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Complex Permittivity Estimation for Each Layer in a Bi-Layer Dielectric Material at Ku-Band Frequencies

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Abstract—In this paper, a new measurement method is proposed to estimate the complex permittivity for each layer in a bi-layer dielectric material using a Ku-band rectangular waveguide WR62. The S_{ij}-parameters at the reference planes in the rectangular waveguide loaded by a bi-layer material sample are measured as a function of frequency using the E8634A Network Analyzer. Also, by applying the transmission lines theory, the expressions for these parameters as a function of complex permittivity of each layer are calculated. The Nelder-Mead algorithm is then used to estimate the complex permittivity of each layer by matching the measured and calculated the S_{ij}-parameters. This method has been validated by estimating, at the Ku-band, the complex permittivity of each layer of three bi-layer dielectric materials. A comparison of estimated values of the complex permittivity obtained from bi-layer measurements and mono-layer measurements is presented.

1. INTRODUCTION

Multi-layer dielectric materials are currently used in microwave integrated circuits and monolithic microwave integrated circuits [1, 2]. By choosing the electromagnetic properties and appropriate thickness for each layer, it is possible to synthesize bi-layer dielectric materials with new electromagnetic properties otherwise not found in a single mono-layer dielectric material [3].

Many measurement techniques have been developed and used in recent years to estimate the complex permittivity of monolayer dielectric materials [4, 7]. These include free space methods, cavity resonators techniques, and transmission line or waveguide techniques. Each technique has its distinct advantages and disadvantages. The free space methods are less accurate because of the unwanted reflection from surrounding objects [4]. The resonant cavity measurement technique is more accurate, but it has narrow band [5]. The Nicholson-Ross technique is widely used [6, 8] to determine the complex permittivity of dielectric material over a wide-band of frequencies accurately.

This technique has the disadvantage of having inaccuracy peaks for dielectric materials with low dielectric losses where the sample length is a multiple of half the wavelength of the rectangular waveguide [8]. To solve this problem, several researchers have combined the Nicholson Ross technique with nonlinear optimization techniques [8, 9]. However, these techniques are not used to estimate the complex permittivity of each layer in a bi-layer dielectric material in wide-band frequencies.

The aim of this work is to propose a new measurement method to estimate the complex permittivity for each layer of a bi-layer dielectric material at Ku-band. The bi-layer dielectric material is loaded in a rectangular waveguide WR62 with dimensions: $a = 15.8\,\text{mm}$ and $b = 7.9\,\text{mm}$. The S_{ij}-parameters are measured as a function of frequency using the Vector Network Analyzer and calculated as a function of the complex permittivity of each layer in a bi-layer dielectric material using transmission lines theory. To estimate the complex permittivity of each layer dielectric material with a specific prior knowledge of the thickness, the square sums of errors between the measured and calculated S_{ij}-parameters are

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minimized using a Nelder-Mead Algorithm [10]. The complex permittivity of each layer in a bi-layer dielectric material such as FR4-Teflon, FR4-Delrin and Teflon-Delrin is determined at the Ku-band frequencies.

2. THEORY

2.1. Direct Problem

This section presents the calculation of the S_{ij}-parameters of a rectangular waveguide loaded with a bi-layer dielectric material as shown in Fig. 1.

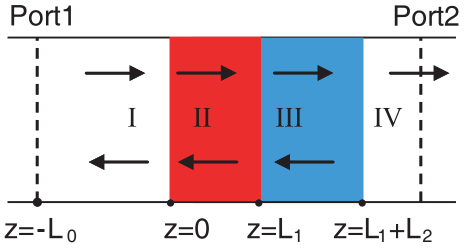

Figure 1. Rectangular waveguide loaded with a bi-layer dielectric material.

The bi-layer dielectric material consists of two layers, where the first layer has complex permittivity ε_{r1} and is located between transverses planes $z = 0$ and $z = L_1$, and the second layer has complex permittivity ε_{r2} and is located between transverse planes $z = L_1$ and $z = L_1 + L_2$.

The S_{ij}-parameter is found by analyzing the electric field at the sample interfaces. Assuming that only the dominant TE_{10} mode propagates in the loaded waveguide (see Fig. 1), the formulation of the S_{ij}-parameters can be expressed as a function of complex permittivity of each layer using transmission lines theory. In regions I, II, III and IV we can write the spatial distribution of the electric field for an incident field normalized to 1 in region I:

$$E_I = 1\exp(-j\gamma_0 z) + A_1\exp(j\gamma_0 z) \tag{1}$$

$$E_{II} = A_2\exp(-j\gamma_1 z) + A_3\exp(j\gamma_1 z) \tag{2}$$

$$E_{III} = A_4\exp(-j\gamma_2 z) + A_5\exp(j\gamma_2 z) \tag{3}$$

$$E_{IV} = A_6\exp(-j\gamma_0 z) \tag{4}$$

where $\gamma_0 = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}}$, $\gamma_i = \sqrt{\varepsilon_{ri}\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}}$ and $\varepsilon_{ri} = \varepsilon'_{ri} - j\varepsilon''_{ri}$ with $i = 1, 2$.

γ_0, γ_1 and γ_2 are the propagation constants in vacuum, first layer and second layer, respectively; (ω) is the angular frequency; (a) is the dimension of the waveguide; c is the speed of light in vacuum. The constants A_i are determined from the boundary conditions.

Tangential component of the electric field is continuous at sample interfaces:

$$E_I(z=0) = E_{II}(z=0) \tag{5}$$

$$E_{II}(z=L_1) = E_{III}(z=L_1) \tag{6}$$

$$E_{III}(z=L_1+L_2) = E_{IV}(z=L_1+L_2) \tag{7}$$

Tangential component of the magnetic field is continuous at sample interfaces:

$$\frac{\partial E_I}{\partial z}(z=0) = \frac{\partial E_{II}}{\partial z}(z=0) \tag{8}$$

$$\frac{\partial E_{II}}{\partial z}(z=L_1) = \frac{\partial E_{III}}{\partial z}(z=L_1) \tag{9}$$

$$\frac{\partial E_{III}}{\partial z}(z=L_1+L_2) = \frac{\partial E_{IV}}{\partial z}(z=L_1+L_2) \tag{10}$$

By application of these boundary conditions, we obtain the following matrix system:

$$\begin{bmatrix} e^{-j\gamma_0 L_1} & -e^{j\gamma_1 L_1} & -e^{-j\gamma_1 L_1} & 0 & 0 & 0 \\ \gamma_0 e^{-j\gamma_0 L_1} & \gamma_1 e^{j\gamma_1 L_1} & -\gamma_1 e^{-j\gamma_1 L_1} & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 0 & -\gamma_1 & \gamma_1 & \gamma_2 & -\gamma_2 & 0 \\ 0 & 0 & 0 & e^{-j\gamma_2 L_2} & e^{j\gamma_2 L_2} & -e^{-j\gamma_0 L_2} \\ 0 & 0 & 0 & -\gamma_2 e^{-j\gamma_1 L_1} & \gamma_2 e^{-j\gamma_1 L_1} & \gamma_0 e^{-j\gamma_0 L_2} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} -e^{j\gamma_0 L_1} \\ \gamma_0 e^{j\gamma_0 L_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

By solving this matrix system, we obtain the equation of S_{11} and S_{12} parameters as a function of the complex permittivities of each layer in the bi-layer material:

$$S_{11}(\varepsilon'_{r1}, \varepsilon''_{r1}, \varepsilon'_{r2}, \varepsilon''_{r2}) = A_1 e^{-2j\gamma_0 L_0} = e^{-2j\gamma_0 L_0} \frac{d_2 + d_1}{d_2 - d_1} \tag{12}$$

$$S_{21}(\varepsilon'_{r1}, \varepsilon''_{r1}, \varepsilon'_{r2}, \varepsilon''_{r2}) = A_6 e^{-j\gamma_0 (L_0 + L_1 + L_2)} = -2e^{-j\gamma_0 L_0} \frac{\gamma_0 \gamma_1 \gamma_2}{(d_2 - d_1). \cos\gamma_1 L_1 . \cos\gamma_2 L_2} \tag{13}$$

where:

$$\begin{aligned} d_1 &= \gamma_1 [\gamma_0 (\gamma_2 - \gamma_1 \tan\gamma_1 L_1 . \tan\gamma_2 L_2) + j\gamma_2 (\gamma_1 \tan\gamma_1 L_1 + \gamma_2 \tan\gamma_2 L_2)] \\ d_2 &= \gamma_0 [\gamma_2 (\gamma_2 \tan\gamma_1 L_1 . \tan\gamma_2 L_2 - \gamma_1) - j\gamma_0 (\gamma_1 \tan\gamma_2 L_2 + \gamma_2 \tan\gamma_1 L_1)] \end{aligned}$$

To calculate S_{12} and S_{22} parameters as a function of complex permittivities of each layer in the bi-layer material, we assume that the fundamental mode TE_{10} is incident in region IV, and we follow the same procedure used to calculate the S_{11} and S_{21}.

$$S_{12}(\varepsilon'_{r1}, \varepsilon''_{r1}, \varepsilon'_{r2}, \varepsilon''_{r2}) = S_{21}(\varepsilon'_{r1}, \varepsilon''_{r1}, \varepsilon'_{r2}, \varepsilon''_{r2}) \tag{14}$$

$$S_{22}(\varepsilon'_{r1}, \varepsilon''_{r1}, \varepsilon'_{r2}, \varepsilon''_{r2}) = \frac{d'_2 + d'_1}{d'_2 - d'_1} \tag{15}$$

where

$$\begin{aligned} d'_1 &= \gamma_2 [\gamma_0 (\gamma_1 - \gamma_2 \tan\gamma_1 L_1 . \tan\gamma_2 L_2) + j\gamma_1 (\gamma_2 \tan\gamma_2 L_2 + \gamma_1 \tan\gamma_1 L_1)] \\ d'_2 &= \gamma_0 [\gamma_1 (\gamma_1 \tan\gamma_2 L_2 . \tan\gamma_1 L_1 - \gamma_2) - j\gamma_0 (\gamma_2 \tan\gamma_1 L_1 + \gamma_1 \tan\gamma_2 L_2)] \end{aligned}$$

For mono-layer dielectric materials, we can choose $L_1 = 0$ and $L_2 = L$, and in this case we have:

$$S_{11}(\varepsilon'_r, \varepsilon''_r) = e^{-2j\gamma_0 L_0} \frac{j(\gamma_0^2 - \gamma^2) \cdot \tan\gamma L}{2\gamma_0 \gamma + j(\gamma_0^2 + \gamma^2) \cdot \tan\gamma L} \tag{16}$$

$$S_{21}(\varepsilon'_r, \varepsilon''_r) = e^{-j\gamma_0 L_0} \frac{2\gamma_0 \gamma}{[2\gamma_0 \gamma + j(\gamma_0^2 + \gamma^2) \cdot \tan\gamma L] \cos\gamma L} \tag{17}$$

$$S_{12}(\varepsilon'_r, \varepsilon''_r) = S_{21}(\varepsilon'_r, \varepsilon''_r) \tag{18}$$

$$S_{22}(\varepsilon'_r, \varepsilon''_r) = \frac{j(\gamma_0^2 - \gamma^2) \cdot \tan\gamma L}{2\gamma_0 \gamma + j(\gamma_0^2 + \gamma^2) \cdot \tan\gamma L} \tag{19}$$

where $\gamma_0 = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}}$ and $\gamma = \sqrt{\varepsilon_r \frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}}$.

2.2. Inverse Problem

This section presents the calculation of the complex permittivity for each layer in a bi-layer dielectric material given with specific prior knowledge of the thickness of each layer. For this raison, we use the Fminsearch function implemented on MATLAB [11] which is based on the Nelder-Mead sequential simplex algorithm [9]. This function solves nonlinear unconstrained multivariable optimization problems, which finds the minimum of a scalar function of several variables from an initial guess of the complex relative permittivity such as $\varepsilon_r' = 1.5$, $\varepsilon_r'' = 0.005\varepsilon_r'$. The error function that we want to minimize with Fminsearch function is the square sums of errors between the measured and calculated S_{ij}-parameters written as follows:

$$f(\varepsilon_{r1}', \varepsilon_{r1}'', \varepsilon_{r2}', \varepsilon_{r2}'') = \sum_{ij} |S_{ijc} - S_{ijm}| \tag{20}$$

3. NUMERICAL RESULTS

To validate the direct problem, the S_{ij}-parameters at the reference planes of the rectangular waveguide in Ku-band loaded by a mono-layer dielectric material Teflon ($\varepsilon_r' = 2.04$, $\varepsilon_r'' = 0.002$) with thickness $L = 1.9$ mm are calculated using the procedure described in Section 2.1 and simulated with HFSS (High Frequency Structure Simulator) software as shown in Fig. 2. It is seen from these results that there is an excellent agreement between calculated and simulated S_{ij}-parameters.

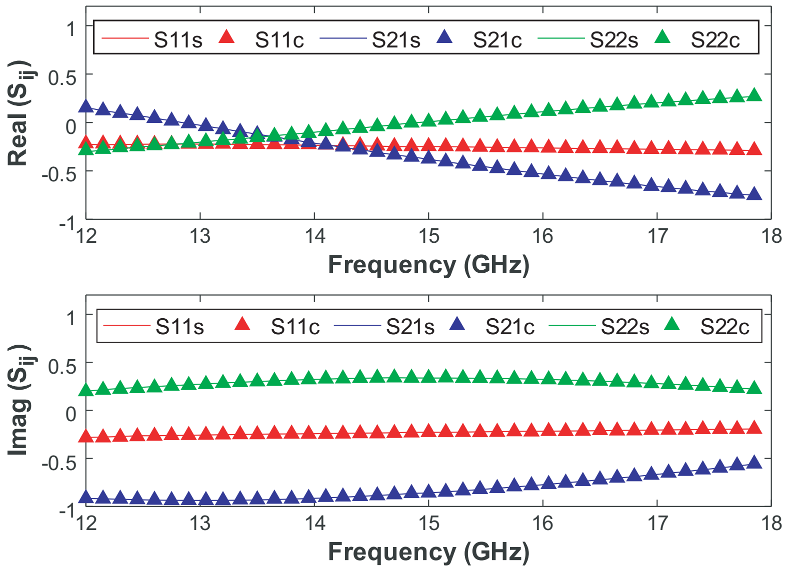

Figure 2. Simulated and calculated S_{ij}-parameters in a rectangular waveguide WR62 ($L_0 + L = 6.7$ mm) loaded by Teflon with thickness $L = 1.9$ mm.

To validate the direct problem of bi-layer dielectric material, the S_{ij}-parameters of a rectangular waveguide in Ku-band loaded by a bi-layer dielectric material formed by FR4 Epoxy ($\varepsilon_{r2} = 4.5 - j0.090$) with thickness $L_2 = 1.5$ mm and Teflon ($\varepsilon_{r1} = 2.04 - j0.002$) with thickness $L_1 = 1.9$ mm are calculated using the procedure described in Section 2.1 and simulated by the use of HFSS software. As can be seen from the results shown in Fig. 3, there is a good agreement between calculated and simulated S_{ij}-parameters.

For the inverse problem, using the procedure described in Sections 2.1 and 2.2, the complex permittivity of mono-layer dielectric material and of each layer in a bi-layer dielectric material was determined in the Ku-band frequencies.

We consider the measurement system shown in Fig. 4. The S_{ij}-parameters at references plane of a Ku-band rectangular waveguide WR62 loaded by a mono or a bi-layer dielectric material were measured using the E8634A Network Analyzer.

First of all, we applied this method to estimate the complex permittivity of mono-layer dielectric material in the Ku-band frequencies. The initial guess of the complex permittivity was $\varepsilon_r = 1.5(1 - j0.005)$. The values of complex permittivity of Teflon, FR4 epoxy and Delrin, with thicknesses of 1.9 mm, 1.5 mm and 2 mm, respectively were determined. For all cases, the obtained results are plotted in Fig. 5.

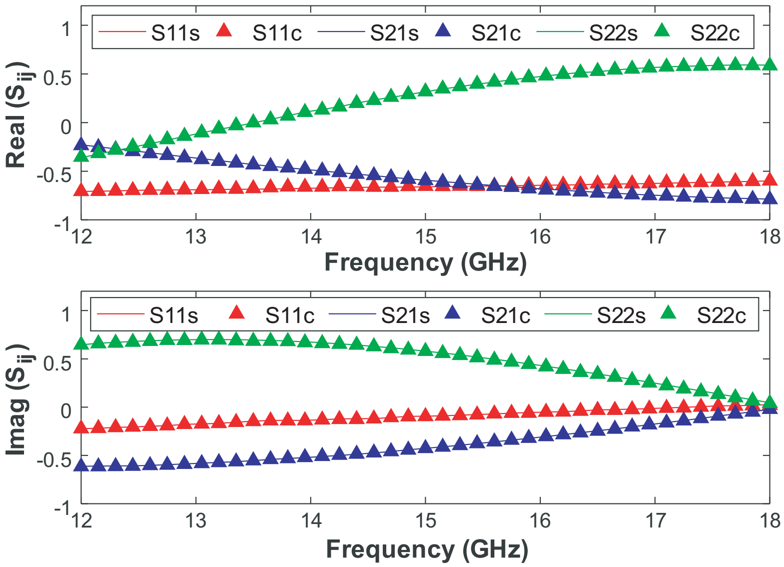

Figure 3. Simulated and calculated S_{ij}-parameters in a rectangular waveguide ($L_0+L_1+L_2 = 6.7\,\text{mm}$) loaded by a bi-layer FR4(1.5 mm) and Teflon(1.9 mm).

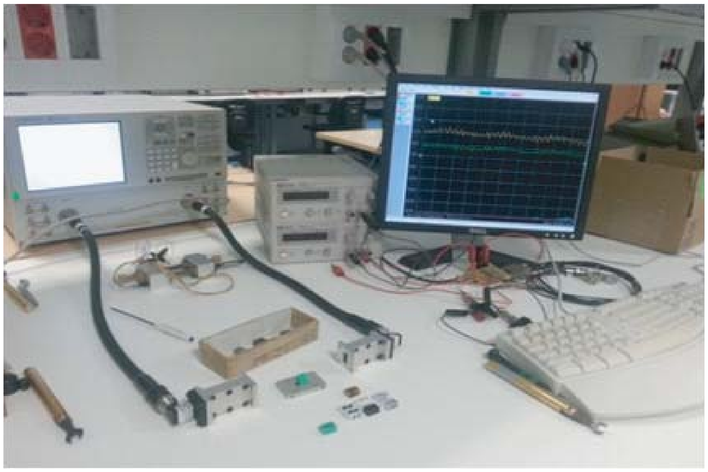

Figure 4. The measurement system.

Table 1. Average complex permittivity and average relative error percentage on the real and imaginary parts of the complex permittivity at Ku-band.

Materials		Measurement		%Error	
Bilayer	Monolayer	Monolayer	Bilayer	ε'_r	ε''_r
FR4-Teflon	FR4	$4.4560 - j0.0969$	$4.5065 - j0.0981$	$< 1.5\%$	$< 7.2\%$
	Teflon	$2.0361 - j0.0035$	$2.0474 - j0.0033$		
FR4-Delrin	FR4	$4.4560 - j0.0969$	$4.5067 - j0.1039$		
	Delrin	$2.9076 - j0.0429$	$2.8812 - j0.0445$		
Teflon-Delrin	Delrin	$2.9076 - j0.0429$	$2.8634 - j0.0433$		
	Teflon	$2.0361 - j0.0035$	$2.0475 - j0.0037$		

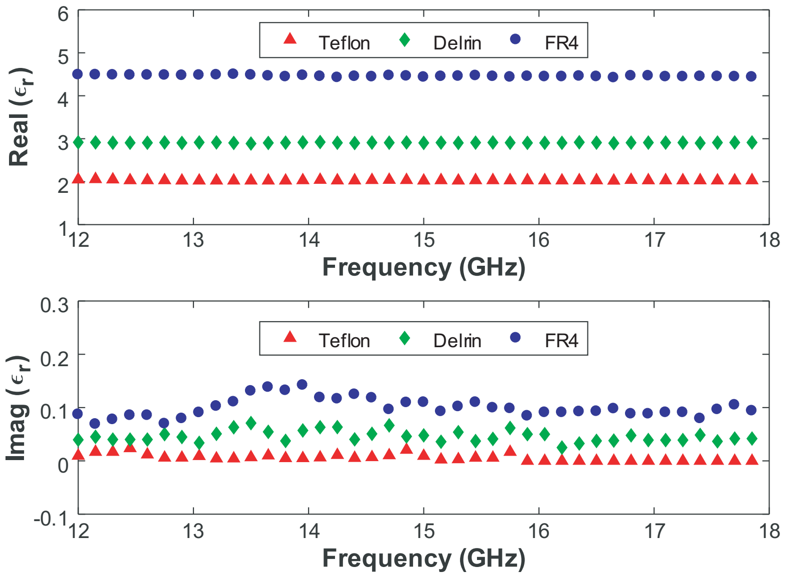

Figure 5. Complex permittivity of the mono-layers Teflon(1.9 mm), Delrin(2 mm) and FR4(1.5 mm) obtained from the S_{ij} measured using the inverse procedure with Nelder-Mead algorithm.

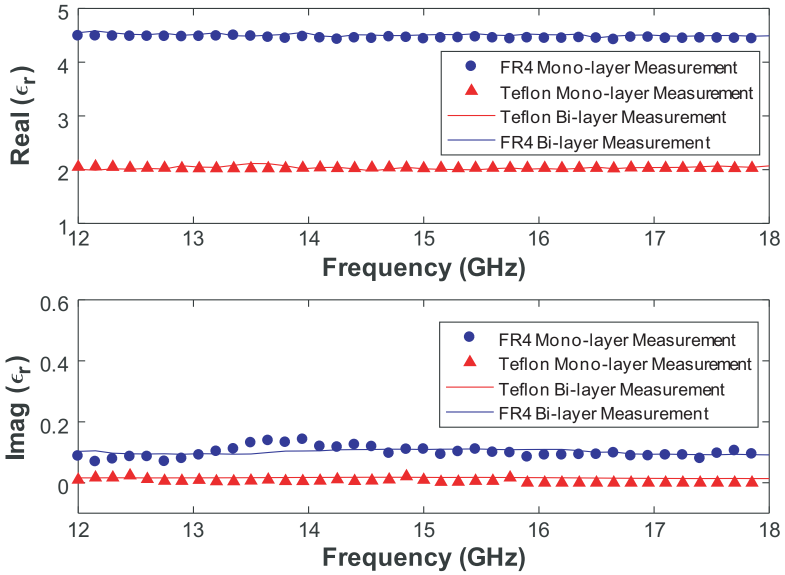

Figure 6. Complex permittivity for each layer in the bi-layer FR4 epoxy(1.5 mm)-Teflon(1.9 mm) obtained from the S_{ij} measured using the inverse procedure with Nelder-Mead algorithm.

The results obtained for the complex permittivity of mono-layers by using the procedure described in this work are in good agreement. We can see that FR4 is a dielectric material with loss tangent about 0.02. However, the Teflon dielectric has a small loss tangent which is around 0.001. For the inverse problem, using the procedure described in Sections 2.1 and 2.2, the complex permittivity for each layer in the bi-layer dielectric material was determined in the Ku-band frequencies. The initial guess of the complex permittivity was $\varepsilon'_{r1} = \varepsilon'_{r2} = 1.5$ and $\varepsilon''_{r1} = 0.005\varepsilon'_{r1}$ and $\varepsilon''_{r2} = 0.01\varepsilon'_{r2}$. The values of dielectric permittivity for each layer in the bi-layer FR4 epoxy-Teflon are determined and plotted in Fig. 6, those of FR4 epoxy-Delrin in Fig. 7, and those of Teflon-Delrin in Fig. 8.

From the results depicted in Figures 6–8, we can conclude that there is a good agreement between the values of the complex permittivities of the mono-layers materials and those of each layer of the

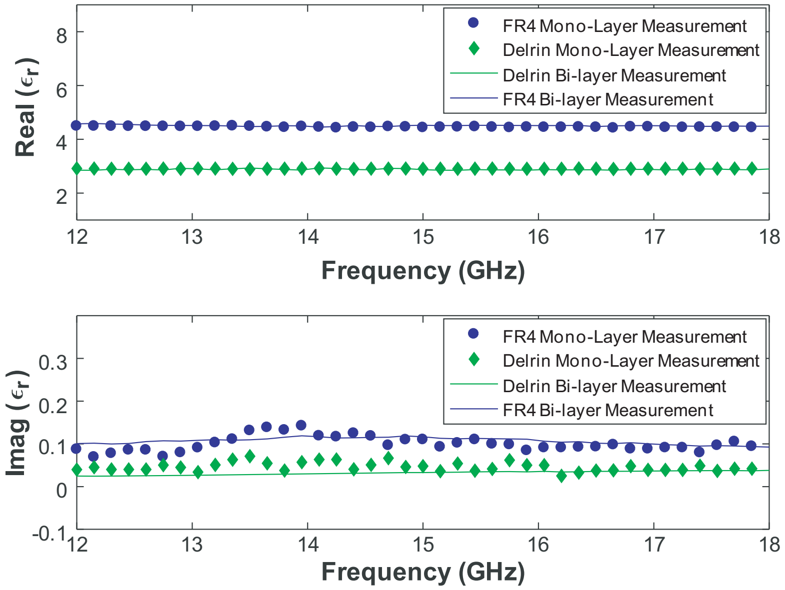

Figure 7. Complex permittivity for each layer in the bi-layer FR4 epoxy(1.5 mm)-Delrin(2 mm) obtained from the S_{ij} measured using the inverse procedure with Nelder-Mead algorithm.

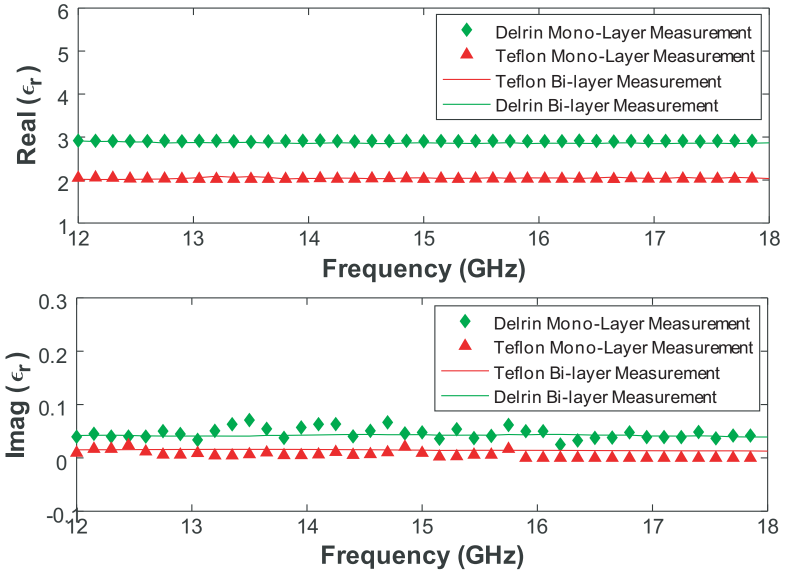

Figure 8. Complex permittivity for each layer in the bi-layer Teflon(1.9 mm)-Delrin(2 mm) obtained from the S_{ij} measured using the inverse procedure with Nelder-Mead algorithm.

bi-layer materials made up from these mono-layers materials.

Table 1 presents the average values and average relative errors of the complex permittivities of each layer of material samples in the Ku-band calculated from monolayer and bi-layer measurements.

The results presented in Table 1 show a good agreement between the average values of the complex permittivities of each layer obtained from the monolayer measurements and those obtained from the bilayer measurements with a small average relative error at the real part of the complex permittivity (lower at 1.5%), and this error can be explained by the presence of air gaps between the individual layers in the bi-layer materials. Because of the low losses of the materials studied, this error can be scarcely higher at the imaginary part ($\leq 7.2\%$).

4. CONCLUSION

In this work, a new measurement method has been presented to estimate the complex permittivity of each layer in a bi-layer dielectric material with a specific prior knowledge of the thickness using a Ku-band rectangular waveguide WR62. The S_{ij}-parameters are measured by Vector Network Analyzer and calculated as a function of complex permittivity of each layer using transmission lines theory. The Nelder-Mead Algorithm has been used to estimate the complex relative permittivity of each layer in a bi-layer dielectric material (or of a mono-layer dielectric material) by matching the calculated value with the measured value of S_{ij}-parameters of a Ku-band rectangular waveguide, loaded by a bi-layer (or a mono-layer) dielectric material. The results obtained from bi-layer measurement are in good agreement with those obtained from mono-layer measurement. This method has been validated using three bi-layer dielectric materials such as FR4-Teflon, FR4-Delrin and Delrin-Teflon. The future work is to adapt this technique to estimate the dielectric properties of magnetic materials.

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