Secrecy Sustainable Transmission Design in Energy Harvesting Enable Relay Networks

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Abstract—In this paper, we investigate the secrecy design in a sustainable relay network, where the relay is energy harvesting enabled and utilizes time switching to harvest wireless power. Specifically, assuming half-duplex amplify-and-forward relaying, we investigate the worst-case secrecy rate maximization by jointly designing the relay beamforming matrix, artificial noise covariance, and the time switching ratio. However, the formulated problem is highly non-convex due to the secrecy rate function and the dynamic relay transmit power constraint. By decoupling the original problem, we propose a two-layer optimization algorithm, where the outer problem is solved by two-dimensional search while the inner problem is solved by semi-definite relaxation. Numerical results show the effectiveness of the proposed scheme.

1. INTRODUCTION

Recently, a newly emerging absorber, named as metamaterial absorber (MA), has aroused great attention [1–3]. Specifically, the multi-band, broad-band, and tunable MAs were investigated in [1, 2], and [3], respectively. Based on amount of theoretically analysis and experiment data, the authors obtained several promising conclusions about MA. The progress in MA makes a new technique named energy harvesting (EH) become more practical, which is considered as one of the most effective approaches for improving the energy efficiency of wireless networks [4]. Since relay is commonly an energy-constrained node, it is critical to require ambient energy supply for supporting transmission in relay networks [5].

Recently, there has been a urgent concern about EH-enabled relay, e.g., the relay can harvest energy from the ambient radio frequency (RF) signal [6]. Specifically, in [7], the authors investigated a novel full duplex wireless-powered relay with self-energy recycling scheme. In [8], the authors investigated the relay beamforming (BF) and power splitting (PS) for EH-enabled amplify-and-forward (AF) relay networks. In [9], the authors investigated the joint power control and energy transfer scheme in EH-enabled AF relay networks. In [10], taking into account imperfect channel state information (CSI), the authors proposed a robust BF and PS scheme.

On the other hand, due to the complex communication environment and the openness of wireless channels, the security issues of relay networks are facing severe challenges [11]. The physical layer security (PLS) technique, which utilizes the features of wireless channel to improve the security, has been proved to be an effective way [12]. Specifically, in [13–15], the authors investigated secure EH-enabled relay methods to maximize the achievable secrecy rate. In [16], the authors investigated a secure EH-enabled relay design to minimize the relay power consumption.

It should be noted that most of these works focus on PS scheme. On the other hand, time switching (TS) is another feasible way to coordinate EH and information decoding (ID) due to simplicity [17–21]. Specifically, in [17], the authors compared the performances of PS and TS methods in EH-enabled
relay networks. In [18], the authors investigated the sum rate performance in multiuser EH-enabled relay networks. Recently, in [19], the authors proposed a new EH relaying protocol, where the source also harvested energy from the relay. However, the secrecy relaying with TS scheme is still rarely investigated. In [20], the authors compared the performances of PS and TS schemes in secrecy EH-enabled relay networks based on perfect CSI. In [21], the authors proposed robust PS and TS methods to minimize the relay power consumption.

Motivated by these observations, in this paper, we investigate robust secrecy design in EH-enabled AF relay networks. Specifically, assuming that TS is utilized by the relay, we investigate the worst-case secrecy rate maximization (SRM) by jointly optimizing the relay BF matrix, artificial noise (AN) covariance, and TS ratio. The signal-to-interference-and-noise ratios (SINRs) for Bob and Eve are jointly determined by multiple parameters, and the transmit power constraint at the relay is dynamic, thus making the problem highly non-convex. To address this problem, we propose to decouple the original problem into two-layer problems, where the outer problem is solved by two-dimensional search while the inner problem is solved by semi-definite relaxation (SDR). Finally, numerical results validate our proposed design.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. System Model

Let us consider a secrecy relay system, in which a transmitter (Alice) sends confidential messages to Bob with the aid of a EH-enabled AF relay (R), under the eavesdrops of an Eve. The system model is shown in Fig. 1. We assume that R is equipped with \( N_r \) antennas while the other nodes are equipped with single antenna, and the direct links from Alice to Bob as well as Eve can be neglected. In addition, we denote \( f, h, \) and \( g \) as the channel vectors from Alice to R, from R to Bob, and from R to Eve, respectively.

![Figure 1](image)

**Figure 1.** The secrecy EH-enabled relay system model.

In the network, R is an EH node equipped with an EH receiver and an ID receiver. We assume that the EH and ID receivers are operating at the same frequency. We also assume that the processing power used by the receive and transmit circuits at R is negligible compared to the transmit power of R. Besides, the process delay at R can be neglected, and the distances from Alice to R and from R to Bob are nearly the same.

The complete transmission block is divided into an EH phase and an information transmission phase via a time switcher. Let \( \alpha \) be the TS ratio allocated to the EH receiver at R, with \( 0 \leq \alpha \leq 1 \). During the initial EH mode, R harvests energy from the received RF signal emitted by Alice during \( \alpha T \) seconds. Then, Alice sends the information carrying signal to R over a period of \( (1 - \alpha)T/2 \) seconds. Lastly, R utilizes the harvested energy to forward the signal to Bob during the remaining part of the time slot. Without loss of generality (W.l.o.g.), we assume that \( T = 1 \).

In the first two phases, Alice sends signal \( s \) satisfying \( \mathbb{E}[|s|^2] = 1 \) to the relay, thus the received
signal at the relay is

\[ y_r = \sqrt{P_s} f s + n_r + n_p, \tag{1} \]

where \( P_s \) is the transmit power at Alice, and \( n_r \) and \( n_p \) are the additive noise and signal processing noise at the relay with variances \( \sigma_r^2 \) and \( \sigma_p^2 \), respectively.

Based on the TS scheme, the amount of energy harvested at the relay is

\[ E = \eta \alpha T \left( P_s \| f \|^2 + \sigma_r^2 \right), \tag{2} \]

where \( \eta \) denotes the energy conversion efficiency. W.l.o.g., we assume that \( \eta = 1 \).

Accordingly, the maximum available power at the relay is

\[ P_{\text{max}} = \frac{E}{(1 - \alpha) T/2} + P_0 = \frac{2\alpha (P_s \| f \|^2 + \sigma_r^2)}{1-\alpha} + P_0, \tag{3} \]

where \( P_0 \) denotes the initial power at the relay.

In the third phase, the relay forwards \( y_r \) via a BF matrix \( W \). In addition, AN is injected to deteriorate the Eve channel. Thus the transmitted signal by the relay is

\[ x_r = W y_r + z, \tag{4} \]

where \( z \) is the AN vector with \( \mathcal{CN}(0, \Sigma) \).

Thus, the received signals at Bob and Eve are

\[ y_b = \sqrt{P_s} h^H W f s + h^H W n_r + h^H W n_p + h^H z + n_b, \tag{5a} \]
\[ y_e = \sqrt{P_s} g^H W f s + g^H W n_r + g^H W n_p + g^H z + n_e, \tag{5b} \]

where \( n_b \) and \( n_e \) are the additive noises at Bob and Eve, with variances \( \sigma_b^2 \) and \( \sigma_e^2 \), respectively.

According to the received signals in Eq. (5), the SINRs at Bob and Eve are, respectively, given by

\[ \Gamma_b = \frac{P_s \| h^H W f \|^2}{\sigma_r^2 \| h^H W \|^2 + \sigma_p^2 \| h^H W \|^2 + \| h^H z \|^2 + \sigma_b^2} = \frac{w^H Q_b w}{w^H R_b w + w^H U_b w + h^H \Sigma h + \sigma_b^2}, \tag{6a} \]
\[ \Gamma_e = \frac{P_s \| g^H W f \|^2}{\sigma_r^2 \| g^H W \|^2 + \sigma_p^2 \| g^H W \|^2 + \| g^H z \|^2 + \sigma_e^2} = \frac{w^H Q_e w}{w^H R_e w + w^H U_e w + g^H \Sigma g + \sigma_e^2}, \tag{6b} \]

where \( w = \text{vec}(W) \), \( Q_b = P_s f^H f T \otimes h h^H \), \( R_b = \sigma_b^2 I \otimes h h^H \), \( U_b = \sigma_b^2 I \otimes h h^H \), \( Q_e = P_s f^H f T \otimes g g^H \), \( R_e = \sigma_e^2 I \otimes g g^H \), and \( U_e = \sigma_e^2 I \otimes g g^H \).

On the other hand, the total power consumption of the relay is

\[ P_r = P_s \| W f \|^2 + \sigma_r^2 \| W \|^2 + \sigma_p^2 \| W \|^2 + \text{Tr}(\Sigma), \tag{7} \]

Thus, the relay transmit power constraint is

\[ \text{Tr}(w^H C w) + \text{Tr}(\Sigma) \leq \frac{2\alpha}{1 - \alpha} \left( P_s \| f \|^2 + \sigma_r^2 \right) + P_0, \tag{8} \]

where \( C = (P_s f f^H + \sigma_b^2 I + \sigma_p^2 I)^T \otimes I \).

### 2.2. Problem Statement

Here, we assume that only imperfect Eve’s CSI can be obtained. Similar to [21], the imperfect CSI is modeled as

\[ \mathcal{G} = \{ g | g = \bar{g} + \Delta g, \| \Delta g \| \leq \epsilon \}, \tag{9} \]

where \( \bar{g} \) denotes the estimate of \( g \); \( \Delta g \) denotes the channel uncertainty; \( \epsilon \) is the size of the bounded error region.

Accordingly, the worst-case secrecy rate is

\[ R_s = \frac{1 - \alpha}{2} \left[ \log_2 (1 + \Gamma_b) - \max_{\bar{g} \in \mathcal{G}} \log_2 (1 + \Gamma_e) \right]. \tag{10} \]
Now, we aim to maximize the worst-case secrecy rate by jointly optimizing the BF matrix, AN covariance and TS ratio. Mathematically, our problem is formulated as

\[
\begin{align*}
\max_{w, \Sigma \geq 0, \ell \geq 1} \quad & R_s \\
\text{s.t.} \quad & (8),
\end{align*}
\]

(11a)

(11b)

Notably, Eq. (11) is hard to handle due to the non-convex in Eqs. (11a) and (11b). Based on this observation, we will design an effective method for Eq. (11) in the following section.

3. A TWO-LAYER METHOD FOR THE SRM PROBLEM

For fixed \( \alpha \), by introducing auxiliary variable \( \beta \), we obtain the following problem

\[
\begin{align*}
\max_{w, \Sigma \geq 0, \beta} \quad & \frac{1-\alpha}{2} \log_2 (1 + \Gamma_b) - \frac{1-\alpha}{2} \log_2 \left( \frac{1}{\beta} \right) \\
\text{s.t.} \quad & \max_{\forall g \in \mathcal{G}} 1 + \Gamma_e \leq \frac{1}{\beta}, \\
& (8),
\end{align*}
\]

(12a)

(12b)

(12c)

Furthermore, for fixed \( \alpha \), the optimal worst-case secrecy rate can be attained via solving the following problem

\[
R_s^* = \max_{\beta} \frac{1-\alpha}{2} \log_2 (\beta + \gamma(\beta)),
\]

(13a)

(13b)

where \( \gamma(\beta) \) is the optimal value of the following inner problem

\[
\gamma(\beta) = \max_{w, \Sigma \geq 0} \frac{w^H Q_b w}{\beta - 1 \left( w^H R_b w + w^H U_b w + h^H \Sigma h + \sigma_b^2 \right)}
\]

(14a)

\[
s.t. \quad w^H Q_e w \leq (\beta - 1) \left( w^H (R_e + U_e) w + g^H \Sigma g + \sigma_e^2 \right), \forall g \in \mathcal{G},
\]

(14b)

(14c)

Besides, the lower bound of \( \beta \) can be determined since

\[
\beta \geq \left( 1 + \frac{w^H Q_b w}{w^H R_b w + w^H U_b w + h^H \Sigma h + \sigma_b^2} \right)^{-1} \geq \left( 1 + \frac{w^H Q_b w}{w^H R_b w} \right)^{-1} \geq \lambda_{\min}\left( (Q_b + R_b) \cdot R_b \right)
\]

(15)

For fixed \( \rho \) and \( \beta \), Eq. (14) can be solved by SDR. Specifically, by denoting \( \mathcal{W} = ww^H \) and neglecting the non-convex constraint \( \text{rank}(\mathcal{W}) = 1 \), Eq. (14) can be transformed as

\[
\max_{\mathcal{W} \geq 0, \Sigma \geq 0} \frac{\beta - 1}{2} \left( \text{Tr} \left( (R_b + U_b) \mathcal{W} \right) + h^H \Sigma h + \sigma_b^2 \right)
\]

(16a)

\[
s.t. \quad \text{Tr} \left( Q_e \mathcal{W} \right) \leq (\beta - 1) \left( \text{Tr} \left( (R_e + U_e) \mathcal{W} \right) + g^H \Sigma g + \sigma_e^2 \right), \forall g \in \mathcal{G},
\]

(16b)

\[
\text{Tr} (\mathcal{C} \mathcal{W}) + \text{Tr} (\Sigma) \leq \frac{2\alpha}{1-\alpha} \left( P_s \|f\|^2 + \sigma_f^2 \right) + P_0,
\]

(16c)

Nextly, to handle the CSI uncertainties in Eq. (16b), we transform Eq. (16b) into the following reformulation

\[
(\beta - 1) \text{Tr} \left( (R_e + U_e) \mathcal{W} \right) - \text{Tr} \left( Q_e \mathcal{W} \right) + (\beta - 1) \left( g^H \Sigma g + \sigma_e^2 \right) \geq 0.
\]

(17)

Furthermore, we introduce the following relationship

\[
\text{Tr} \left( Q_e \mathcal{W} \right) = g^H \left( \sum_{\ell=1}^{N_r} x_\ell \left( P_s \|f\|^2 \otimes I_{N_r} \right) \mathcal{W} \mathcal{E}_\ell^H \right) g,
\]

(18a)

\[
\text{Tr} \left( R \mathcal{W} \right) = \sigma_f^2 g^H \left( \sum_{\ell=1}^{N_r} x_\ell \mathcal{W} \mathcal{E}_\ell^H \right) g,
\]

(18b)
where $\mathbf{E}_\ell \triangleq [\mathbf{0}, \ldots, \mathbf{I}_{N_r}, \ldots, \mathbf{0}] \in \mathbb{C}^{N_r \times N_r^2}$ is a zero matrix, except for the columns ranging from $N_r^2 + (\ell - 1)N_r + 1$ to $N_r^2 + \ell N_r$, which form an identity matrix.

Thus, Eq. (17) can be equivalently rewritten as
\begin{equation}
\mathbf{g}^H \mathbf{x} + (\beta^{-1} - 1) \sigma_e^2 \geq 0,
\end{equation}
where
\begin{equation}
\mathbf{X} \triangleq (\beta^{-1} - 1) \left(\sigma_r^2 + \sigma_e^2\right) \sum_{\ell=1}^{N_r} \mathbf{E}_\ell \mathbf{W} \mathbf{E}_\ell^H - \sum_{\ell=1}^{N_r} \mathbf{E}_\ell \left( P_s \mathbf{f}^T \mathbf{f}^T \otimes \mathbf{I}_{N_r} \right) \mathbf{W} \mathbf{E}_\ell^H + (\beta^{-1} - 1) \mathbf{\Sigma}.
\end{equation}

In the following, we will introduce the S-lemma to transform Eq. (19) into deterministic form.

**Lemma 1** (S-lemma [22]): Define the function
\begin{equation}
f_j (\mathbf{x}) = \mathbf{x}^H \mathbf{A}_j \mathbf{x} + 2\text{Re} \left\{ \mathbf{b}_j^H \mathbf{x} \right\} + c_j, \quad j = 1, 2
\end{equation}
where $\mathbf{A}_j = \mathbf{A}_j^H \in \mathbb{C}^{n \times n}$, $\mathbf{b}_j \in \mathbb{C}^{n \times 1}$, and $c_j \in \mathbb{R}$. The implication $f_1 (\mathbf{x}) \leq 0 \Rightarrow f_2 (\mathbf{x}) \leq 0$ holds if and only if there exists $\lambda \geq 0$ such that
\begin{equation}
\lambda \begin{bmatrix}
\mathbf{A}_1 & \mathbf{b}_1 \\
\mathbf{b}_1^H & c_1
\end{bmatrix} - \begin{bmatrix}
\mathbf{A}_2 & \mathbf{b}_2 \\
\mathbf{b}_2^H & c_2
\end{bmatrix} \succeq 0,
\end{equation}
provided that there exists a point $\mathbf{x}_0$ such that $f_1 (\mathbf{x}_0) < 0$.

From the S-lemma, Eq. (19) can be transformed into the following linear matrix inequality (LMI)
\begin{equation}
\begin{bmatrix}
\lambda \mathbf{I} + \mathbf{X} & \mathbf{X} \mathbf{g} \\
\mathbf{g}^H \mathbf{X} & -\lambda \epsilon^2 + \mathbf{g}^H \mathbf{X} \mathbf{g} + (\beta^{-1} - 1) \sigma_e^2
\end{bmatrix} \succeq 0,
\end{equation}
where $\lambda \geq 0$ is a introduced auxiliary variable.

Combining these steps, we obtain the following SDR problem
\begin{equation}
\begin{aligned}
\max_{\mathbf{W} \succeq 0, \mathbf{\Sigma} \succeq 0} & \quad \text{Tr} \left( \mathbf{Q}_b \mathbf{W} \right) \\
\text{s.t.} & \quad (16c), (21).
\end{aligned}
\end{equation}

Via the Charnes-Cooper Transformation [23], e.g., denoting $\tilde{\mathbf{W}} = \xi \mathbf{W}$, $\tilde{\mathbf{\Sigma}} = \xi \mathbf{\Sigma}$, $\tilde{\lambda} = \xi \lambda$ with $\xi \geq 0$, Eq. (22) can be equivalently reformulated as
\begin{equation}
\begin{aligned}
\max_{\tilde{\mathbf{W}} \succeq 0, \tilde{\mathbf{\Sigma}} \succeq 0, \xi, \tilde{\lambda}} & \quad \text{Tr} \left( \mathbf{Q}_b \tilde{\mathbf{W}} \right) \\
\text{s.t.} & \quad \text{Tr} \left( (\mathbf{R}_b + \mathbf{U}_b) \tilde{\mathbf{W}} \right) + \mathbf{h}^H \tilde{\mathbf{\Sigma}} \mathbf{h} + \xi \sigma_b^2 = \beta,
\end{aligned}
\end{equation}
\begin{equation}
\begin{bmatrix}
\tilde{\lambda} \mathbf{I} + \tilde{\mathbf{X}} & \tilde{\mathbf{X}} \tilde{\mathbf{g}} \\
\tilde{\mathbf{g}}^H \tilde{\mathbf{X}} & -\tilde{\lambda} \epsilon^2 + \tilde{\mathbf{g}}^H \tilde{\mathbf{X}} \tilde{\mathbf{g}} + \xi (\beta^{-1} - 1) \sigma_e^2
\end{bmatrix} \succeq 0,
\end{equation}
\begin{equation}
\text{Tr} \left( \mathbf{C} \tilde{\mathbf{W}} \right) + \text{Tr} \left( \tilde{\mathbf{\Sigma}} \right) \leq \frac{2\alpha \xi}{1 - \alpha} \left( P_s \| \mathbf{f} \|^2 + \sigma_b^2 \right) + \xi P_0,
\end{equation}
where
\begin{equation}
\tilde{\mathbf{X}} \triangleq (\beta^{-1} - 1) \left(\sigma_r^2 + \sigma_e^2\right) \sum_{\ell=1}^{N_r} \mathbf{E}_\ell \tilde{\mathbf{W}} \mathbf{E}_\ell^H - \sum_{\ell=1}^{N_r} \mathbf{E}_\ell \left( P_s \mathbf{f}^T \mathbf{f}^T \otimes \mathbf{I}_{N_r} \right) \tilde{\mathbf{W}} \mathbf{E}_\ell^H + (\beta^{-1} - 1) \tilde{\mathbf{\Sigma}}.
\end{equation}

Eq. (23) can be effectively solved by the toolbox CVX [24].

To sum up, for fixed $\alpha$ and $\beta$, the optimal $\left( \mathbf{W}^*, \mathbf{\Sigma}^* \right)$ can be obtained via solving Eq. (23), while the optimal $\alpha$ and $\beta$ can be solved by two-dimensional search since $\alpha \in [0, 1]$ and $\beta \in [\lambda_{\min}((\mathbf{Q}_b + \mathbf{R}_b), \mathbf{R}_b), 1]$.
Since $\mathbf{W}^*$ does not always satisfy $\text{rank}(\mathbf{W}^*) = 1$, we utilize the Gaussian Randomization (GR) to extract the optimal $\mathbf{w}^*$ to Eq. (14). Specifically, we obtain multiple $\mathbf{w}$ via the following equation

$$\mathbf{w} = \sqrt{\text{Tr} (\mathbf{W}^*)} \|\mathbf{W}^* \mathbf{v}\|^{-1} \mathbf{W}^* \mathbf{v}$$

(25)

where $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is the randomly generated complex Gaussian vector. Then, we choose the optimal $\mathbf{w}^*$, which maximizes Eq. (16a) and satisfies Eq. (16b).

4. SIMULATION RESULTS

In this section, we evaluate the performance of our design through Monte Carlo simulations. The simulation settings are assumed as follows: $N_r = 6$, $P_s = 20$ dBW, $P_0 = 20$ dBW, and $\sigma_f^2 = \sigma_p^2 = \sigma_b^2 = \sigma_e^2 = -50$ dBm. Each entry of $\mathbf{f}$, $\mathbf{h}$ and $\bar{\mathbf{g}}$ is randomly generated by $\mathcal{CN}(0, 10^{-4})$, and the CSI uncertainty is $\epsilon^2 = 10^{-8}$. In addition, we compare our algorithm with the following methods: 1) the no AN method, e.g., setting $\Sigma = \mathbf{0}$ by only optimizing $\mathbf{w}$ and $\alpha$; 2) the traditional secrecy AF relay method, which can be seen as a special case of our model via setting $\alpha = 0$ by only optimizing $\mathbf{w}$ and $\Sigma$; 3) the fixed TS radio method, e.g., setting a fixed TS radio $\alpha = 0.5$ by only optimizing $\mathbf{w}$ and $\Sigma$; 4) the method in [21]. The five methods are labeled as “the proposed method”, “no AN method”, “traditional secrecy AF”, “the fixed TS method”, and “the method in [21]”, respectively.

Firstly, we show the worst-case secrecy rate versus the transmit power $P_s$ of Alice in Fig. 2. From Fig. 2, we can see that $R_s$ increases with the increasing of $P_s$ for all these methods. Our design outperforms the traditional secrecy AF relay method and fixed TS relay method, since TS gives the relay more flexibility to tradeoff between EH and information transmission. Besides, the no AN method suffers the worst performance, which shows the effect of AN on security.

![Figure 2. The worst-case secrecy versus the transmit power.](image1)

![Figure 3. The worst-case secrecy versus the relay initial power.](image2)

Secondly, we show the worst-case secrecy rate versus the initial power at the relay $P_0$ in Fig. 3. From Fig. 3, we can see that $R_s$ increases with the increasing of $P_0$. Again, our design outperforms the other methods. Furthermore, by comparing Fig. 3 with Fig. 2, we find that the slope of the curves in Fig. 2 is steeper than the curves in Fig. 3 for all these methods, which shows that $P_s$ has more effect on the secrecy performance, since increasing $P_s$ not only enhances the signal strength, but also increases the available power of the relay.

Lastly, we show the worst-case secrecy rate versus the relay antenna number $N_r$ in Fig. 4. From Fig. 4, we can see that $R_s$ increases with the increase of $N_r$ for all these methods, especially for the methods which are EH-enabled. The increase of $N_r$ not only increases the spatial freedom to relay the
information and interfere the Eve, but also adds the relay’s capability to harvest wireless energy, thus lead to more achievable power at the relay.

5. CONCLUSION

In this paper, we have investigated an AN-aided robust BF and TS design for a secure sustainable AF relay network. Specifically, we assume that TS is employed and formulated the worst-case secrecy rate maximization design. We propose a two-layer optimization algorithm which combines two-dimensional search and SDR to solve the highly non-convex problem. Numerical results validate the effectiveness of our proposed scheme.

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