributed surface impedance (both constant and variable along the vibrator length) [11–16]. The length of impedance vibrators can be both shorter or longer than that of a perfectly conducting vibrator. This is particularly important when there exist restrictions on radiator dimensions. Here we present a mathematical model of the impedance vibrator systems in the free space, characterized by the surface impedances and by its distribution functions along the vibrators length.

2. FORMULATION OF THE PROBLEM AND SOLUTION OF INTEGRAL EQUATIONS FOR THE CURRENTS

Consider a system consisting of N parallel impedance vibrators in the free space. Let $2L_n$ and r_n be the length and radius of the n-th vibrator, respectively. The vibrators centers in the Cartesian coordinate system are z_n, x_n, y_n. The projection of electric fields $E_{0s_n}(s_n)$ of extraneous sources on the n-th vibrator axis ($n = 1, 2, \ldots, N$) can be decomposed into two parts relative to the geometric center of the vibrator: a symmetric $E^s_{0s_n}(s_n)$ and anti-symmetric $E^a_{0s_n}(s_n)$ marked by the superscripts s and a. The variable s_n is the local coordinate along the axis of the n-th vibrator. A system of integral equations relative to vibrators currents $J_n(s_n)$ can be written as follows [11]

$$\sum_{n=1}^{N}\left(\frac{d^2}{ds_m^2}+k^2\right)\int_{-L_n}^{L_n} J_n(s'_n)G_{s_m}(s_m,s'_n)\,ds'_n = -i\omega[E_{0s_m}(s_m)+z_{im}(s_m)J_m(s_m)], \quad m=1,2,\ldots,N, \quad (1)$$

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where $z_{im}(s_m) = r_{im} + ix_{im}(s_m)$ is the internal complex impedance per unit length [Ohm/m] of the m-th vibrator, which may be variable along the vibrator length; $k = 2\pi/\lambda$, λ is the wavelength in free space; ω is the circular frequency.

Since the fields of extraneous source are presented by two components, each vibrator current consists of two terms, symmetric and antisymmetric, i.e., $J_n(s_n) = J_n^s(s_n) + J_n^a(s_n)$. Let us now present the vibrator currents as a product of the unknown complex amplitudes and predefined scalar function $f_{nq}^{s,a}(s_n)(q = 0, 1, \ldots, Q)$ as

$$J_n^{s,a}(s) = \sum_{q=0}^{Q} J_{nq}^{s,a} f_{nq}^{s,a}(s_n), \quad f_{nq}^{s,a}(\pm L_n) = 0. \tag{2}$$

Then the system of Equation (1) can be written as

$$\sum_{n=1}^{N}\sum_{q=0}^{Q}\left(\frac{d^2}{ds_m^2}+k^2\right)\int_{-L_n}^{L_n}\left[\begin{array}{c} J_{nq}^s f_{nq}^s(s_n') \\ +J_{nq}^a f_{nq}^a(s_n')\end{array}\right] G_{s_m}(s_m, s_n')ds_n' - i\omega z_{im}(s_m)\sum_{p=0}^{Q}\left[J_{mq}^s f_{mq}^s(s_m) + J_{mq}^a f_{mq}^a(s_m)\right]$$
$$= -i\omega[E_{0s_m}^s(s_m) + E_{0s_m}^a(s_m)]. \tag{3}$$

Let us multiply, following the generalized method of induced EMF [11, 13], the left- and right-hand sides of Equation (3) by $f_{mp}^s(s_m)$ and $f_{mp}^a(s_m)$ $(p = 0, 1, \ldots, Q)$ and integrate results over the length of the vibrators. Thus, we arrive at the system of linear algebraic equations for the current amplitudes J_{nq}^s and J_{nq}^a

$$\left\{\begin{array}{l} \sum\limits_{n=1}^{N}\sum\limits_{q=0}^{Q}\left[J_{nq}^s\left(Z_{mn,pq}^{ss}+\delta_{mn}\tilde{Z}_{m,pq}^{ss}\right)+J_{nq}^a\left(Z_{mn,pq}^{sa}+\delta_{mn}\tilde{Z}_{m,pq}^{sa}\right)\right] = -\dfrac{i\omega}{2k}E_{0\,mp}^s, \\ \sum\limits_{n=1}^{N}\sum\limits_{q=0}^{Q}\left[J_{nq}^s\left(Z_{mn,pq}^{as}+\delta_{mn}\tilde{Z}_{m,pq}^{as}\right)+J_{nq}^a\left(Z_{mn,pq}^{aa}+\delta_{mn}\tilde{Z}_{m,pq}^{aa}\right)\right] = -\dfrac{i\omega}{2k}E_{0\,mp}^a, \end{array}\right. \tag{4}$$

where

$$Z_{mn,pq}^{\left\{\begin{smallmatrix} ss\\ aa\\ sa\\ as\end{smallmatrix}\right\}} = \frac{1}{2k}\int_{-L_m}^{L_m} f_{mp}^{\left\{\begin{smallmatrix} s\\ a\\ s\\ a\end{smallmatrix}\right\}}(s_m)\left[\left(\frac{d^2}{ds_m^2}+k^2\right)\int_{-L_n}^{L_n} f_{nq}^{\left\{\begin{smallmatrix} s\\ a\\ a\\ s\end{smallmatrix}\right\}}(s_n')G_{s_m}(s_m,s_n')ds_n'\right]ds_m,$$

$$\tilde{Z}_{m,pq}^{\left\{\begin{smallmatrix} ss\\ aa\\ sa\\ as\end{smallmatrix}\right\}} = -\frac{i\omega}{2k}\int_{-L_m}^{L_m} f_{mp}^{\left\{\begin{smallmatrix} s\\ a\\ s\\ a\end{smallmatrix}\right\}}(s_m) f_{mq}^{\left\{\begin{smallmatrix} s\\ a\\ a\\ s\end{smallmatrix}\right\}}(s_m)z_{im}(s_m)ds_m,$$

$$E_{0\,mp}^{s,a} = \int_{-L_m}^{L_m} f_{mp}^{s,a}E_{0s_m}^{s,a}(s_m)ds_m, \quad \delta_{mn} = \left\{\begin{array}{l} 1 \text{ if } m = n, \\ 0 \text{ if } m \neq n\,. \end{array}\right.$$

As an example, let us consider the Yagi-Uda antenna [2–10] (Fig. 1).

Let the vibrators be arranged so that their central points are on the z-axis of the Cartesian coordinate system, and the longitudinal axes of the vibrators are oriented parallel to the x-axis. Let us enumerate the vibrators $n = 1, 2, \ldots, N$ by their position on the z-axis, so that $n = 1$ and $n = 2$ correspond to the active vibrator and reflector, respectively, and the remaining vibrators are directors. The active vibrator $(n = 1)$ is excited at its center $(s_1 = 0)$ by δ-generator of harmonic oscillations with voltage amplitude V_0. Thus, the projection of the electric field of extraneous sources on the longitudinal axis of the active vibrator has only symmetric component $E_{0s_1}(s_1) = E_{0s_1}^s(s_1) = V_0\delta(s_1)$ and the fields $E_{0s_n}(s_n) = 0$ for $n = 2, 3, \ldots, N$. Let us approximate the current distribution on

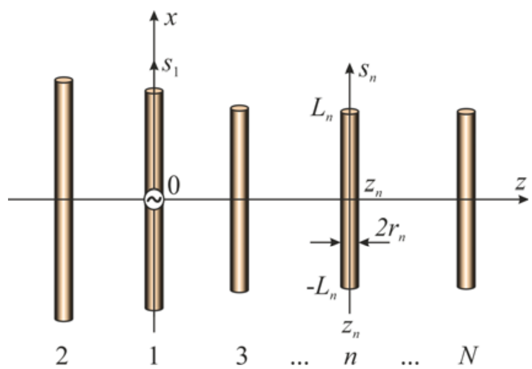


Figure 1. The configuration of Yagi-Uda antenna.

the active vibrator by functions $f_{10}^s(s_1) = \sin\tilde{k}(L_1 - |s_1|)$ and $f_{11}^s(s_1) = \cos\tilde{k}s_1 - \cos\tilde{k}L_1$, while the currents on the passive vibrators by functions $f_{n1}^s(s_n) = \cos\tilde{k}s_n - \cos\tilde{k}L_n$ $(n = 2, 3, \ldots, N)$. Here $\tilde{k} = k - \frac{i2\pi z_{in}^{av}}{Z_0\Omega}$, $z_{in}^{av} = \frac{1}{2L_n}\int\limits_{-L_n}^{L_n} z_{in}(s_n)ds_n$ is the mean internal impedance along the vibrator length [11], $Z_0 = 120\pi$ Ohm and $\Omega = 2\ln(2L_n/r_n)$. The current distribution functions were obtained as solutions of the integral equation for the current on a solitary impedance vibrator by the averaging method [11, 12]. The impedance distribution along the vibrators can be represented by $z_{in}(s_n) = z_{in}^{av}\,\phi_n(s_n)$, where the distribution functions $\phi_n(s_n)$ are normalized so that mean values over the vibrator length were equal to unit. In general case, the normalized surface impedance of the vibrator $\bar{Z}_{Sn} = 2\pi r_n z_{in}/Z_0$ can be complex so that $\bar{Z}_{S_n} = \bar{R}_{S_n} + i\bar{X}_{Sn}$. If $\bar{X}_{Sn} > 0$ the impedance is of inductive type and $\bar{X}_{Sn} = kr_nC_{Ln}$, and if $\bar{X}_{Sn} < 0$, the impedance is of capacitive type and $\bar{X}_{Sn} = -C_{Cn}/(kr_n)$ where the constants C_{Ln} and C_{Cn} are defined by the vibrator dimensions and physical parameters of the vibrator material. Formulas defining specific realizations of the vibrator surface impedance are given in Appendix A.

The radiation pattern of Yagi-Uda antenna (Fig. 1) can be written as follows

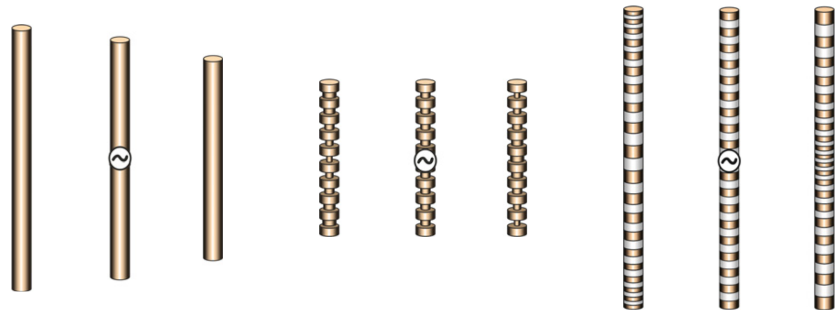
$$F(\theta,\varphi) = \frac{1}{F_m}\sin\left(\arccos\left(\sin\theta\cos\varphi\right)\right)\sum_{n=1}^{N}\left[e^{ikz_n\cos\theta}\int\limits_{-L_n}^{L_n} J_n(s_n)e^{iks_n\sin\theta\cos\varphi}ds_n\right],$$

where $1/F_m$ is the normalizing factor. The radiation patterns in $\vec{E}$- and $\vec{H}$-vector's planes may be defined as $F_E = F(\theta, \varphi = 0°)$ and $F_H = F(\theta, \varphi = 90°)$, respectively.

3. NUMERICAL RESULTS

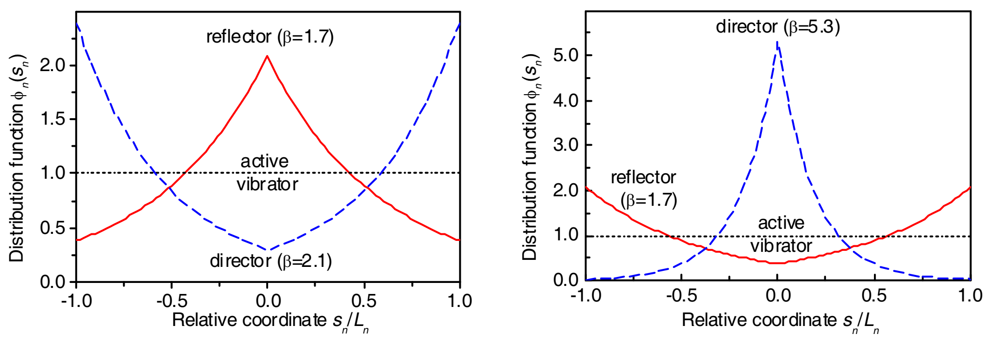
It is known that the characteristics of the Yagi-Uda antenna can be varied by adjusting the lengths of vibrators and distances between them. To obtain axial radiation of the Yagi-Uda antenna composed of perfectly conducting vibrators, the directors should be slightly shorter and the reflector should be longer than the active vibrator (Fig. 2(a)). Recent studies have shown that the reactance of the vibrators in the Yagi-Uda array required to obtain the necessary input parameters and radiation characteristics can also be achieved by variable the magnitude and distribution function of the surface impedance of the fixed length vibrator. The electrical length of the impedance vibrators can be both shorter and longer than that of the perfectly conducting vibrator.

The inductive surface impedance reduces the vibrator resonant length; therefore, the Yagi-Uda antenna with impedance vibrators can be used, when the weight and size parameters are significant. The capacitive impedance and, hence, longer vibrators can increase the antenna input resistance. Let us consider several functions defining distribution of the vibrator surface impedance, namely: constant distribution function (CDF), and decreasing (DDF) and increasing (IDF) distribution functions. All these function have equal mean and are symmetrical relative to the vibrator center. The inductive impedance with DDF and the capacitive impedance with IDF increase the vibrator resonant length



(a) (b) (c)

Figure 2. Three-element Yagi-Uda antenna array: (a) perfectly conducting vibrators ($2L_1 = 0.44\lambda_0$, $2L_2 = 0.5\lambda_0$, $2L_3 = 0.38\lambda_0$); (b) inductive impedance vibrators ($2L_n = 0.35\lambda_0$); (c) capacitive impedance vibrators ($2L_n = 0.65\lambda_0$).



(a) (b)

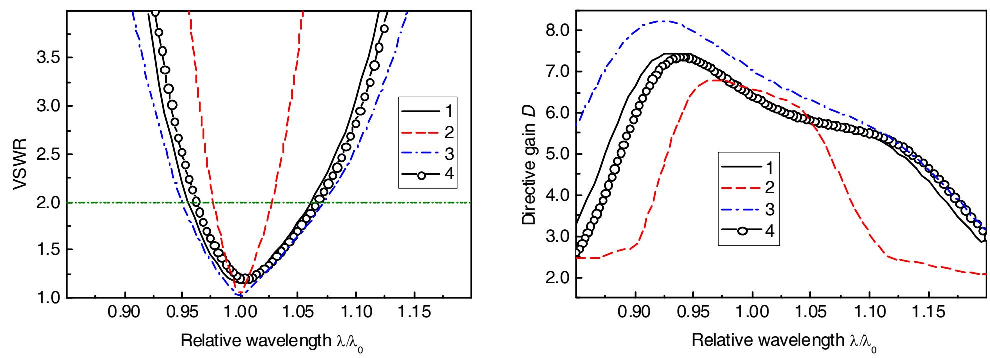
Figure 3. The distribution functions of the surface impedance of vibrators in Yagi-Uda arrays: (a) the inductive impedance; (b) the capacitive impedance (reflector, director, active vibrator).

relative to the impedance with CDF, while the inductive impedance with IDF and the capacitive impedance with DDF decrease the resonant length. That is, the DDF increases impedance while the IDF decreases it. This effect can be used during the design of antenna arrays. Let's illustrate this possibility by numerical calculations of electrodynamic characteristics for a three-element Yagi-Uda antenna array with the equal-length vibrators and the variable inductive impedance $\bar{Z}_{Sn}(s_n) = i\bar{X}_{Sn}^{av}\phi_n(s_n)$ (Fig. 2(b)) and capacitive impedance $\bar{Z}_{Sn}(s_n) = -i\bar{X}_{Sn}^{av}\phi_n(s_n)$ (Fig. 2(c)). Since the normalized surface impedances $\bar{X}_{Sn}^{av}$ of all vibrators in the array are equal, phases of currents in the radiators to ensure the axial radiation can be obtained by selection of the surface impedance distribution functions. The geometric parameters of array elements and impedances $\bar{X}_{Sn}^{av}$ should be selected so as to match the antenna input impedance at operating wavelength λ_0 and the feeder line characteristic impedance and to provide a low voltage standing-wave ratio (VSWR) in the feeder line of the active vibrator.

Consider the exponentially decreasing and exponentially increasing distribution functions $\phi_n(s_n) = \alpha\exp[-\beta|s_n|/L_n]$ and $\phi_n(s_n) = \alpha\exp[-\beta(|s_n|/L_n - 1)]$, where $\alpha = \beta/[1-\exp(-\beta)]$ is the normalization factor, and β is the arbitrary dimensionless constant. The functions $\phi_n(s_n)$ for the three antennas shown in Fig. 2 are plotted in Fig. 3.

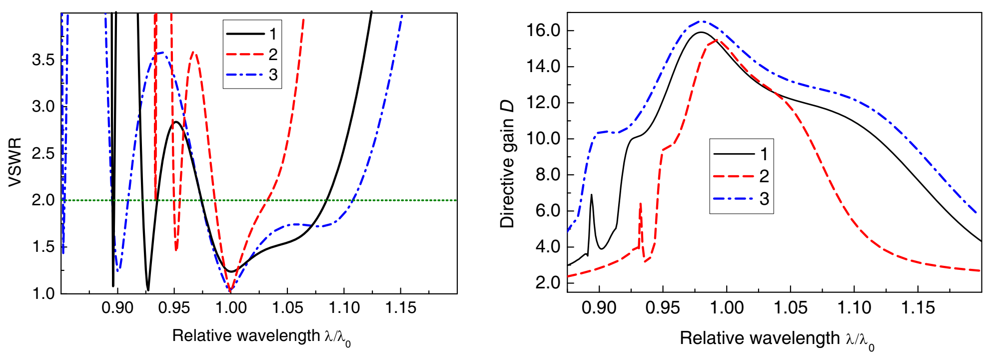
The plots of VSWR in feeder lines with wave resistance $W = 50\,\text{Ohm}$ (curves 1, 4), $W = 25\,\text{Ohm}$ (curve 2), and $W = 75\,\text{Ohm}$ (curve 3) and directivity D versus the wavelength are shown in Fig. 4 for the three-element Yagi-Uda arrays (Fig. 2) with the impedance distribution presented in the Fig. 3 ($C_{Ln}^{av} = 1.448$, $C_{Cn}^{av} = 5.466 \times 10^{-3}$, $r_n = 0.01\lambda_0$, $z_2 = -0.25\lambda_0$, $z_3 = 0.2\lambda_0$).

As can be seen from the plots in Fig. 4(a), the antenna array with impedance vibrators and the feed line can be better matched than the antenna with perfectly conducting vibrators. Fig. 5 shows similar plots for seven-element arrays with perfectly conducting vibrators ($2L_1 = 0.45\lambda_0$, $2L_2 = 0.5\lambda_0$, $2L_{3-7} = 0.4\lambda_0$, $W = 50\,\text{Ohm}$), with impedance vibrators (variable impedance of inductive type:



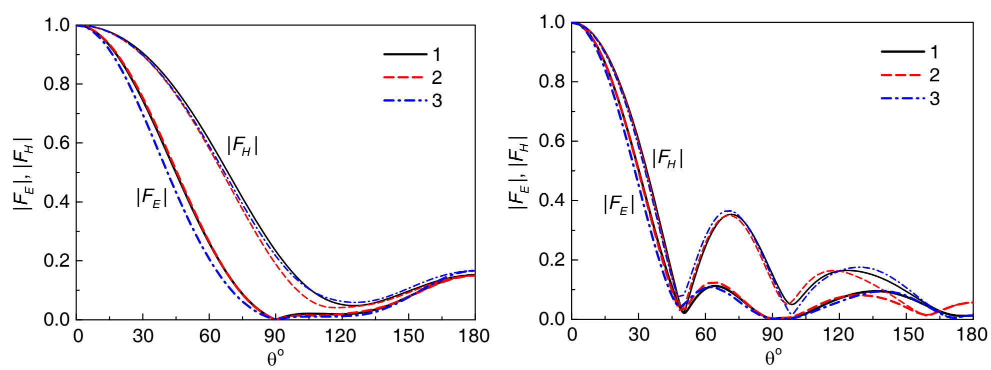
(a) (b)

Figure 4. VSWR and D versus the wavelength for the three-element Yagi-Uda antenna: curves 1, 4 — perfectly conducting vibrators, curve 4 was obtained by the method of moments with piecewise constant basis vibrators; curve 2 — vibrators with variable impedance of inductive type; curve 3 — vibrators with variable capacitive impedance.



(a) (b)

Figure 5. VSWR and D versus the wavelength for the seven-element Yagi-Uda antenna: curve 1 — perfectly conducting vibrators, curve 2 — vibrators with variable impedance of inductive type; curve 3 — vibrators with variable capacitive impedance.



(a) (b)

Figure 6. The radiation patterns of the (a) three-element and (b) seven-element Yagi-Uda antennas: curves 1 — perfectly conducting vibrators, curves 2 — vibrators with variable impedance of inductive type; curves 3 — vibrators with variable capacitive impedance.

$2L_n = 0.35\lambda_0$, $C_{Ln}^{av} = 1.464$, $\beta = 1.7$ for the reflector and directors, $W = 25\,\text{Ohm}$), and with impedance vibrators (variable impedance of capacitive type: $2L_n = 0.65\lambda_0$, $C_{Cn}^{av} = 5.341 \times 10^{-3}$, $\beta = 1.7$ for the reflector and $\beta = 4.5$ for the directors, $W = 75\,\text{Ohm}$). The other parameters are as follows: $r_n = 0.01\lambda_0$, $z_2 = -0.25\lambda_0$, $z_3 = 0.2\lambda_0$, and the distances between the directors are $0.2\lambda_0$.

As can be seen from the plots in Fig. 4(a) and Fig. 5(a), the array with the impedance vibrators is better matched with feed line than perfectly conducting vibrators. The inductive impedance decreases and capacitive impedance increases the array operating band defined using VSWR (for example, at the level VSWR = 2) and the directivity D as compared with the case $\bar{Z}_{Sn} = 0$.

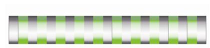
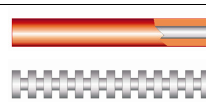
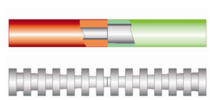
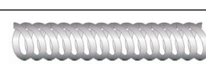
The radiation patterns of the Yagi-Uda antennas at $\lambda = \lambda_0$ are shown in Fig. 6.

4. CONCLUSION

The problem solution presented in the paper can be used as a basis for multi-parameter optimization of electrodynamic characteristics of radiating multi-element structures built on vibrators with variable distributed surface impedance. The distinctive peculiarity of the method, proposed by the authors, is the use of the approximating functions, resulting from the integral equation solution for the current by the asymptotic averaging method, in the current distribution along the impedance vibrator. The ground of rightness and correctness of such an approach is represented in the format of comparative analysis with the calculated results by the method of moments. One would note that the new conception of the generalized method of induced EMF, keeping all known advantages of numerical-analytical methods in comparison with direct numerical methods, extends to the cases of the vibrator with the impedance, variable along its length, and the impedance vibrators systems rather simply. Thus the proposed generalized method of induced EMF allows to widen the boundaries of numerical-analytical investigations of practically significant problems of the impedance vibrators application sufficiently.

APPENDIX A. SURFACE IMPEDANCE OF VIBRATORS

Formulas determining the distributed surface impedance of electrically thin vibrators (material parameters are: permittivity ε, permeability μ, and conductivity σ) have the following form

No	The vibrator design	Vibrator model	Impedance
1	Solid metal cylinder. The radius satisfy inequality $r >> \Delta^0$, Δ^0 is skin layer thickness.		$\bar{Z}_S = \frac{1+i}{120\pi\sigma\Delta^0}$
2	Metallized dielectric cylinder. Metal layer thickness is $h_R << \Delta^0$.		$\bar{Z}_S = \frac{1}{120\pi\sigma h_R + ikr(\varepsilon - 1)/2}$
3	Metal-dielectric cylinder. L_1 is the thickness of a metal discs, L_2 is the thickness of a dielectric disks.		$\bar{Z}_S = -i\,\frac{L_2}{L_1+L_2}\frac{2}{kr\varepsilon}$
4	Magnetodielectric metalized cylinder. r_i is the radius of internal conducting cylinder.		$\bar{Z}_S = \frac{1}{120\pi\sigma h_R - i/kr\mu \ln(r/r_i)}$
5	Metal cylinder coated with magnetodielectric layer, which thickness is $r - r_i$, or corrugated cylinder $(L_1+L_2) << \lambda$, where L_1 is crests thickness where $Z_S = 0$, L_2 is the notch width where $Z_S \neq 0$.		$\bar{Z}_S = i/kr\mu \ln(r/r_i)$
			$\bar{Z}_S(s) = \bar{Z}_S\phi(s)$
6	Metal monofilar helix. r is helix radius $kr << 1$, ψ is winding angle.		$\bar{Z}_S = (i/2)kr\,\mathrm{ctg}^2\psi$

Formulas for surface impedances of vibrators are derived in the frame of the impedance concept [11] and valid for thin cylinders $|(k\sqrt{\varepsilon\mu}r)^2 \ln(k\sqrt{\varepsilon\mu}r_i)| \ll 1$ both for finite and infinite cylinders, located in the hollow electrodynamic volume. If vibrators are in a material medium with parameters ε_1 and μ_1, all above formulas must contain the factor $\sqrt{\mu_1/\varepsilon_1}$.

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