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Electromagnetic Levitation of Nonmagnetic Disc

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Abstract—The paper presents analytical solution of nonmagnetic and conductive disc levitation problem. The alternating magnetic field exerts eddy currents in conductive disc and levitation force, subsequently. The electromagnetic field and eddy currents distributions are determined. The force acting upon nonmagnetic disc (Lorentz, Maxwell, coenergy methods) and power losses (Joule volume integral, Poynting surface integral methods) are evaluated. For example, levitation force and power losses versus field frequency are figured out. Additionally, an optimization task for power losses at constant disc volume is solved.

1. INTRODUCTION

In many engineering applications, solutions of Maxwell equations are required over a wide parameters range. The nature of the analytical solution provides a set of general information about influence of admissible parameters on integral quantities. Often, the solution is obtained for wide range of frequency, conductivity and other parameters. Moreover, with respect to the taken assumptions it can be treated as an accurate solution of given problem. In practice, the analytical solution can be used for two main purposes. Firstly, analytical solution is a benchmark test for numerical procedures. Secondly, the analytical solution constitutes a start point for numerical methods that solve nonlinear problems. Moreover, analytical solution could minimize the computational effort by designing hybrid solutions of both analytical and numerical procedures.

The objective of this paper is to present analytical solution and state some conclusions for nonmagnetic disc levitation in alternating magnetic field. The present paper is structured as follows: analysis of excitation in axial field, superposition with radial field, levitation force evaluation by three methods (Lorentz, Maxwell, coenergy), checking power balance and giving examples of the proposed approach.

The novelty of the solution presented is taking into account both axial and radial magnetic fields inductions.

2. ELECTROMAGNETIC LEVITATION

Electromagnetic levitation is a phenomenon that may lead to raising an object in gravitational field by exerting electromagnetic force [1, 3, 4]. Electromagnetic levitation systems have received wide attention recently because of their practical importance in many engineering systems such as high-speed trains, frictionless bearings, vibration isolation of sensitive actuators, levitation of molten metal in induction furnaces [2, 4, 6–9].

Particularly, the levitation could be caused by:

- Inhomogeneity force component and/or hysteresis phenomenon (magnetic levitation),

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- Lorentz force acting upon currents induced in an conductive object (electromagnetic levitation).
- Poynting component (electromagnetic field momentum component) that is out of technical interests due to significantly high field frequency needed.

Magnetic levitation caused by inhomogeneity force component appears on outer surface of ferromagnetic objects where the reluctivity changes [10–15]. Magnetic levitation can also be caused by magnets (permanent, electromagnets) especially in maglev techniques. However, the nonmagnetic objects may levitate mostly in alternating magnetic field as a result of electromagnetic induction in conductive nonmagnetic object (e.g., disc). The levitation force is a result of the induced eddy currents in magnetic field. In order to obtain levitation force of needed value, the magnetic field has to be designed in a certain way. There are necessary axial and radial fields for causing the levitation. It should be pointed out that the eddy currents in conductive object lead to power losses, and consecutively, the object (feedstock, droplet) may be molten without any pot.

3. MAGNETIC FIELD EXCITATION

Electromagnetic levitation may appear while Lorentz forces could lift an object in the presence of gravitation field. Lorentz forces act on currents induced in conductive object, e.g., cylinder (disc) in magnetic field. For nonmagnetic object

$$\mu = \mu_0, \tag{1}$$

only Lorentz force can lift them in gravitation field. Due to assumption that there are forces induced by magnetic hysteresis, permanent magnets, magnetic anisotropy and reluctivity change [1, 3, 11, 12, 16–18]. The electromagnetic levitation is immanently connected with the presence of power losses. The power losses can melt the levitating object as well without putting them into any pot.

Let us consider a cylinder (disc, plate) in which symmetry axis z is vertical, i.e., parallel to gravitational field lines — Fig. 1. The axial magnetic field B_0 is induced by certain coils, and this design is another problem [4, 6, 8].

The magnetic field is harmonic at frequency f. The complex notation for force calculation is applied. For axial field ($B_\phi = 0$, $B_\rho = 0$, derivatives vanish with respect to the angle $\partial/\partial\varphi = 0$, Fig.

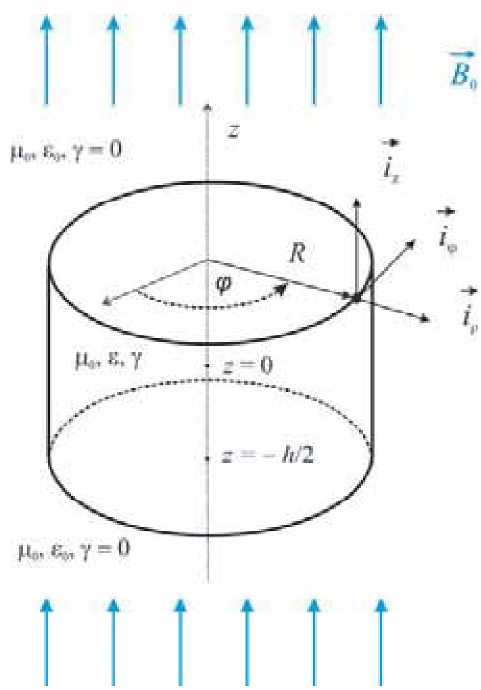

Figure 1. Cylinder (disc) in axial magnetic field.

1), the following can be written

$$B_\rho = \frac{\partial A_z}{\rho\partial\varphi} - \frac{\partial A_\varphi}{\partial z} = -\frac{\partial A_\varphi}{\partial z} = 0, \tag{2}$$

$$B_\varphi = \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} = -\frac{\partial A_z}{\partial \rho} = 0, \tag{3}$$

$$B_z = \frac{\partial(\rho A_\varphi)}{\rho\partial\rho} - \frac{\partial A_\rho}{\rho\partial\varphi} = \frac{\partial(\rho A_\varphi)}{\rho\partial\rho}. \tag{4}$$

Thus the magnetic flux density can be described by only one component of magnetic vector potential A_ϕ

$$\vec{B} = \frac{\vec{i}_z}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho}, \tag{5}$$

and it follows

$$\left(\mathrm{curl}\vec{B}\right)_\varphi = -\vec{i}_\varphi\frac{\partial(\rho B_z)}{\partial\rho} = -\vec{i}_\varphi\frac{\partial}{\partial\rho}\left(\frac{\partial(\rho A_\varphi)}{\rho\partial\rho}\right). \tag{6}$$

Eq. (6) and Ampère-Maxwell law for isotropic, homogeneous, stationary and linear disc lead to the second order differential equation

$$\frac{\partial^2 A_\varphi}{\partial\rho^2} + \frac{\partial A_\varphi}{\rho\partial\rho} - \frac{1}{\rho^2}A_\varphi = -\mu(J_\varphi + sD_\varphi), \tag{7}$$

where J_ϕ denotes the conduction current, sD_ϕ the displacement current, $s = j2\pi f$, and f the frequency of field. For magnetic potential vector tangential component, Bessel equation takes the form

$$\frac{\partial^2 A_\varphi}{\partial\rho^2} + \frac{\partial A_\varphi}{\rho\partial\rho} - \left(\Gamma^2 + \frac{1}{\rho^2}\right)A_\varphi = 0, \tag{8}$$

with the solution [5, 19–21] as follows

$$A_\varphi(\rho) = a\mathrm{I}_1(\Gamma\rho) + b\mathrm{K}_1(\Gamma\rho), \tag{9}$$

where $\mathrm{I}_1(\,)$, $\mathrm{K}_1(\,)$ are the Bessel functions of the third order [5], a, b the constants, and $\Gamma = \sqrt{s\mu\gamma + s^2\mu\varepsilon}$. Constant b must be set to $b = 0$ because function $\mathrm{K}_1(\,)$ and its derivative have got singularity for $\rho = 0$. Magnetic flux density for conductive region is given by the following relation

$$\vec{B} = -\vec{i}_\rho\frac{\partial A_\varphi}{\partial z} + \frac{\vec{i}_z}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho} = a\frac{\vec{i}_z}{\rho}\frac{\partial(\rho I_1(\Gamma\rho))}{\partial\rho} = \vec{i}_z a\Gamma I_0(\Gamma\rho), \tag{10}$$

where one of the equalities for derivative of Bessel function $\mathrm{I}_0(\,)$ is applied [5].

For nonconductive region $\rho > R$ is satisfied

$$\left(\mathrm{curl}\vec{B}\right)_\varphi = -\vec{i}_\varphi\frac{\partial(\rho B_z)}{\partial\rho} = -\vec{i}_\varphi\frac{\partial}{\partial\rho}\left(\frac{\partial(\rho A_\varphi)}{\rho\partial\rho}\right) = 0, \tag{11}$$

thus tangential component A_ϕ of magnetic potential vector is given by the relation

$$A_\varphi = \frac{1}{2}c\rho + \frac{d}{\rho}, \tag{12}$$

where c and d are integration constants. The solution in Eq. (12) can be validated by putting them directly into Eq. (11). For nonconductive region $\rho > R$ magnetic flux density is as follows

$$\vec{B} = -\vec{i}_\rho\frac{\partial A_\varphi}{\partial z} + \frac{\vec{i}_z}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho} = \vec{i}_z c. \tag{13}$$

Constant c is equal to magnetic flux density axial component $c = B_0$ forced along axis z. Constants a, d are evaluated by two continuity conditions for: radial components of magnetic flux density and tangential components of magnetic field strength at the boundary of disc for $\rho = R$, hence resulting in

$$a = B_0/(\Gamma\mathrm{I}_0(\Gamma R)), \tag{14}$$

$$d = R(a\mathrm{I}_1(\Gamma R) - 0.5B_0 R). \tag{15}$$

Axial magnetic field does not lead to levitation. The axial field induces current I_2 and exerts only Lorentz force F_ρ oriented radially (Fig. 2). Moreover, the inhomogeneity force does not appear due to the lack of change of reluctivity at the bottom and top bases of cylinder (disc).

In order to cause disc levitation, a field with radial component is needed, i.e., passing through side surface of disc (cylinder) as shown in Fig. 3. Both magnetic fields axial and radial component can exert Lorentz force (levitation force). Let us assume the presence of a second magnetic field, denoted by index 2, with components as follows

$$B_{2\rho} = C\rho/2, \quad B_{2\varphi} = 0, \quad B_{2z} = -Cz, \tag{16}$$

where $z = 0$ for the centre of disc (Fig. 3). It should be emphasized that the field distribution in Eq. (16) is valid in the disc volume and nearby only.

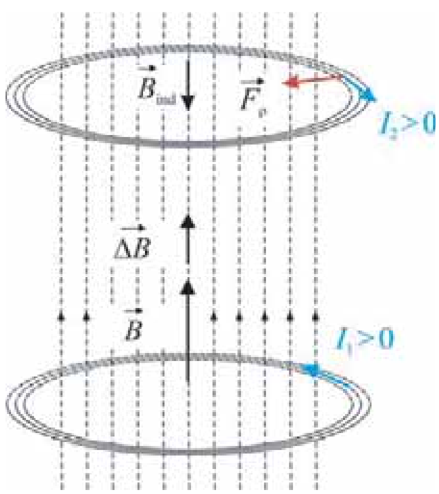

Figure 2. Lorentz force $\vec{F}_\rho$ exerted by axial magnetic field (magnetic flux density change $\Delta\vec{B}$ exerts current I_2 that subsequently leads to Lorentz force; $\vec{B}_{\text{ind}}$ magnetic flux density exerted by current I_2).

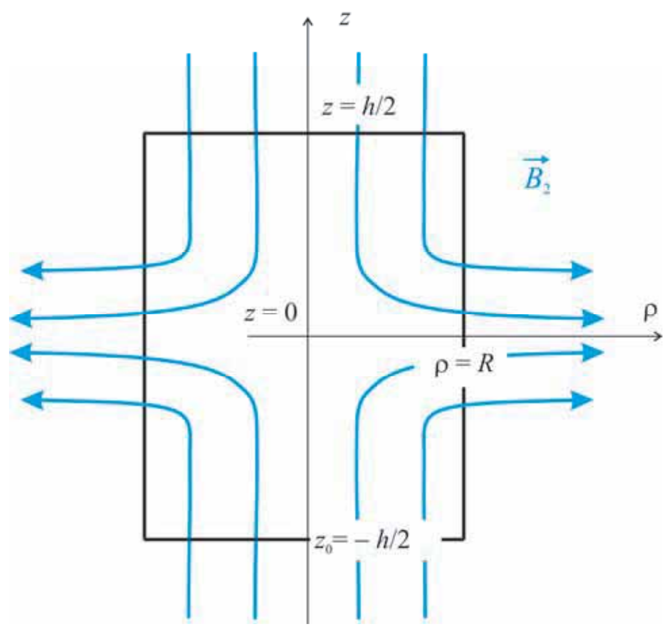

Figure 3. Second magnetic field given by Eq. (16) — the cross-section sketch (h disc height).

Exerting magnetic field with both axial and radial components, i.e., design of the coils shape, is a technical problem which sometimes is difficult to solve [4, 6, 8]. The main idea of constructing such an excitation system is depicted in Fig. 4. The designed number and shapes of the coils do not depend only on electromagnetic field requirements, e.g., the supply circuit and mechanical problems significantly influence the coils design, too.

The second magnetic field (with radial component) given by Eq. (16) has got the divergence equal to zero. The curl of the second magnetic field is equal to zero, i.e., the radial field is irrotational. It is easy to check this by calculating the curl of field in Eq. (16) taking into account Eq. (1). As a consequence of the assumption in Eq. (16) — according to the Faraday law after integrating over ρ — the electric field strength longitudinal component is as follows

$$E_{2\varphi} = \frac{s}{2}Cz\rho + \frac{D}{\rho}. \tag{17}$$

Constant $D = 0$ because the electric field is limited for $\rho < R$. Furthermore, constant C describes modulus and phase of the radial field with index 2. If both fields, axial and radial, are excited by the

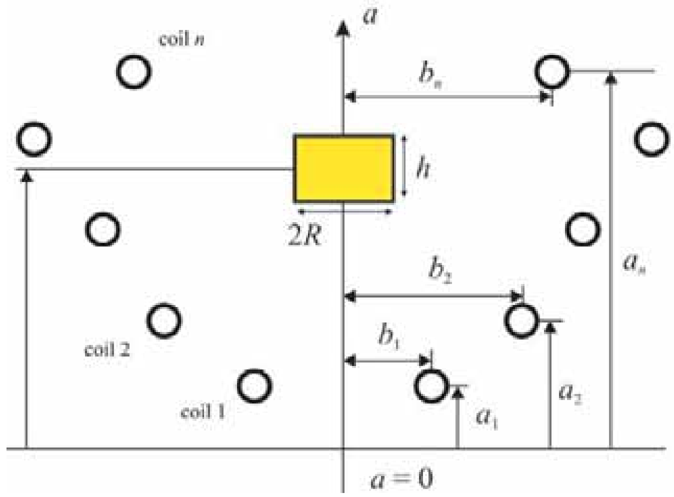

Figure 4. Main idea of excitation system construction for levitation system [4, 6, 8].

same coils, the phases of both fields are equal. Constant C value is depended on the number of coils and current in excitation coils (Fig. 4). The radial magnetic field also causes additional eddy currents in conductive disc. Namely, according to Eqs. (1) and (16) the radial magnetic field strength rotation equals zero. Hence, the Ampère-Maxwell law in the form of

$$\mathrm{curl}\vec{H}_2 = \vec{J}_2 + \gamma\vec{E}_2 + s\varepsilon\vec{E}_2, \tag{18}$$

determines the additional eddy currents density $\vec{J}_2$ as given below

$$\vec{J}_2 = -(\gamma + s\varepsilon)\vec{E}_2. \tag{19}$$

This current density $\vec{J}_2$ is much less than the currents density induced by axial magnetic field, and as a consequence, their power losses are significantly less than the power losses caused by axial field.

Finally, the total magnetic and electric fields are the sum of the two fields resulting from axial and radial excitations, respectively.

4. LEVITATION FORCE AND POWER BALANCE FOR NONMAGNETIC DISC

Force acting on nonmagnetic disc of height h, radius R and magnetic permeability $\mu = \mu_0$ is calculated by means of:

a) Maxwell tensor,
b) coenergy function, and
c) Lorentz force density.

It should be underlined that the inhomogeneity component does not appear because the disc is nonmagnetic.

a) Maxwell tensor for cylindrical co-ordinate system z axis component

$$\vec{\sigma}_z = -H_z\vec{B} + \vec{i}_z e_\mu. \tag{20}$$

leads to the total force acting on disc by integrating them over closed surface including the disc [Appendix, 1, 18]. The density of total force is equal to

$$f_z = -\,\mathrm{div}(\vec{\sigma}_z), \tag{21}$$

thus the volume integral thereof is as follows

$$F_{Mz} = \int\limits_V f_z \mathrm{d}V = -\int\limits_V \mathrm{div}(\vec{\sigma}_z)\mathrm{d}V = -\int\limits_S \vec{\sigma}_z \mathrm{d}\vec{S}. \tag{22}$$

The surface integral

$$F_{\mathrm{M}z} = \int\limits_S \left(H_z \vec{B} - \vec{i}_z e_\mu \right) \cdot \mathrm{d}\vec{S}, \tag{23}$$

is the sum of integrals over two bases areas and side area, and it is given below for complex vectors

$$F_{\mathrm{M}z} = \frac{\pi \mathrm{R}}{\mu_0} \int\limits_{z_0}^{z_0+h} \mathrm{Re}\{B_z^* B_\rho\} \mathrm{d}z + \frac{\pi}{2\mu_0} \int\limits_0^R \left(|B_z|^2 \Big|_{z_0+h} - |B_z|^2 \Big|_{z_0} \right) \rho \mathrm{d}\rho, \tag{24}$$

where $z_0 = -h/2$ is axial coordinate of lower base of the disc.

b) The total force can also be calculated by partial derivative of magnetic coenergy function W_C with respect to z [1, 14, 15, 17, 20, 22]

$$F_{\mathrm{C}z} = \frac{\partial W_{\mathrm{C}}}{\partial z}\bigg|_{\vec{J}=\mathrm{const}} = \int\limits_V \vec{J} \frac{\partial \vec{A}}{\partial z} \mathrm{d}V. \tag{25}$$

The complex vectors can be applied in equations

$$F_{\mathrm{C}z} = \pi \int\limits_{z_0}^{z_0+h} \int\limits_0^R \mathrm{Re}\left\{ (\gamma E_\varphi + J_{2\varphi}) \frac{\mathrm{d}A_{2\varphi}^*}{\mathrm{d}z} \right\} \rho \mathrm{d}\rho \mathrm{d}z. \tag{26}$$

c) The physical reason of exerting the levitation force constitutes the Lorentz force, hence for complex vectors

$$F_{\mathrm{L}z} = \int\limits_V \mathrm{Re}\left\{ \left(\vec{J} \times \vec{B}^* \right)_z \right\} \mathrm{d}V, \tag{27}$$

which can be developed in the form of two integrals

$$F_{\mathrm{L}z} = -\pi \int\limits_{z_0}^{z_0+h} \int\limits_0^R \mathrm{Re}\left\{ (\gamma E_\varphi + J_{2\varphi}) B_\rho^* \right\} \rho \mathrm{d}\rho \mathrm{d}z. \tag{28}$$

The three methods: Maxwell tensor, coenergy and Lorentz force density rigorously lead to the same value

$$F_{\mathrm{M}z} = F_{\mathrm{C}z} = F_{\mathrm{L}z}, \tag{29}$$

if the displacement current density module $|sD_\phi|$ can be neglected in comparison with the conduction current density module $|J_\phi|$

$$|sD_\varphi| \ll |J_\varphi|. \tag{30}$$

In such a case, the so-called Poynting force component [Appendix, 17–19, 22]

$$\vec{f}_{\mathrm{P}} = \frac{\partial \left(\vec{D} \times \vec{B} \right)}{\partial t}, \tag{31}$$

vanishes $f_{Pz} \to 0$. Additionally, for checking the previous calculations the Poynting force component is calculated by means of complex vectors

$$F_{\mathrm{P}z} = -2\pi \int\limits_{z_0}^{z_0+h} \int\limits_0^R \mathrm{Re}\left\{ \mathrm{Re}(s) \varepsilon E_\varphi B_\rho^* \right\} \rho \mathrm{d}\rho \mathrm{d}z, \tag{32}$$

and always remains equal to zero for considered fields frequencies.

The calculations of forces bring the conclusions that maximal levitation force $F_{Mz} = F_{Lz}$ is oriented vertically upwards (i.e., towards z axis in Fig. 1) for real C and $C \cdot B_0 > 0$. T is condition resulting from Eqs. (14)–(16).

Moreover, for checking the accuracy of the analytical solutions proposed, power balance is controlled. Electromagnetic field power balance components [1, 15, 20] by means of complex vectors are as follows:

- power losses

$$P_\gamma = \pi \int\limits_{z_0}^{z_0+h} \int\limits_0^R \left[E_\varphi \left(\gamma E_\varphi^* + J_{2\varphi}^*\right)\right] \rho \mathrm{d}\rho \mathrm{d}z, \tag{33}$$

- change in time of magnetic energy

$$E_\mu = \frac{\pi}{2\mu} \int\limits_{z_0}^{z_0+h} \int\limits_0^R \left\{|B_\rho|^2 + |B_z|^2\right\} \rho \mathrm{d}\rho \mathrm{d}z, \tag{34}$$

which satisfies the balance equation [1, 3, 10, 11, 18, 19]

$$S_{\mathrm{P}} = P_\gamma + 2sE_\mu, \tag{35}$$

where $s = j2\pi f$.

Poynting vector surface integral S_{P} over the outer areas of the disc (the normal is oriented inside) equals

$$S_{\mathrm{P}} = \pi \int\limits_0^R \left\{ \left(E_\varphi H_\rho^*\right)\big|_{z_0+h} - \left(E_\varphi H_\rho^*\right)\big|_{z_0} \right\} \rho \mathrm{d}\rho - \pi R \int\limits_{z_0}^{z_0+h} \mathrm{Re}\left\{E_\varphi H_z^*\right\} \mathrm{d}z. \tag{36}$$

The power losses are an important quantity, which determines temperature rise of levitating disc and enable the evaluateion of approximate time of reaching the melting point for the disc (i.e., to become a liquid).

All the above presented analytical solutions can be treated as benchmark tasks for numerical methods [1, 18, 21, 23].

5. EXAMPLES

For example, for nonmagnetic disc as follows: height $h = 2\,\mathrm{mm}$, radius $R = 5\,\mathrm{mm}$, conductivity $\gamma = 35 \cdot 10^6\,\mathrm{S \cdot m^{-1}}$ at the fields frequency $f = 0.1\,\mathrm{MHz}$, axial magnetic flux density $B_0 = 0.7\,\mathrm{T}$, constant for radial field $C = 0.7\,\mathrm{T \cdot m^{-1}}$ (phases of both fields are the same) the levitation force is positive: $F_{Mz} = F_{Lz} = F_{Cz} = 29\,\mathrm{mN}$. Power losses are equal to $P_\gamma = 1007\,\mathrm{kW}$, which confirms the real part of Poynting vector surface integral $S_P = (1007 + j1036)\,\mathrm{VA}$. Magnetic field energy is equal to $E_\mu = 0.82\,\mathrm{mJ}$.

While disc is levitating, it is heated thoroughly, and its temperature rises rapidly. For aluminum disc density $d = 2.7\,10^3\,\mathrm{kg/m^3}$, specific heat $c_w = 900\,\mathrm{J/(kg \cdot K)}$ and time $\Delta t = 1\,\mathrm{ms}$, the rise of temperature approximately equals $\Delta T = 2.6\,\mathrm{K}$ and enables melt of the aluminum disc ($t_{\mathrm{meltingAl}} = 660°\mathrm{C}$) in a few minutes. The average power volume density in disc area is equal to about $6.4\,10^9\,\mathrm{W \cdot m^{-3}}$.

In Fig. 5, levitation force (calculated by means of Lorentz and Maxwell methods) vs. field frequency is presented. It should be pointed out that the levitation force reaches a certain maximal value (about 11 mN in Fig. 5). Hence, it results in that only cylinder (disc) of limited mass can levitate in a given excitation field.

Fig. 5 shows that the levitation force increases with the frequency and converges monotonically to maximal (limit) value

$$F_{\mathrm{L\,max}} = \pi \frac{B_0 C}{2\mu_0} R^2 h, \tag{37}$$

for $|\Gamma R| \gg 1$ as it results directly from integral in Eq. (28) taking into account Eqs. (12), (16) and approximation

$$I_\nu(z) \approx \frac{\exp(z)}{\sqrt{2\pi z}}, \tag{38}$$

which is valid for arguments with great modulus $|z|$ for any ν.

In Fig. 6 there are presented power losses evaluated by two methods: Joule-Lenz volume integral and Poynting vector surface integral. Additionally, Fig. 7 presents the ratio in Eq. (39) of power losses to

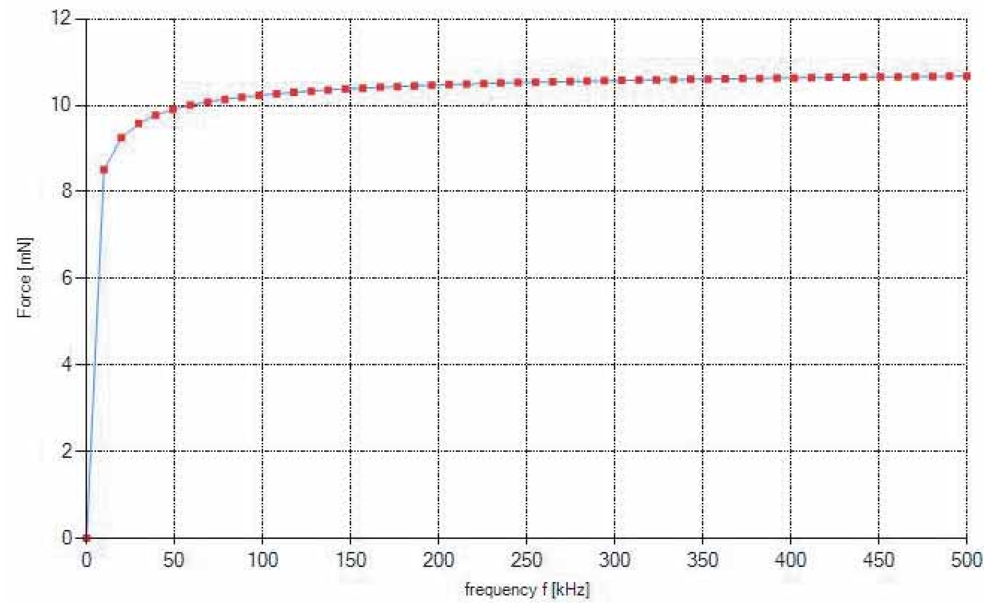

Figure 5. Levitation force calculated by methods: Maxwell (points) and Lorentz (line) for nonmagnetic disc ($h = 2\,\mathrm{mm}$, $R = 3\,\mathrm{mm}$, $\gamma = 56 \cdot 10^6\,\mathrm{S \cdot m^{-1}}$, $f_{\max} = 0.5\,\mathrm{MHz}$, $B_0 = 0.7\,\mathrm{T}$, $C = 0.7\,\mathrm{T \cdot m^{-1}}$) vs. frequency.

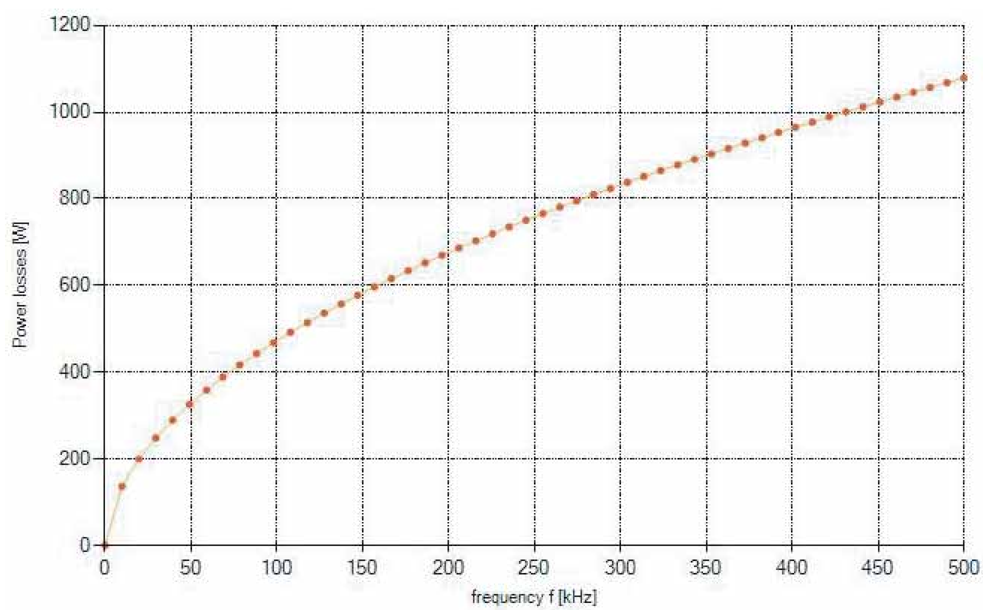

Figure 6. Power losses calculated by methods: Joule-Lenz (line) and Poynting (points) for nonmagnetic disc ($h = 2\,\mathrm{mm}$, $R = 3\,\mathrm{mm}$, $\gamma = 56 \cdot 10^6\,\mathrm{S \cdot m^{-1}}$, maximal frequency $f_{\max} = 0.5\,\mathrm{MHz}$, $B_0 = 0.7\,\mathrm{T}$, $C = 0.7\,\mathrm{T \cdot m^{-1}}$) vs. frequency.

square root of field frequency vs. frequency. It is shown in Fig. 7 that the power losses are approximately proportional to frequency square root near upper limit of considered frequency interval, i.e.,

$$\frac{P_\gamma}{\sqrt{f}} \approx \text{const.} \tag{39}$$

The limit property in Eq. (39) results from the approximation in Eq. (38). Assuming that power losses determine mainly axially exerted field in Eq. (9) with constant in Eq. (12),

$$P_\gamma \approx \gamma\,|s|^2\,\frac{\mu^2}{\mu_0^2}\frac{2\pi B_0^2 h}{|\Gamma|^2\,\mathrm{I}_0(\Gamma R)\mathrm{I}_0^*(\Gamma R)}\int\limits_0^R \frac{\exp(2\alpha\rho)}{2\pi\,|\Gamma|\,\rho}\rho\mathrm{d}\rho, \tag{40}$$

where $\Gamma = \alpha + \mathrm{j}\alpha = \sqrt{\pi f \mu\gamma}(1+\mathrm{j})$, and finally

$$P_\gamma \approx \pi\gamma\omega^2\frac{\mu^2}{\mu_0^2}\frac{B_0^2 R h}{\alpha\,|\Gamma|^2}(1-\exp(-2\alpha R)) \approx \pi\frac{B_0^2}{\mu_0^2}\frac{\alpha}{\gamma}Rh, \tag{41}$$

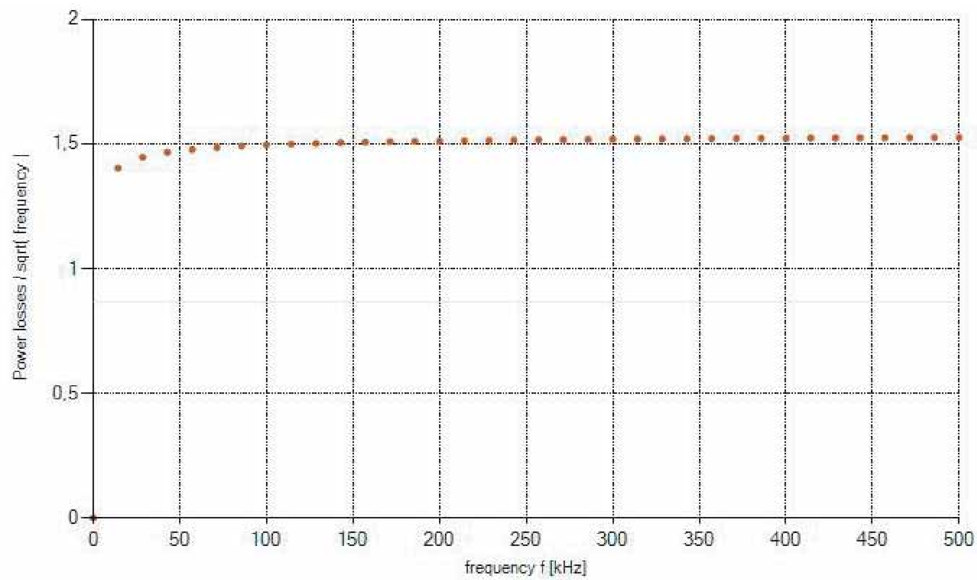

Figure 7. Ratio of power losses to square root of frequency for nonmagnetic disc ($h = 2\,\mathrm{mm}$, $R = 3\,\mathrm{mm}$, $\gamma = 56\cdot 10^6\,\mathrm{S\cdot m^{-1}}$, maximal frequency $f_{\max} = 0.5\,\mathrm{MHz}$, $B_0 = 0.7\,\mathrm{T}$, $C = 0.7\,\mathrm{T\cdot m^{-1}}$) vs. frequency.

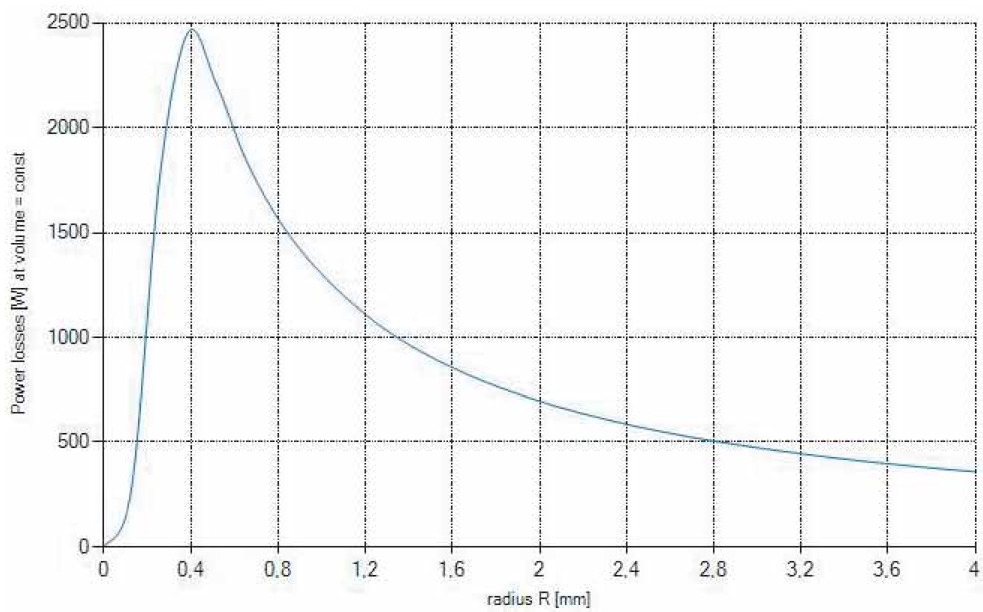

Figure 8. Optimization of nonmagnetic disc shape at the volume condition $V = \mathrm{const}$ ($V = 56.5\cdot 10^{-9}\,\mathrm{m^3} = \mathrm{const}$, $\gamma = 56\cdot 10^6\,\mathrm{S\cdot m^{-1}}$, $f = 0.1\,\mathrm{MHz}$, $B_0 = 0.7\,\mathrm{T}$, $C = 0.7\,\mathrm{T\cdot m^{-1}}$) vs. disc radius R.

that proves and generalizes the limit property in Eq. (39).

Based on the presented analytical solution of the levitation task, it is easy to find the optimal shape of the disc with respect to power losses. Let's assume that the disc volume $V = \pi R^2 h$ is constant, thus the power losses vs. disc radius R can be presented as shown in Fig. 8. Hence, the radius (and the height) of the disc for the maximal power losses value can be found at given volume of the disc.

6. CONCLUSIONS

Nonmagnetic disc is presented. Excitation by magnetic field is assumed with axial (vertical) and radial (horizontal) components. Maxwell's equations are solved in cylindrical co-ordinate system by separation of variables method.

The levitation force and power losses are evaluated. The electromagnetic field power balance is checked. The efficient solutions are obtained over a large frequency interval and different parameters, which is not easy to reach by numerical methods.

The solutions enable the reduction of computational cost, e.g., at high field frequencies. The limit values of force and power losses are calculated.

There is evaluated maximal levitation force (Lorentz component) — Eq. (37). Limit property for power losses is proved — Eq. (41).

Despite apparent simplicity of the solutions, some tests are provided in order to validate analysis of levitation force and power losses. Firstly, three methods for levitation force calculations are applied: Maxwell, coenergy and Lorentz (Fig. 5, Eq. (29)). All three methods yield the same force value, and there appears no distinction. Secondly, the power losses are calculated by both Joule-Lenz volume integral and surface integral of Poynting vector (Fig. 6, Eq. (35)).

The analyses have brought the following conclusion:

- the analytical solutions obtained from levitation process give opportunities to approach of both levitation and melting processes by robust and rapid toll;
- the industrial problems of melting metals can be predicted by the developed model for wide range of frequency, conductivity, diameter and other parameters of molten objects;
- analytical solutions shown can be treated as a benchmark tasks for numerical methods;
- nowadays, the rapid model of levitation phenomenon may also be involved in algorithms for 3D analyses.

APPENDIX A. ELECTROMAGNETIC FORCE DENSITY AND ITS COMPONENTS

However, the rearranging of Maxwell equations seems not interesting, and it should be presented formally. Electromagnetic field force volume density in curvilinear orthogonal co-ordinate system (u, v, w) can be presented with the help of Maxwell equation

$$\mathrm{curl}\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{A1}$$

and Lorentz force density in the form of

$$\vec{f}_{\mathrm{L}} = \rho\vec{E} + \vec{J} \times \vec{B}, \tag{A2}$$

where $\vec{J}$ is the forced current density which satisfies, ρ the charge density, $\vec{E}$ the electric field strength, and $\vec{B}$ the magnetic flux density. Two components of Lorentz force

$$\vec{f}_{\mathrm{L}} = \vec{E}\mathrm{div}\vec{D} + \left(\mathrm{curl}\vec{H} - \dot{\vec{D}}\right) \times \vec{B}, \tag{A3}$$

can be rearranged in the form of

$$\vec{f}_{\mathrm{L}} = \mathrm{curl}\vec{H} \times \vec{B} + \vec{H}\mathrm{div}\vec{B} - \frac{\partial\left(\vec{D} \times \vec{B}\right)}{\partial t} + \vec{D} \times \mathrm{curl}\vec{E} + \vec{E}\mathrm{div}\vec{D}, \tag{A4}$$

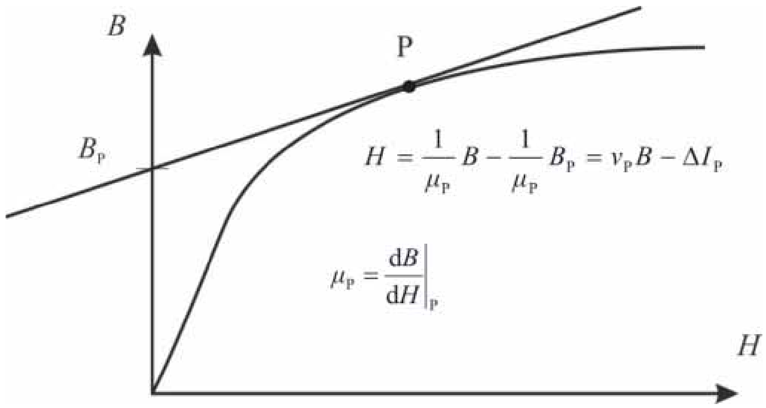

Figure A1. Definition of term ΔI_u for particular locus.

where term $\vec{H}\mathrm{div}\vec{B} = 0$ is added for mathematical symmetry.

Let us present constitutive relation in the general form of

$$H_u = \nu_{uw} B_w - \Delta I_u, \tag{A5}$$

where reluctivities ν_{uw} can be asymmetrical, and ΔI_u u^{th} component of magnetization vector describes either permanent magnets or hysteresis phenomena (Fig. A1).

The first and second components of the right-hand side of Eq. (A2) can be written as follows

$$\mathrm{curl}\vec{H} \times \vec{B} + \vec{H}\mathrm{div}\vec{B} = \vec{i}_u \mathrm{div}_{|u|}(\vec{\sigma}_{\mu u}) - \vec{\Delta}_\mu - \vec{N}_\mu - \vec{Q}_\mu - \vec{M}_\mu, \tag{A6}$$

L_u is Lame coefficient for u^{th} coordinate, no summation due to u

$$\mathrm{div}_u(\cdot) = L_{|u|}^{-1}\mathrm{div}\left(L_u(\cdot)\right), \tag{A7}$$

$$\vec{\sigma}_{\mu u} = -H_u\vec{B} + \vec{i}_u e_\mu = -H_u\vec{B} + \vec{i}_u\left(\frac{1}{2}\vec{H}\vec{B}\right), \tag{A8}$$

$$\vec{N}_\mu = \frac{1}{2}B_u B_w \mathrm{grad}(\nu_{uw}), \tag{A9}$$

inhomogenous force component

$$\vec{Q}_\mu = \frac{1}{2}\mathrm{grad}(B_u\Delta I_u) - B_u\mathrm{grad}(\Delta I_u), \tag{A10}$$

hysteresis force component,

$$\vec{M}_\mu = \frac{1}{2}(\nu_{vu} - \nu_{uv})B_v\mathrm{grad}(B_u), \tag{A11}$$

anisotropy component, and an auxiliary vector in the form of

$$\vec{\Delta}_\mu = \frac{1}{2}B_v H_v \vec{i}_u \frac{\partial \ln(L_v^2/|L|)}{L_u \partial x_u}, \tag{A12}$$

for orthogonal curvilinear coordinate system ($L = L_u L_v L_w$ denotes multiplication of Lame coefficients).

The fourth and fifth components on the right-hand side of Eq. (A4) can be rearranged in the same manner as the first and second in Eq. (A2)

$$\mathrm{curl}\tilde{\mathrm{E}} \times \vec{D} + \vec{E}\mathrm{div}\vec{D} = \vec{i}_u \mathrm{div}_{|u|}(\vec{\sigma}_{\varepsilon u}) - \vec{\Delta}_\varepsilon - \vec{N}_\varepsilon - \vec{Q}_\varepsilon - \vec{M}_\varepsilon, \tag{A13}$$

where the constitutive relation (electric permittivities ε_{uw} can be asymmetrical) is defined as follows

$$D_u = \varepsilon_{uw} E_w - \Delta P_u, \tag{A14}$$

where ΔP_u is u^{th} component of electric polarization vector, and

$$\vec{\sigma}_{\varepsilon u} = -E_u\vec{D} + \vec{i}_u e_\varepsilon = -E_u\vec{D} + \vec{i}_u\left(\frac{1}{2}\vec{E}\vec{D}\right). \tag{A15}$$

$$\vec{N}_\varepsilon = -\frac{1}{2}E_u E_w \mathrm{grad}(\varepsilon_{uw}), \tag{A16}$$

$$\vec{Q}_\varepsilon = \frac{1}{2}\mathrm{grad}(\Delta P_u E_u) - E_u \mathrm{grad}\left(\Delta P_u\right), \tag{A17}$$

$$\vec{M}_\varepsilon = -\frac{1}{2}(\varepsilon_{vu} - \varepsilon_{uv})E_v \mathrm{grad}(E_u), \tag{A18}$$

and an auxiliary vector

$$\vec{\Delta}_\varepsilon = \frac{1}{2}D_v E_v \vec{i}_u \frac{\partial \ln(L_v^2/|L|)}{L_u \partial x_u}. \tag{A19}$$

Hence, Eq. (A4) takes the form of

$$\vec{f}_\mathrm{L} = -\frac{\partial\left(\vec{D}\times\vec{B}\right)}{\partial t} - \vec{i}_u \mathrm{div}_{|u|}\left(\vec{\sigma}_u\right) - \vec{\Delta} - \vec{N} - \vec{Q} - \vec{M}, \tag{A20}$$

where the components for electric and magnetic field are summarized subsequently

$$\vec{\sigma}_u = -E_u\vec{D} - H_u\vec{B} + \vec{i}_u e = -E_u\vec{D} - H_u\vec{B} + \vec{i}_u\left(\frac{1}{2}\vec{E}\vec{D} + \frac{1}{2}\vec{H}\vec{B}\right), \tag{A21}$$

$$\vec{N} = \vec{N}_\varepsilon + \vec{N}_\mu = -\frac{1}{2}E_u E_w \mathrm{grad}(\varepsilon_{uw}) + \frac{1}{2}B_u B_w \mathrm{grad}(\nu_{uw}), \tag{A22}$$

$$\vec{Q} = \vec{Q}_\varepsilon + \vec{Q}_\mu, \tag{A23}$$

$$\vec{\Delta} = \left(\frac{1}{2}D_v E_v + \frac{1}{2}B_v H_v\right)\vec{i}_u \frac{\partial \ln\left(L_v^2/|L|\right)}{L_u \partial x_u}, \tag{A24}$$

$$\vec{M} = \frac{1}{2}(\nu_{vu} - \nu_{uv})B_v \mathrm{grad}(B_u) - \frac{1}{2}(\varepsilon_{vu} - \varepsilon_{uv})E_v \mathrm{grad}(E_u) \tag{A25}$$

and Poynting force density is denoted in the form of [1, 2, 9, 15, 16, 20].

$$\vec{f}_\mathrm{P} = \frac{\partial\left(\vec{D}\times\vec{B}\right)}{\partial t}. \tag{A26}$$

The total force density is given as follows

$$\vec{f} = \vec{f}_\mathrm{L} + \vec{f}_\mathrm{P} + \vec{N} + \vec{Q} + \vec{M}. \tag{A27}$$

According to Eq. (A20), total force can be written by Maxwell stress tensor in the following form

$$\vec{f} = -\vec{i}_u \mathrm{div}_{|u|}\left(\vec{\sigma}_u\right) - \vec{\Delta}. \tag{A28}$$

Conclusion: If the following can be omitted:

a) Poynting force $\vec{f}_\mathrm{P} = 0$ (field at low frequency),

b) Inhomogeneous force $\vec{N} = 0$ (region is homogeneous with respect to ε_{vu} and ν_{vu}),

c) Hysteresis component $\vec{Q} = 0$ (no hysteresis phenomenon, no permanent magnets), and most important for the new extended necessary and sufficient condition:

d) Anisotropy component $\vec{M} = 0$ for region with symmetrical matrices ($u \neq v$)

$$\varepsilon_{vu} \neq \varepsilon_{uv}, \quad \nu_{vu} \neq \nu_{uv}. \tag{A29}$$

the Lorentz force equals the total electromagnetic field force.

However, the rearranging of equations seems not interesting, and it should be presented rigorously.

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