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Characteristics of Scattering for Anisotropic Particles in Photoelectric Electromagnetic Beam

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Abstract—The basic wave types of electromagnetic field propagation in anisotropic media are obtained. Based on the orthogonality relation between the vector wave functions and the orthogonality of trigonometric functions, etc., the expressions of zero order scattering fields and first-order scattered fields of arbitrary electromagnetic beam are presented. A stochastic system identification model for electromagnetic beam scattering by anisotropic particles is established. In the S wave band, the relationships respectively between the scattering field expansion coefficients, the basic wave types of the particle field and the tensor of dielectric constant are studied, and their validity of the model is verified. Taking the elliptical Gaussian beam as an example, the beam scattering characteristics of anisotropic media particles are investigated. The used method is simple, exploring a new approach of researching the electromagnetic beam scattering characteristics from anisotropic medium targets.

1. INTRODUCTION

In recent years, with the development of modern radar system, modern stealth, remote sensing and other relative technical fields, the research of electromagnetic scattering of anisotropic targets has attracted much attention. When the electromagnetic wave or transient wave propagates in an anisotropic medium, there are two or more than two propagation vectors, and their magnitude depends on the polarization of the electric field [1, 2], which is the main difficulty in the study of anisotropic target scattering. The electromagnetic wave propagation through a stratified inhomogeneous anisotropic medium is investigated [3]. The scattering properties of isotropic sphere particles illuminated by a plane wave have been studied in detail, called the Mie theory [4]. However, the analytical theory of scattering by anisotropic sphere particles is not perfect. The electromagnetic scattering from two-dimensional anisotropic targets can be approximately solved, and some numerical results [5] can be obtained by using integral equations. The scattered field of objects of uniaxial crystal may be approximately researched using genetic algorithm and scalar equation [6, 7]. The scattering property of uniaxial anisotropic multilayer dielectric sphere [8, 9] of an ordinary direction of optical axis is also researched by using the method of expanding the incident wave with ordinary wave and extraordinary wave. It is a meaningful attempt to expand electromagnetic wave in anisotropic medium by spherical vector wave functions [10], which provide a new research technique for anisotropic target scattering. When a material is an anisotropic medium, the radiation problem in the medium can be studied by using a new Lorenz gauge [11]. It is still a blank to apply this method to electromagnetic scattering. Utilizing TDFD and other numerical methods, characteristics of both infinite flat plasma and anisotropic infinite elliptical cylinder target [12, 13] can be solved when they are illuminated by a plane electromagnetic wave. In order to solve the problem of electromagnetic wave propagation and scattering in the lossless anisotropic media, the multi-scale theory of electromagnetic field is proposed [14, 15] and applied to research

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Rayleigh scattering characteristics both of anisotropic magnetized plasma ball and an anisotropic ellipsoid. Many scholars have done a lot of work and attempt in the aspect of the beam formation and its angular spectrum expansion for the anisotropic media [16–18]. The existing papers mainly focus on the research work of isotropic particles and a few uniaxial media particles under a plane wave irradiation. Few studies have reported the beam scattering by anisotropic particles. In some papers, the scattering of uniaxial particles is studied by using relative element vector wave functions which are obtained in isotropic medium. Because the orthogonality of unit vector wave functions is established based on that the electromagnetic potentials and electromagnetic fields in the isotropic space satisfy the same wave equation, but in anisotropic space, they do not satisfy the same formation of differential equation, using unit vector wave functions to research uniaxial particle scattering is inconsistent in theory, and it is difficult to carry out the studies of anisotropic medium scattering. This paper, on the basis of the existing research, studies photoelectric beam scattering characteristics of anisotropic particles by utilizing the methods of Taylor series expansion and system identification. The fundamental wave types of anisotropic particles are obtained, and the system identification model of anisotropic particle photoelectric beam scattering is established. Taking elliptical Gaussian photoelectric beam as an example, the beam scattering characteristics of the anisotropic medium particles are studied, and the validity of the obtained results is verified. Time harmonic factor $e^{j\omega t}$ is used in this paper and is omitted.

2. SCATTERING CHARACTERISTICS OF ANISOTROPIC PARTICLES

2.1. Basic Waves in Anisotropic Media

Assuming that the medium is anisotropic in electricity and isotropic in magnetism, μ denotes the permeability, and the dielectric constant of dyadic form is written as

$$\boldsymbol{\varepsilon} = \varepsilon_0 \left(\varepsilon_x \mathbf{e}_1\mathbf{e}_1 + \varepsilon_y \mathbf{e}_2\mathbf{e}_2 + \varepsilon_z \mathbf{e}_3\mathbf{e}_3\right) \tag{1}$$

where $\mathbf{e}_i$ is the unite vector of the Cartesian coordinate system, and the relation $\mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3$ among the vectors is satisfied. Let $\mathbf{E} = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}$ be a plane wave of an anisotropic medium. It is obtained from Maxwell's equation that

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = -\omega^2 \mu \mathbf{D} \tag{2}$$

where $\mathbf{k}$, ω are respectively the propagation vector and angular frequency. When the electromagnetic wave propagates along the z axis and is polarized in the direction of $\mathbf{e}_1$, the electromagnetic wave may be written as

$$\mathbf{E} = \mathbf{e}_1 E_0 e^{-jkz}$$

The magnitude of the propagating vector is obtained after putting above in Eq. (2)

$$k_1 = k_0 \sqrt{\varepsilon_x} \tag{3}$$

where $k_0 = \omega\sqrt{\varepsilon_0 \mu}$, being the magnitude of propagating vector in free space. When the electromagnetic wave propagates along the z axis and is polarized in the direction of $\mathbf{e}_2$, in the same manner, it is obtained as

$$k_2 = k_0 \sqrt{\varepsilon_y} \tag{4}$$

It can be seen from Eqs. (3) and (4) that when a plane electromagnetic wave propagates along the Z axis in an anisotropic medium, the magnitude of the wave propagating vector is only related to the dielectric constant in the polarization direction and is independent of the dielectric constant in the other directions. When the plane electromagnetic wave is polarized in x direction and propagates in the y-z plane, it can be written as

$$\mathbf{E} = \mathbf{e}_1 E_0 e^{-j(k_z z + k_y y)}$$

In the same manner, we obtain the magnitude of propagating vector

$$k = \sqrt{k_y^2 + k_z^2} = k_0 \sqrt{\varepsilon_x} \tag{5}$$

Expression (5) shows that when the electromagnetic wave propagates in any direction, the magnitude of the wave vector is only determined by the dielectric constant of the polarization direction. The three coordinate axes in a Cartesian coordinate system have the same position, so this conclusion is of universal significance. There are generally three components of an arbitrary polarized electric wave vector, so the propagation of plane electromagnetic wave in an anisotropic medium has three basic propagation vectors, and their magnitudes are

$$\begin{cases} k_1 = k_0\sqrt{\varepsilon_x} \\ k_2 = k_0\sqrt{\varepsilon_y} \\ k_3 = k_0\sqrt{\varepsilon_z} \end{cases} \tag{6}$$

Taking the X direction polarization as an example, the basic propagation wave mode is

$$E_x = A_{xy}e^{-jk_0\sqrt{\varepsilon_x}y} + A_{xz}e^{-jk_0\sqrt{\varepsilon_x}z}$$

where the first letter of the subscript of the coefficient represents the direction of polarization, and the second letter represents the direction of propagation. Similarly the following is obtained for y-polarization and z-polarization.

$$\begin{aligned} E_y &= A_{yx}e^{-jk_0\sqrt{\varepsilon_y}x} + A_{yz}e^{-jk_0\sqrt{\varepsilon_y}z} \\ E_z &= A_{zx}e^{-jk_0\sqrt{\varepsilon_z}x} + A_{zy}e^{-jk_0\sqrt{\varepsilon_z}y} \end{aligned}$$

It can be seen that there is orthogonality between the basic wave modes of different propagation directions and of different coordinates. Therefore, the electromagnetic wave in an anisotropic medium may be expressed with basic wave modes

$$\begin{aligned} \mathbf{E} = &\ \mathbf{e}_1\left(A_{xy}e^{-jk_0\sqrt{\varepsilon_x}y} + A_{xz}e^{-jk_0\sqrt{\varepsilon_x}z}\right) + \mathbf{e}_2\left(A_{yx}e^{-jk_0\sqrt{\varepsilon_y}x} + A_{yz}e^{-jk_0\sqrt{\varepsilon_y}z}\right) \\ &+\mathbf{e}_3\left(A_{zx}e^{-jk_0\sqrt{\varepsilon_z}x} + A_{zy}e^{-jk_0\sqrt{\varepsilon_z}y}\right) \end{aligned} \tag{7}$$

where there are six coefficients to be determined. Typical particles are generally spheres, and in the spherical coordinate system expression (7) is developed as, omitted the radial component

$$\begin{aligned} E_\theta =& \cos\theta\cos\phi\left(A_{xy}e^{-jk_0\sqrt{\varepsilon_x}r\sin\theta\sin\phi} + A_{xz}e^{-jk_0\sqrt{\varepsilon_x}r\cos\theta}\right) \\ &+ \cos\theta\sin\phi\left(A_{yx}e^{-jk_0\sqrt{\varepsilon_y}r\sin\theta\cos\phi} + A_{yz}e^{-jk_0\sqrt{\varepsilon_y}r\cos\theta}\right) \\ &- \sin\theta\left(A_{zx}e^{-jk_0\sqrt{\varepsilon_z}r\sin\theta\cos\phi} + A_{zy}e^{-jk_0\sqrt{\varepsilon_z}r\sin\theta\sin\phi}\right) \\ E_\varphi =& \cos\phi\left(A_{yx}e^{-jk_0\sqrt{\varepsilon_y}r\sin\theta\cos\phi} + A_{yz}e^{-jk_0\sqrt{\varepsilon_y}r\cos\theta}\right) \\ &- \sin\phi\left(A_{xy}e^{-jk_0\sqrt{\varepsilon_x}r\sin\theta\sin\phi} + A_{xz}e^{-jk_0\sqrt{\varepsilon_x}r\cos\theta}\right) \end{aligned} \tag{8}$$

Formula (8) can be used to represent the electric field inside an anisotropic medium, and the corresponding magnetic field can be obtained by substituting it in the Maxwell's equation. It can be seen that there are six coefficients in θ component. We will start with this component to study the scattering properties of anisotropic particles.

2.2. System Identification Model of Scattering for Target of Anisotropic Media

The scattered field of isotropic spherical particles can be obtained with the Mie theory irradiated by a plane electromagnetic wave. The scattered field of an anisotropic particle in a photoelectric beam is related not only to the magnitude and dielectric constant of the particle, but also to the expression of the beam. Supposing that the incident beam propagating along the z-axis in the particle coordinate system and is polarized in the x-direction, it may be written as

$$\mathbf{E} = \mathbf{e}_1 f(x, y, z)e^{-jk_0 z} \tag{9}$$

There is a sphere with radius of a, and its center is at the origin of the coordinate system. Dielectric constant of dyadic form is given as Eq. (1). For the purpose of research convenience, Eq. (9) is expanded with the Taylor series in the domain near the coordinate origin. The zero order of coordinates is obtained as $f_0e^{-jk_0z}$, and the three first-orders of coordinates are $f_x'xe^{-jk_0z}$, $f_y'ye^{-jk_0z}$ and $f_z'ze^{-jk_0z}$ respectively. Then those terms are expanded with spherical vector functions, and finally expressions of the scattering field are developed by using the boundary condition of electromagnetic field on the surface of the sphere. The specific process is as follows, and zero term is

$$\mathbf{E}_0 = f_0 \sum_{n=1} \left[a_{1n}^2 \mathbf{M}_{1n}^2 + b_{1n}^1 \mathbf{N}_{1n}^1\right] \tag{10}$$

The corresponding scattered field is

$$\mathbf{E}_s = f_0 \sum_{n=1} \left[a_{1ns}^2 \mathbf{M}_{1n}^2 + b_{1ns}^1 \mathbf{N}_{1n}^1\right] \tag{11}$$

First-order of x, y and z and their corresponding scattered fields are

$$\mathbf{E}_x = f_x' \sum_{n=2} \left[a_{2nx}^2 \mathbf{M}_{2n}^2 + b_{2nx}^1 \mathbf{N}_{2n}^1\right] \tag{12}$$

$$\mathbf{E}_{xs} = f_x' \sum_{n=2} \left[a_{2nxs}^2 \mathbf{M}_{2n}^2 + b_{2nxs}^1 \mathbf{N}_{2n}^1\right] \tag{13}$$

$$\mathbf{E}_y = f_y' \sum_{n=2} \left[a_{2ny}^1 \mathbf{M}_{2n}^1 + b_{2ny}^2 \mathbf{N}_{2n}^2\right] \tag{14}$$

$$\mathbf{E}_{ys} = f_y' \sum_{n=2} \left[a_{2nys}^1 \mathbf{M}_{2n}^1 + b_{2nys}^2 \mathbf{N}_{2n}^2\right] \tag{15}$$

$$\mathbf{E}_z = f_z' \sum_{n=1} \left[a_{1nz}^2 \mathbf{M}_{1n}^2 + b_{1nz}^1 \mathbf{N}_{1n}^1\right] \tag{16}$$

$$\mathbf{E}_{zs} = f_z' \sum_{n=1} \left[a_{1nzs}^2 \mathbf{M}_{1n}^2 + b_{1nzs}^1 \mathbf{N}_{1n}^1\right] \tag{17}$$

The above Eqs. (10), (12), (14) and (16) represent the incident fields, and Eqs. (11), (13), (15) and (17) represent the scattered fields. The following is an example of Eqs. (10) and (11) to illustrate the establishment of the scattering model with the stochastic system method and the solution of scattering coefficients a_{1ns}^2, b_{1ns}^1 and amplitude A_{ij} of the internal field expression for the anisotropic particles. The processes of obtaining the solution of the corresponding coefficients in Eqs. (12) to (17) are similar, but the corresponding substitutions are made in the scattering model, and the specific substitutions and steps will be given subsequently.

On surface of the sphere, the electromagnetic field satisfies the continuous condition of tangential component. Because Eq. (11) represents the spherical electromagnetic wave propagating outward, it takes

$$z_n(kr) = h_n^{(2)}(k_0r)$$

On surface of the sphere, the tangential component of the electric field is continuous, i.e.,

$$E_{0\theta} + E_{s\theta} = E_\theta \tag{18}$$

The first item of the left end of Eq. (18) is the component of the incident field, and the second is the component of the scattered field. The right is the component of the inner field of the anisotropic particle, and its expression can be seen in the first formula of Eq. (8). Based on calculation of the Mie theory and numerical calculation of scattering of the conductor sphere, it is found that the contribution of the scattered field mainly comes from the first 10 terms of the series of the scattered field, especially in the case of small particle scattering [20]. Because the scattered field of a spherical particle is a series formation, the field is a deterministic magnitude, so mathematically, the series is a convergent series. It is known that in the convergent series, the magnitude of the consequent term is always less than the magnitude of the former term. The terms of $n > 10$ may be neglected in application just as the Taylor series used in actual applications, and generally only the first two or three terms are used. Further simulation shows that the effect of truncation n on the amplitude of A_{ij} has little contribution. If more accurate scattering amplitude of A_{ij} is needed, it is only necessary to increase the truncation number

n in the program. In this paper we assume that the scattered field series is truncated in $n = 10$, so we obtain

$$\begin{aligned} E_{0\theta} &= f_0 \sum_n \left[a_{1ns}^2 \left(-\mathbf{M}_{1n}^2\right) + b_{1ns}^1 \left(-\mathbf{N}_{1n}^1\right)\right]\Big|_\theta + \cos\theta\cos\phi \left(A_{xy} e^{-jk_0\sqrt{\varepsilon_x} r \sin\theta\sin\phi} + A_{xz} e^{-jk_0\sqrt{\varepsilon_x} r\cos\theta}\right) \\ &\quad + \cos\theta\sin\phi \left(A_{yx} e^{-jk_0\sqrt{\varepsilon_y} r\sin\theta\cos\phi} + A_{yz} e^{-jk_0\sqrt{\varepsilon_y} r\cos\theta}\right) \\ &\quad - \sin\theta \left(A_{zx} e^{-jk_0\sqrt{\varepsilon_z} r\sin\theta\cos\phi} + A_{zy} e^{-jk_0\sqrt{\varepsilon_z} r\sin\theta\sin\phi}\right) \end{aligned}$$

The left side in above expression is the incident field, and there are $10 + 10 + 6 = 26$ coefficients in the right side to be determined. Obviously using boundary conditions to obtain solutions is impossible. In order to solve the 26 coefficients, the known variables of the right side as the inputs of the system, and the incident field in the left as the outputs of the system, 26 unknown coefficients may be considered as the parameters to be determined. A model of stochastic system identification for electromagnetic scattering on surface of the sphere is developed as shown in Fig. 1.

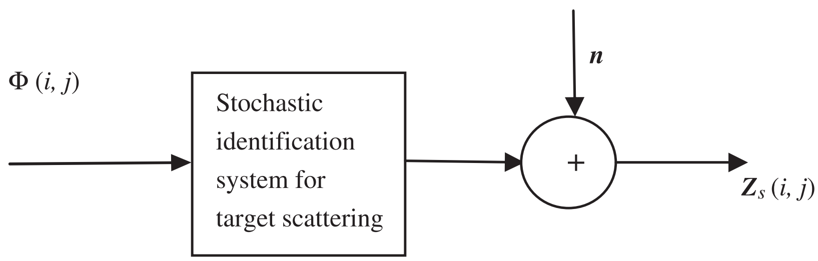

Figure 1. Stochastic system identification model for target scattering.

In Fig. 1, $\mathbf{\Phi}(i,j)$ is the input variable matrix of this identification system, and $\mathbf{Z}(i,j)$ is the output variable matrix. Let $\hat{\mathbf{P}}$ be the estimated value of the parameter to be identified, and $\mathbf{n}$ denotes random process error with mean zero. The results of N time measurements can be written as follows

$$\begin{aligned} &\mathbf{Z} = \mathbf{\Phi}\mathbf{P} + \mathbf{n} \\ &\mathbf{Z} = \left[\begin{array}{cccc} E_{0\theta}(\theta_1,\varphi_1) & E_{0\theta}(\theta_2,\varphi_2) & \cdots & E_{0\theta}(\theta_N,\varphi_N) \end{array}\right]^t, \quad \mathbf{n} = \left[\begin{array}{cccc} n(1) & n(2) & \cdots & n(N) \end{array}\right]^t \\ &\mathbf{P} = \left[\begin{array}{ccc} \overbrace{a_{11s}^2, a_{12s}^2, \ldots, a_{110s}^2}^{10} & \overbrace{b_{11s}^1, b_{12s}^1, \ldots, b_{110s}^1}^{10} & \overbrace{A_{xy}, A_{xz}, A_{yx}, \ldots, A_{zy}}^{6} \end{array}\right]^t \end{aligned} \tag{19}$$

where $n(i)$ is the error caused in the i-th time measurement

$$\mathbf{\Phi} = \left[\begin{array}{c} \overbrace{\begin{array}{cccc} -\mathbf{M}_{11}^2(1) & -\mathbf{M}_{12}^2(1) & \ldots & -\mathbf{M}_{110}^2(1) \\ -\mathbf{M}_{11}^2(2) & -\mathbf{M}_{12}^2(2) & \ldots & -\mathbf{M}_{110}^2(2) \\ \vdots & \vdots & \ldots & \vdots \\ -\mathbf{M}_{11}^2(N) & -\mathbf{M}_{12}^2(N) & \ldots & -\mathbf{M}_{110}^2(N) \end{array}}^{10\times f_0} \quad \overbrace{\begin{array}{cccc} -\mathbf{N}_{11}^1(1) & -\mathbf{N}_{12}^1(1) & \ldots & -\mathbf{N}_{110}^1(1) \\ -\mathbf{N}_{11}^1(2) & -\mathbf{N}_{12}^1(2) & \ldots & -\mathbf{N}_{110}^1(2) \\ \vdots & \vdots & \ldots & \vdots \\ -\mathbf{N}_{11}^1(N) & -\mathbf{N}_{12}^1(N) & \ldots & -\mathbf{N}_{110}^1(N) \end{array}}^{10\times f_0} \\ \overbrace{\begin{array}{cccc} \cos\theta_1\cos\phi_1 e^{-jk_0\sqrt{\varepsilon_x} a\sin\theta_1\sin\phi_1} & \cos\theta_1\cos\phi_1 e^{-jk_0\sqrt{\varepsilon_x} a\cos\theta_1} & \ldots & -\sin\theta_1 e^{-jk_0\sqrt{\varepsilon_z} a\sin\theta_1\sin\phi_1} \\ \cos\theta_2\cos\phi_2 e^{-jk_0\sqrt{\varepsilon_x} a\sin\theta_2\sin\phi_2} & \cos\theta_2\cos\phi_2 e^{-jk_0\sqrt{\varepsilon_x} a\cos\theta_2} & \ldots & -\sin\theta_2 e^{-jk_0\sqrt{\varepsilon_z} a\sin\theta_2\sin\phi_2} \\ \vdots & \vdots & \vdots & \vdots \\ \cos\theta_N\cos\phi_N e^{-jk_0\sqrt{\varepsilon_x} a\sin\theta_N\sin\phi_N} & \cos\theta_N\cos\phi_N e^{-jk_0\sqrt{\varepsilon_x} a\cos\theta_N} & \ldots & -\sin\theta_N e^{-jk_0\sqrt{\varepsilon_z} a\sin\theta_N\sin\phi_N} \end{array}}^{6} \end{array}\right]$$

The least square solution of Eq. (19) is given as

$$\hat{\mathbf{P}} = \left[\mathrm{Re}\left(\mathbf{\Phi}^t\mathbf{\Phi}^*\right)\right]^{-1}\mathrm{Re}\left(\mathbf{\Phi}^t\mathbf{Z}^*\right) \tag{20}$$

The scattered field of the anisotropic target can be obtained by extracting the first and second terms of matrix $\hat{\mathbf{P}}$ and then substituting them in Eq. (11).

When Eqs. (12) and (13) are used to modeling, the parameter matrix will be changed as follows

$$\mathbf{P}=\left[\overbrace{0,a^2_{22xs},\ldots,a^2_{210xs}}^{10}\ \overbrace{0,b^1_{22xs},\ldots,b^1_{210xs}}^{10}\ \overbrace{A_{xy},A_{xz},A_{yx},\ldots,A_{zy}}^{6}\right]^t$$

The output variable matrix of the scattering model is revised

$$\mathbf{Z}=\left[\ E_{x\theta}(\theta_1,\varphi_1)\quad E_{x\theta}(\theta_2,\varphi_2)\quad\cdots\quad E_{x\theta}(\theta_N,\varphi_N)\ \right]^t$$

And replace the first 20 columns of the input variable matrix $\mathbf{\Phi}$ with spherical vector wave functions $\mathbf{M}^2_{2n}$ and $\mathbf{N}^1_{2n}$, and then being multiplied with f'_x.

When Eqs. (14) and (15) are used to establish the scattering model, the parameter matrix to be identified is modified as

$$\mathbf{P}=\left[\overbrace{0,a^1_{22ys},\ldots,a^1_{210ys}}^{10}\ \overbrace{0,b^2_{22ys},\ldots,b^2_{210ys}}^{10}\ \overbrace{A_{xy},A_{xz},A_{yx},\ldots,A_{zy}}^{6}\right]^t$$

The output variable matrix of the scattering model is modified as

$$\mathbf{Z}=\left[\ E_{y\theta}(\theta_1,\varphi_1)\quad E_{y\theta}(\theta_2,\varphi_2)\quad\cdots\quad E_{y\theta}(\theta_N,\varphi_N)\ \right]^t$$

And replace the first 20 columns of input variable matrix $\mathbf{\Phi}$ with spherical vector wave functions $\mathbf{M}^1_{2n}$ and $\mathbf{N}^2_{2n}$, and then being multiplied by f'_y.

When Eqs. (16) and (17) are used to establish the scattering model, the parameter matrix to be identified is modified as

$$\mathbf{P}=\left[\overbrace{a^2_{11zs},a^2_{12zs},\ldots,a^2_{110zs}}^{10}\ \overbrace{b^1_{11zs},b^1_{12zs},\ldots,b^1_{110zs}}^{10}\ \overbrace{A_{xy},A_{xz},A_{yx},\ldots,A_{zy}}^{6}\right]^t$$

The output variable matrix of the scattering model is modified as

$$\mathbf{Z}=\left[\ E_{z\theta}(\theta_1,\varphi_1)\quad E_{z\theta}(\theta_2,\varphi_2)\quad\cdots\quad E_{z\theta}(\theta_N,\varphi_N)\ \right]^t$$

And replace the first 20 columns of input variable matrix $\mathbf{\Phi}$ with spherical vector wave functions $\mathbf{M}^2_{1n}$ and $\mathbf{N}^1_{1n}$, and then being multiplied by f'_z.

For higher-order items such as $f''_x x^2 e^{-jk_0 z}$, $f''_{xy} xy e^{-jk_0 z}$, $f''_y y^2 e^{-jk_0 z}$, the specific methods for determining their scattered fields are similar to that of the former, being limited in length and are no longer researched in detail. After the scattering coefficients are obtained from Eqs. (11), (13), (15) and (17), the formula of scattered field for an anisotropic medium particle is

$$\mathbf{E}_s=\sum_n\left[a^2_{1ns}\mathbf{M}^2_{1n}+b^1_{1ns}\mathbf{N}^1_{1n}+a^2_{2nxs}\mathbf{M}^2_{2n}+b^1_{2nxs}\mathbf{N}^1_{2n}+a^1_{2nys}\mathbf{M}^1_{2n}+b^2_{2nys}\mathbf{N}^2_{2n}+a^2_{1nzs}\mathbf{M}^2_{1n}+b^1_{1nzs}\mathbf{N}^1_{1n}\right]\quad(21)$$

The scattering cross section σ is also developed from Eq. (21), and its specific process is no longer researched in detail. On surface of the sphere with a radius of 0.8 meters, 1600 observation points are chosen, as shown in Fig. 2, and the operating frequency is 3 GHz. Following is the numerical results of coefficients a^2_{1ns}, b^1_{1ns} as the 0-order field being incident on the particle.

Figure 3 and Fig. 4 show variations of the first 8 terms of the coefficients with respect to the element of the permittivity tensor in the zeroth order field, and Fig. 5 shows the variation of the amplitudes of internal fields. Fig. 3 shows that the coefficient a^2_{1s} of the scattered field does not change with the change of the dielectric constant of the polarization direction, and coefficient a^2_{2s} decreases with the increase of dielectric constant. When $n > 3$, the changes of other coefficients are relatively complex and generally small. Fig. 4 shows the variation of coefficient b^1_{ns} with respect to dielectric constant of the polarization direction of the incident field.

We can see that coefficient b^1_{1s} decreases with the increase of the dielectric constant, and b^1_{2s} has a change of the opposite. When $n > 3$, the coefficients are small, which are similar to the result of the Mie theory. In Fig. 5, the relations of the amplitudes of different subscripts with number n are

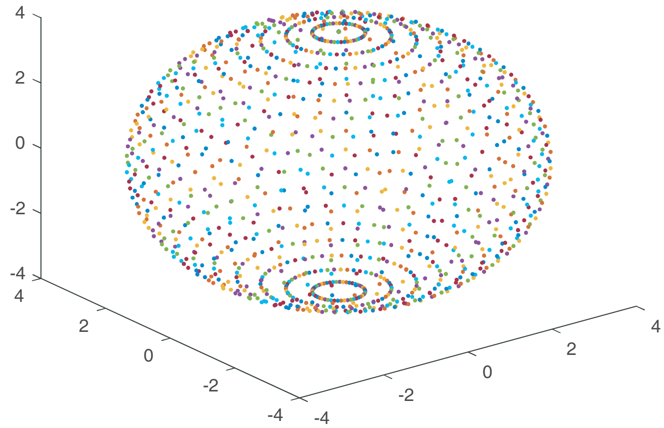

Figure 2. Distribution of observation points on the sphere.

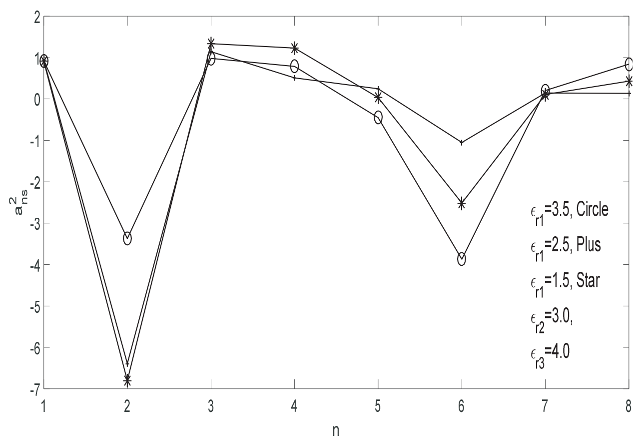

Figure 3. Variation of scattering coefficient a^2_{ns} with n and ε_{r1}.

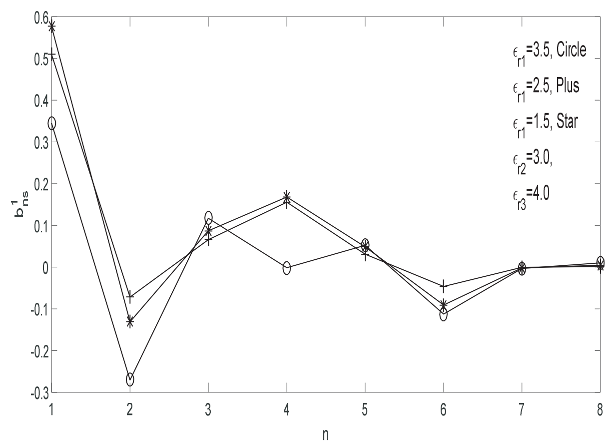

Figure 4. Variation of scattering coefficient b^1_{ns} with n and ε_{r1}.

$A_{xy} \leftrightarrow 1$, $A_{xz} \leftrightarrow 2$, $A_{yx} \leftrightarrow 3$, $A_{yz} \leftrightarrow 4$, $A_{zx} \leftrightarrow 5$, $A_{zy} \leftrightarrow 6$. The number of $n = 1$ and $n = 2$ show respectively the basic wave mode's amplitudes, the x-directional polarization propagating along both the y-direction and z-direction, and the minus sign indicates that there is π phase difference. It can be seen that the amplitude of $n = 2$ is the largest, which indicates that the basic wave mode similar to the state of the polarization and propagation of incident wave plays a decisive role. The magnitudes corresponding to $n = 3$, $n = 4$ show the amplitudes of y-polarization, which are almost zero, and results corresponding to $n = 5$, $n = 6$ show the basic mode of polarizing in z-direction, which indicates the existence of cross polarization in the anisotropic medium. Further simulation indicates that effects of the dielectric constant in the other direction on coefficients of the scattered field expansion are very small.

With the second term xe^{-jk_0z} of the series incident on the anisotropic particle, Fig. 6 shows the numerical result of the coefficient a^2_{nxs}, and its variation is similar to a^2_{ns} in Fig. 3; however, the magnitude is three orders smaller in magnitude than the a^2_{ns}. The variation of Fig. 7 is similar to that of Fig. 4 and also has a decrease of two orders in magnitude. Fig. 8 and Fig. 5 show the changes in the internal field of anisotropic particles, and the magnitude shown in Fig. 8 is also smaller than that shown in Fig. 5 by three orders in magnitude. These results indicate that in the study of beam scattering of anisotropic particles, the high-order terms of series expansions for a photoelectric beam may be ignored

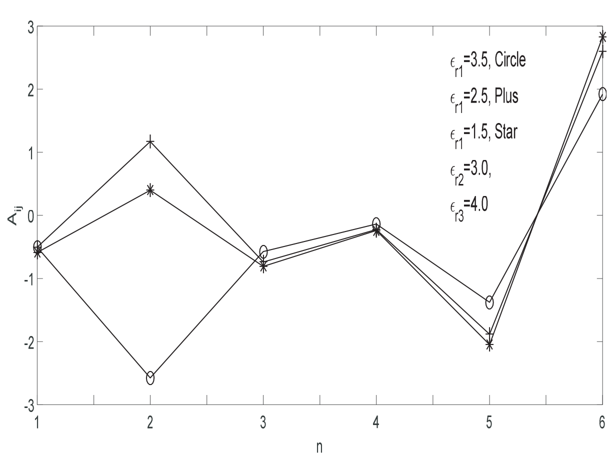

Figure 5. Amplitude A_{ij} of internal field with n and ε_{r1}.

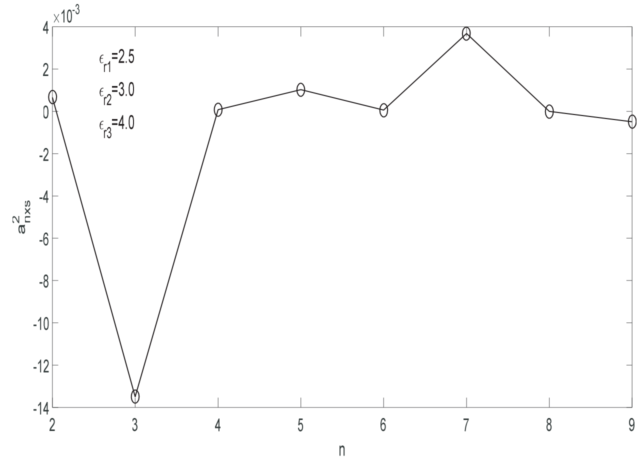

Figure 6. Variation of scattering coefficient a^2_{nxs} with n.

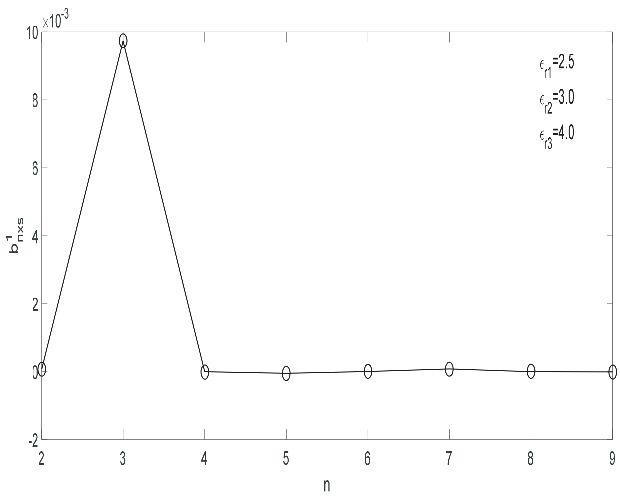

Figure 7. Variation of scattering coefficient b^1_{nxs} with n.

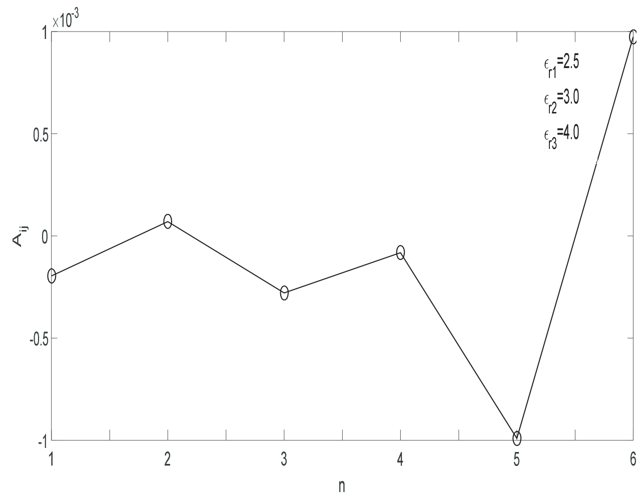

Figure 8. Amplitude of internal field A_{ij} with n.

in application. It is also shown that the amplitudes of basic wave modes, corresponding to $n = 1$ up to $n = 4$, are almost unchanged. The last two coefficients in Fig. 8 correspond to amplitudes of the basic modes of polarization in z-direction and propagating in the directions of x and y, respectively, indicating that there exists a cross polarization in the anisotropic particles.

When the third and fourth terms ye^{-jk_0z} and ze^{-jk_0z} of the series expansion respectively irradiate the anisotropic particles, the expansion coefficients of the scattered field and variation of the inner field of the particles are not very different from those of the second term xe^{-jk_0z} irradiating the particles and are no longer discussed.

System identification is of inputting some signals to the system or model, measuring the corresponding outputs or responses, then mathematically processing the inputs and outputs, and finally system parameters or model parameters are determined. This method has very high reliability in application and has been successfully applied in electromagnetic field [1, 2]. Since the measurements and studies of anisotropic particle scattering are relatively limited, we take the same target magnitude and electromagnetic wave frequency as that used in previous one. The system identification is used to calculate the scattering of an isotropic spherical particle in a plane wave and is compared with

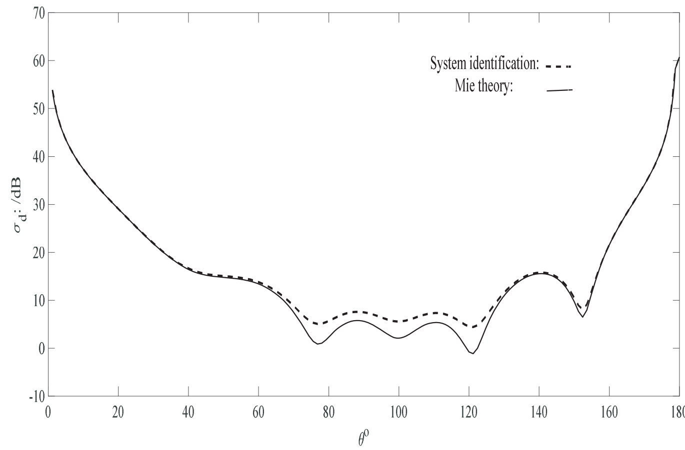

Figure 9. Comparison of the results of stochastic system method with the results of the literature.

the famous Mie theory. The differential scattering cross sections are shown in Fig. 9. It can be seen that results of the two methods are in good agreement, which shows the effectiveness of the proposed algorithm.

2.3. Scattering of Elliptical Gaussian Beams by Anisotropic Particles

In [5, 19], the expression of the elliptical Gaussian beam in a particle coordinate system is given assumed that the electric field polarizing in x-direction and propagating along z-direction. The amplitude of the zero-order field of series expansion of the elliptical Gaussian beam is thus obtained as

$$f\left(0,0,0\right)=E_0\sqrt{-Q_{x0}Q_{y0}}e^{-ikd} \tag{22}$$

The amplitudes of the first-order fields are

$$f_z=z\left(E_0\frac{\dfrac{1}{kW_{0y}^2}+\dfrac{1}{kW_{0x}^2}-i\dfrac{4d}{k^2W_{0y}^2W_{0x}^2}}{\left(i\left(\dfrac{2d}{kW_{0y}^2}+\dfrac{2d}{kW_{0x}^2}\right)+\dfrac{4d^2}{k^2W_{0y}^2W_{0x}^2}-1\right)^{\frac{3}{2}}}e^{-ikd}\right) \tag{23}$$
$$f_x=0\quad f_y=0$$

where

$$Q_{y0}=\left(i+\frac{2d}{kW_{0y}^2}\right)^{-1},\quad Q_{x0}=\left(i+\frac{2d}{kW_{0x}^2}\right)^{-1}$$

W_{0y} and W_{0x} are waist widths of the beams respectively in y-direction and x-direction, and d is the distance between the particle and the beam. The scattered field of the anisotropic particle is acquired after putting Eqs. (22) and (23) in Eq. (21). According to [21], the beam waist width is taken as $W_{0y}=1000\lambda$ and $W_{0x}=900\lambda$. The beam irradiation distance of $d=20\,\mathrm{m}$ is used, and the observation point parameters are $r=200\,\mathrm{m}$ and $\phi=0.25\pi$. Some of the results are as follows.

Variations of σ_d with the dielectric constant tensor element are shown in Fig. 10 to Fig. 12, and the unit of y-axis is in decibels. It can be seen that the influence of the dielectric constant of the direction of incident wave polarization on the σ_d is different from that induced by the elements of other directions. The dielectric constant of polarization direction mainly affects forward scattering and backscattering, but this effect is not a positive proportional relation but a complex function of the dielectric constant, and it also indicates that the scattering of the target is mainly a kind of dipole

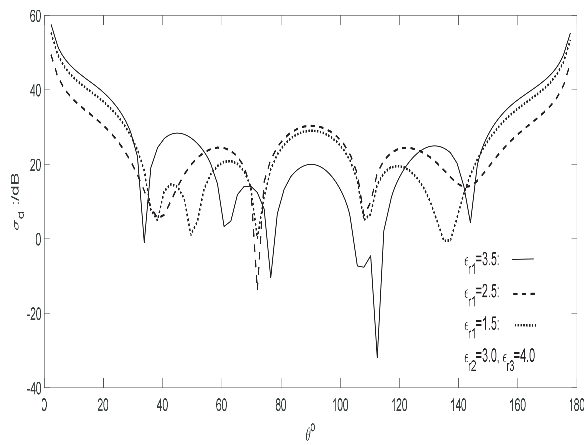

Figure 10. Variation of differential scattering cross section σ_d with ε_{r1}.

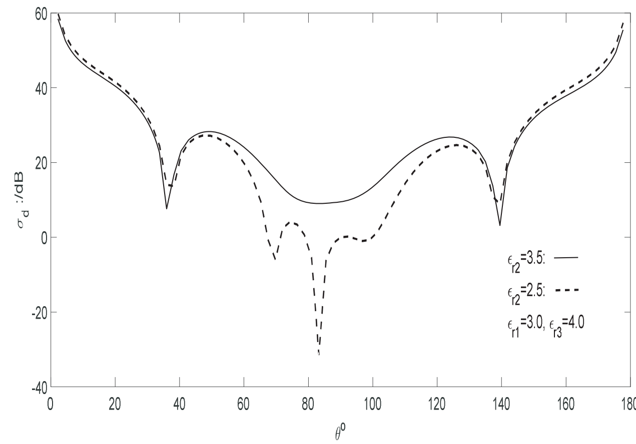

Figure 11. Variation of differential scattering cross section σ_d with ε_{r2}.

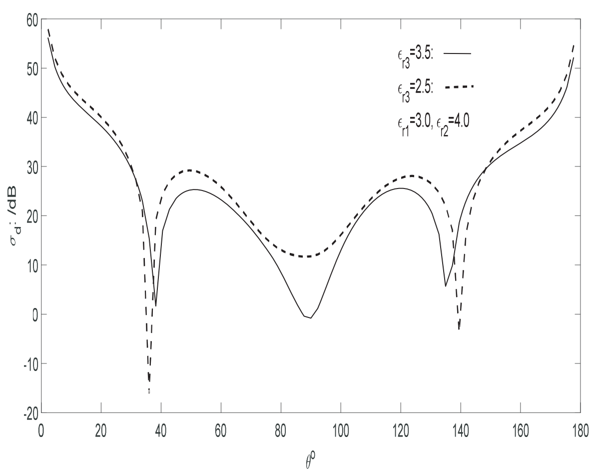

Figure 12. Variation of differential scattering cross section σ_d with ε_{r3}.

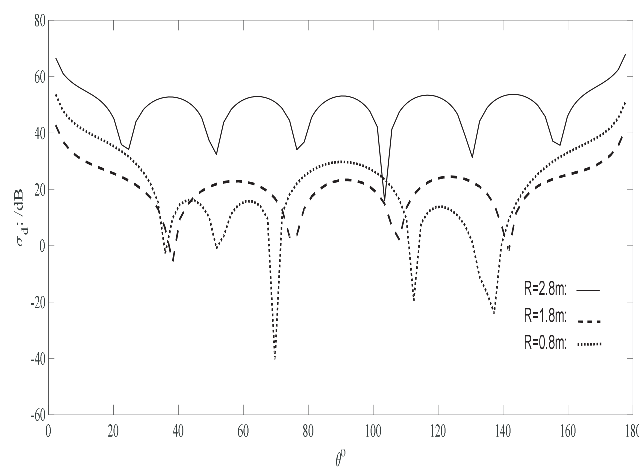

Figure 13. Variation of differential scattering cross section σ_d with radius R.

radiation. The dielectric constants of the other two directions have little contribution to forward scattering and backscattering, but they have a certain influence on scattering in the space between the forward and backward regions, near the space of $\theta = 0.5\pi$. The dipole radiation theory shows that when the observation azimuth is perpendicular to the electric dipole, the radiation is the strongest. A strong radiation in the area of $\theta = 0.5\pi$ indicates that there is a cross polarization in anisotropic targets. Since the direction of the total electric field in the target depends mainly on the polarization direction of the incident field, the polarization charge depends on the polarization intensity vector, and the polarization intensity vector also depends on the dielectric constant of direction in polarization and the total electric field. The polarized charge is influenced by the dielectric constant in the direction of polarization. The additional electric field generated by the polarized charge and the incident field constitute the scattering source of the target, which leads to a big influence of the dielectric constant in polarization on the forward differential scattering and backward differential scattering. Fig. 13 shows the relation between scattering of points and the particle's radius in S-band for an anisotropic particle. The scattering increases generally with the increase of the radius, but this increase is not a proportional relationship, which indicates the complexity of the scattering of anisotropic particles.

3. CONCLUSION

The basic wave patterns of electromagnetic field propagation in anisotropic dielectric particles are studied, and the expressions of the zero-order field and the first-order fields of coordinates are obtained based on the orthogonal relations among vector wave functions and that of trigonometric functions. Based on the basic electromagnetic wave type of anisotropic dielectric particles and Taylor series of arbitrary electromagnetic wave beams, a stochastic system identification model for anisotropic beam scattering is developed. Based on this established model, the relations for the expansion coefficients of first-order scattered field, zero-order field and basic wave modes of the anisotropic medium particle with the tensor of dielectric constant are studied in S-band. The reason of the scattering result is explained from the theory of electromagnetism, and the validity of this established model is verified. Taking elliptical Gaussian beam as an example, the beam scattering characteristics of anisotropic dielectric particles are studied, and it is found that the dielectric constant in the direction of the beam polarization has a great influence on forward scattering and backward scattering. The method is not limited by the magnitude of particle and beam frequency, and it has certain significance in analyzing the scattering characteristics of anisotropic medium particle being irradiated by the electromagnetic wave beams.

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