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Coupling Analysis of Non-Parallel Transmission Lines Excited by Ambient Wave Using a Time Domain Hybrid Method

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Abstract—A time domain hybrid method is presented to solve the coupling problem of non-parallel transmission lines (NPTLs) excited by ambient wave efficiently, which consists of transmission line (TL) equations, finite-difference time-domain (FDTD) method, and interpolation techniques. In this method, NPTLs are firstly divided into multiple independent transmission line segments according to the FDTD grids. Then TL equations are applied to build the coupling models of these TL segments, which rely on the calculation precisions of per unit length (p.u.l) distribution parameters of NPTLs and equivalent sources of TL equations. Thus, the p.u.l parameters of NPTLs are derived from empirical formulas, and the equivalent sources are obtained by some linear interpolation schemes of electric fields on the edges of FDTD grids. Finally, the difference scheme of FDTD is utilized to discretize the TL equations to obtain the voltages and currents on NPTLs and terminal loads. The significant feature of this hybrid method is embodied by its synchronous calculations of space electromagnetic fields and transient responses on NPTLs in time domain. The accuracy of this presented method is testified by the numerical simulations of plane wave coupling to NPTLs on the ground and in the shielded cavity by comparing with FDTD-SPICE method and CST software.

1. INTRODUCTION

With the rapid development of wireless communications, the working frequencies and integration degrees of electronic devices are becoming higher and higher. As a result, these devices are more susceptible to space electromagnetic fields. Transmission lines (TLs) widely used in these devices are the main paths of space electromagnetic fields coupling to the sensitive circuits of the lines, which may disturb or damage the normal operations of these circuits. Limited to the positions of the circuits in these devices, transmission lines cannot always keep parallel with other lines, which can be called as non-parallel transmission lines (NPTLs). Therefore, to guarantee the safety of these devices, studying the coupling problem of NPTLs excited by ambient wave is a significant way.

The preferred method used for the coupling analysis of NPTLs is full wave algorithms, such as finite-difference time-domain (FDTD) method [1], method of moments (MOM) [2] and finite element method (FEM) [3]. Unfortunately, the prominent feature of these methods is that the targets should be meshed in fine grids when they contain local electrically small structures, which should occupy large memories. In conclusion, full wave algorithms may not be suitable for the coupling analysis of electronic devices obviously, because transmission lines are the important components of these devices, which are fine structures.

Hence, studying field-to-line numerical methods is greatly necessary. Fortunately, several efficient hybrid methods have been developed for decades. Among these methods, Baum-Liu-Tesche (BLT) equation was first proposed in 1978 [4]. It aims at building the relationships of the voltages and

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currents at the nodes of transmission lines by scattering and transmission matrices, which are solved by matrix operation to obtain the responses on these nodes. However, traditional BLT equation [5–8] is a frequency-domain method, which is not suitable for the coupling simulation when ambient wave is a broadband signal. Although this method has been extended to time domain [9], it needs a number of convolution operations, which should decrease the computation efficiency of this method. Thus, to better deal with the coupling problems of broadband signals acting on TLs, efficient time domain algorithms are necessary to be studied. Finite-difference time-domain simulation program with integrated circuit emphasis (FDTD-SPICE) and FDTD-TL methods are the two classical numerical methods in time domain. In FDTD-SPICE method [10–14], the excitation fields of TLs are computed by the FDTD method firstly, and then the SPICE equivalent circuit model of TLs are established and solved by SPICE software to obtain the transient responses on the loads of TLs. Unfortunately, it needs a number of theoretical derivations, and the calculations of electromagnetic fields and transient responses on the loads are obtained separately. The significant feature of FDTD-TL method [15–17] compared with FDTD-SPICE method is that it can realize the co-calculations of space electromagnetic fields and transient responses on TLs and loads. In this method, the space electromagnetic fields are computed by the FDTD method, which are imported into the TL equations as distribution sources. Then the TL equations are solved by the difference scheme of FDTD method to obtain the responses on the lines. However, it is just suitable for parallel transmission lines currently.

Therefore, this paper presents an efficient time domain hybrid method based on FDTD-TL method, which can be well applied to the coupling analysis of ambient wave to non-parallel transmission lines. Compared with FDTD-TL method, this presented method is improved in two aspects: (1) the p.u.l distribution parameters suitable for NPTLs are derived from empirical formulas; (2) the interpolation schemes used for the calculations of equivalent sources of NPTLs are presented. In addition, this method can realize the synchronous calculations of space electromagnetic field radiation and transient responses on the NPTLs.

2. THEORY OF THE TIME DOMAIN HYBRID METHOD

To explain the theory of this presented method clearly, a typical model of NPTLs on the ground is considered, as shown in Fig. 1. The ground is assumed as a perfect conductor (PEC) plane and located on the xy plane of cartesian coordinate system. These transmission lines with different heights and oblique angles (named as $\theta_1, \theta_2, \ldots, \theta_N$) are parallel to the ground. The horizontal projections of these lines are assumed to be the same. Here, the radiation of NPTLs under the illumination of ambient wave can be ignored, because the distances between the NPTLs and the ground are electrically small compared with the minimum wavelength of the wave generally.

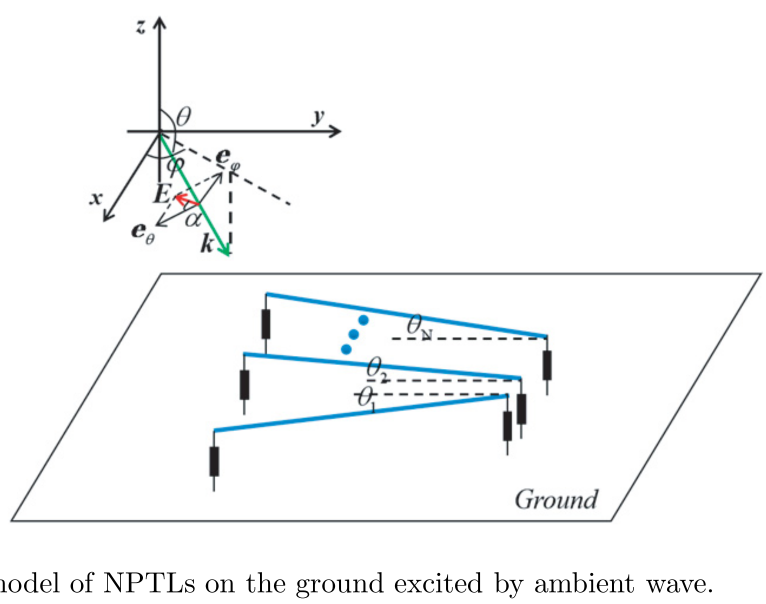

Figure 1. Typical model of NPTLs on the ground excited by ambient wave.

In this method, establishing the corresponding transmission line equations for the coupling analysis of NPTLs is the most important and initial step, which can be expressed as

$$\frac{\partial}{\partial y}\mathbf{V}(y,t) + \mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(y,t) = \mathbf{V}_F(y,t) \tag{1}$$

$$\frac{\partial}{\partial y}\mathbf{I}(y,t) + \mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(y,t) = \mathbf{I}_\mathrm{F}(y,t) \tag{2}$$

where $\mathbf{V}(y,t)$ and $\mathbf{I}(y,t)$ are the voltage and current vectors of NPTLs, respectively. $\mathbf{L}$ and $\mathbf{C}$ represent the per unit length (p.u.l) inductance and capacitance matrices of NPTLs, respectively. $\mathbf{V}_F(y,t)$ and $\mathbf{I}_F(y,t)$ stand for the equivalent distribution voltage and current sources, respectively, which are described as

$$\mathbf{V}_\mathrm{F}(y,t) = -\frac{\partial}{\partial y}\mathbf{E}_\mathrm{T}(y,t) + \mathbf{E}_\mathrm{L}(y,t) \tag{3}$$

$$\mathbf{I}_\mathrm{F}(y,t) = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{E}_\mathrm{T}(y,t) \tag{4}$$

$\mathbf{E}_\mathrm{T}(y,t)$ and $\mathbf{E}_\mathrm{L}(y,t)$ are obtained from the space electromagnetic fields, which are written as

$$[\mathbf{E}_\mathrm{T}(y,t)]_i = \int_0^{h_i} \mathbf{e}_z^{ex}(x,y,z,t)\,dz \tag{5}$$

$$[\mathbf{E}_\mathrm{L}(y,t)]_i = \mathbf{e}_{l_i}^{ex}(x,y,h_i,t) - \mathbf{e}_{l_i}^{ex}(x,y,0,t) \tag{6}$$

where i stands for the i-th line, h_i the height of the i-th line to the ground, and l_i the direction of the i-th line. e_l^{ex} and e_z^{ex} are the incident electric fields along and perpendicular to the NPTLs, respectively. The NPTLs can be removed when calculating the electromagnetic fields around them, because the equivalent sources of TL equations are not relevant to the scattering fields of NPTLs.

Obviously, it can be observed from the TL equations that the precision of TL equations is determined by the accurate calculations of p.u.l distribution parameters and equivalent sources of NPTLs, which will be introduced in detail as follows.

2.1. Calculation of Per Unit Length Parameters for NPTLs

First, the NPTLs are divided into N_L segments according to the FDTD grids, as shown in Fig. 2.

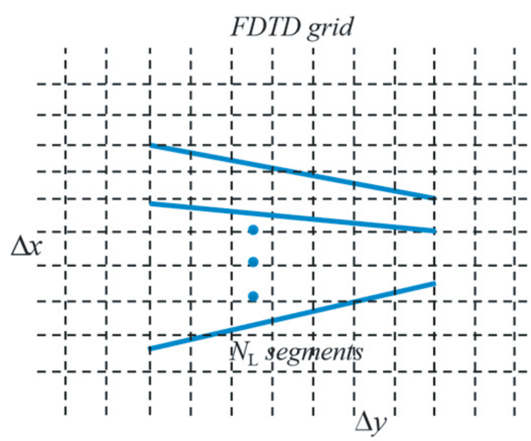

Figure 2. The division of NPTLs by FDTD grids.

The empirical formulas of inductance parameter matrix for parallel TLs can be found from [18], which are expressed as

$$L_{ii} = \mu_0 \ln(2h_i/r_i)/2\pi \tag{7}$$

$$L_{ij} = \mu_0 \ln\left(1 + 4h_ih_j/d_{ij}^2\right)/4\pi \tag{8}$$

where i and j stand for the i-th and j-th transmission lines, respectively. L_{ii} and L_{ij} represent the self and mutual inductances of the TLs, respectively. r_i is the radius of the i-th line. h_i, h_j, and d_{ij} are the heights of the i-th and j-th lines and the distance between the two lines, respectively.

Because the self inductance is independent of the distance between the lines, Equation (7) is still suitable for the calculation of self inductances of NPTLs. On the other hand, mutual inductances are closely related to the distances between the lines, and as a result, Equation (8) is not applicable to the calculations of mutual inductances of NPTLs, because the space distances of NPTLs are changed continually. Under this circumstance, to obtain the mutual inductances of each segment of NPTLs, it should be further divided into multiple small elements, which can be seen as approximately parallel transmission lines, as shown in Fig. 3. The number of elements is set as M, and the distances between these elements are assumed as $d_1, d_2, \ldots, d_M$. Then the mutual inductances of each segment of NPTLs can be obtained from the sum average of the mutual inductances of these elements, which can be expressed as

$$L_{ij} = \sum_{k=1}^{k=M} \left[\mu_0 \ln\left(1 + 4h_i h_j / d_k^2\right) / 4\pi\right] / M \tag{9}$$

Then the capacitance parameter matrix of NPTLs can be calculated by $\mathbf{C} = \mu_0 \varepsilon_0 \mathbf{L}^{-1}$.

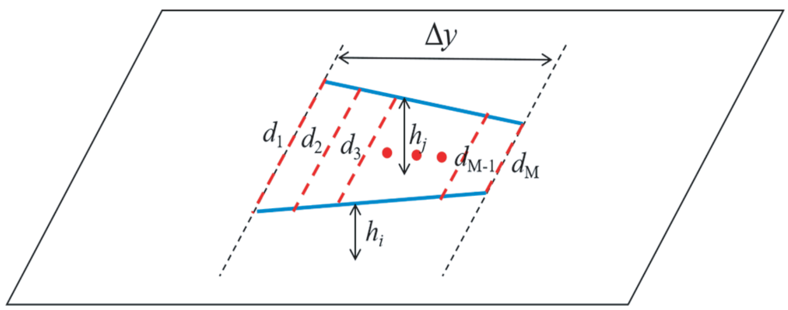

Figure 3. The calculation of mutual inductance of NPTLs.

2.2. Calculation of Equivalent Distribution Sources for NPTLs

Because the distances between these lines of NPTLs are changed, the lines cannot be located on the edges of FDTD grids. Therefore, the incident electric fields needed by the equivalent distribution sources of NPTLs cannot be obtained from the electric fields on FDTD girds directly, which must be calculated by interpolation schemes.

First, to get the values of $\mathbf{E}_\mathrm{L}(y,t)$ in distribution voltage source, the electric fields along the NPTLs should be ascertained. To explain this solution process clearly, we take one line of NPTLs as an example, and it is assumed that the starting and ending points of the line are on the edges of FDTD grids, as shown in Fig. 4. Then the center point of each segment of this line is on the central plane of FDTD gird where the point is located. The electric field along the line can be expressed by the vector product form as $\mathbf{E} \cdot \mathbf{e}_l = E_x \cdot a_x \mathbf{e}_x + E_y \cdot a_y \mathbf{e}_y$, where $\mathbf{e}_x$ and $\mathbf{e}_y$ stand for the unit vectors of x and y axes, respectively. a_x and a_y represent the decomposed proportions of electric field along the line in x and y axes, respectively, which are written as $a_x = \sin\theta$ and $a_y = \cos\theta$, where θ is the angle between the line and horizontal direction. E_x and E_y are the electric field components at the center point of each segment, which should be obtained by the interpolation of four adjacent electric fields on the FDTD grids. Taking E_y as an example, the interpolation scheme can be expressed as

$$E_y = (1-\eta)\left[\xi E_{y1} + (1-\xi) E_{y3}\right] + \eta\left[\xi E_{y2} + (1-\xi) E_{y4}\right] \tag{10}$$

where E_{y1}, E_{y2}, E_{y3}, and E_{y4} are the electric fields on FDTD grids. η and ξ are the scales of center point in the FDTD grid along x and z directions.

Second, the values of $\mathbf{E}_T(y,t)$ in distribution voltage and current sources must be confirmed, which are contributed by the integration of electric fields E_z^{inc} perpendicular to NPTLs at the starting and

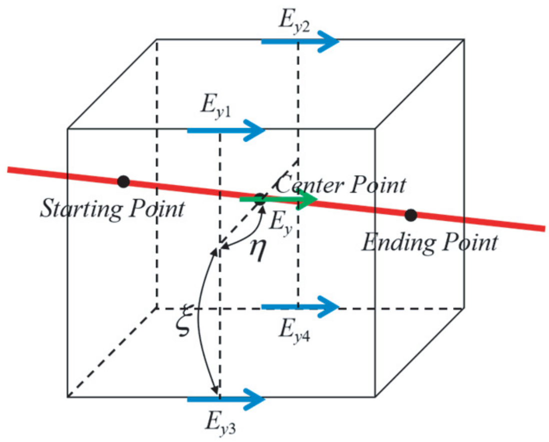

Figure 4. Interpolation scheme for the electric fields at the center point of each segment.

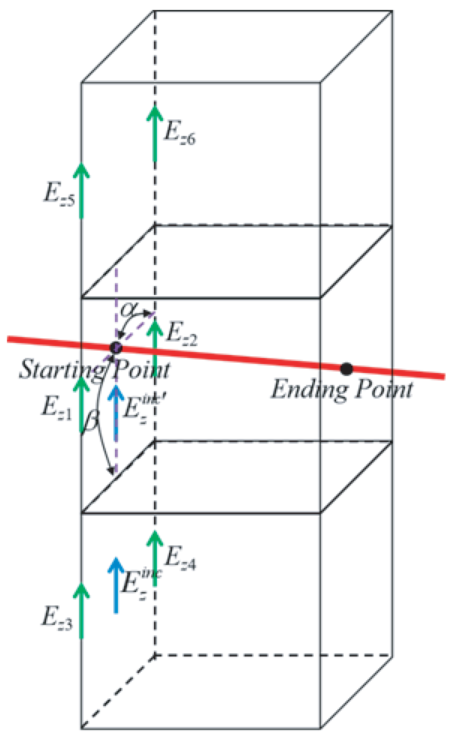

Figure 5. Interpolation scheme for the electric fields at starting and ending points of each segment.

ending points of each segment of NPTLs, as shown in Fig. 5. Obviously, they can be interpolated from two adjacent electric fields E_z on FDTD grids, which can be expressed as

$$E_z^{inc} = \alpha E_{z3} + (1-\alpha)\, E_{z4} \tag{11}$$

Note that the electric fields $E_z^{inc'}$ near the lines should be interpolated from four adjacent electric fields E_z on FDTD grids. The interpolation scheme can be described as

$$E_z^{inc'} = \begin{cases} [(0.5{+}\beta)\,[\alpha E_{z1} + (1-\alpha)\, E_{z2}] + (0.5{-}\beta)\,[\alpha E_{z3} + (1-\alpha)\, E_{z4}]] & \beta < 0.5 \\ [(1.5{-}\beta)\,[\alpha E_{z1} + (1-\alpha)\, E_{z2}] + (\beta{-}0.5)\,[\alpha E_{z5} + (1-\alpha)\, E_{z6}]] & \beta \geq 0.5 \end{cases} \tag{12}$$

where α and β are the scales of starting or ending points in the FDTD grid along x and z directions.

After the TL equations are established, the difference scheme of FDTD is applied to solve the TL equations to obtain the transient voltages and currents on the lines, which can reach the goal of synchronous calculations of space electromagnetic fields and transient responses on NPTLs. The iteration formulas of currents and voltages on the NPTLs are expressed as

$$\mathbf{I}^{n+\frac{1}{2}}\left(k+\frac{1}{2}\right)=\left[\frac{\mathbf{R}(k)}{2}+\frac{\mathbf{L}(k)}{\Delta t}\right]^{-1}\left(\begin{array}{l}\left[\frac{\mathbf{L}(k)}{\Delta t}-\frac{\mathbf{R}(k)}{2}\right]\mathbf{I}^{n-\frac{1}{2}}\left(k+\frac{1}{2}\right)-\frac{\mathbf{V}^{n}(k+1)-\mathbf{V}^{n}(k)}{\Delta z}\\ -\frac{\mathbf{E}_T^{n}(k+1)-\mathbf{E}_T^{n}(k)}{\Delta z}+\mathbf{E}_L^{n}\left(k+\frac{1}{2}\right)\end{array}\right) \tag{13}$$

$$\mathbf{V}^{n+1}(k)=\left[\frac{\mathbf{G}(k)}{2}+\frac{\mathbf{C}(k)}{\Delta t}\right]^{-1}\left(\begin{array}{l}\left[\frac{\mathbf{C}(k)}{\Delta t}-\frac{\mathbf{G}(k)}{2}\right]\mathbf{V}^{n}(k)-\frac{\mathbf{I}^{n+\frac{1}{2}}\left(k+\frac{1}{2}\right)-\mathbf{I}^{n+\frac{1}{2}}\left(k-\frac{1}{2}\right)}{\Delta z}\\ -\mathbf{C}\frac{\mathbf{E}_T^{n+1}(k)-\mathbf{E}_T^{n}(k)}{\Delta t}\end{array}\right) \tag{14}$$

where k represents the positions of voltage and current nodes on the NPTLs. Δz and Δt are the space and time steps used by the FDTD method, respectively.

3. NUMERICAL SIMULATION

Two cases about NPTLs on the PEC ground and in the shielded cavity are employed to test the accuracy of this presented method by comparing with the FDTD-SPICE method and commercial software CST using microwave studio.

The first case is the coupling analysis of NPTLs on the PEC ground excited by ambient wave, as shown in Fig. 6. The dimension of the ground is $0.4\,\mathrm{m}\times 1.0\,\mathrm{m}$. The height and radius of the three lines are $1.1\,\mathrm{cm}$ and $1\,\mathrm{mm}$, respectively. If the original position of the ground is set as $(0,0,0)$, the positions of nodes #1, #2, #3, #4, #5, and #6 at the starting and ending ports of the lines are $(0.15\,\mathrm{m}, 0.1\,\mathrm{m}, 0.011\,\mathrm{m})$, $(0.19\,\mathrm{m}, 0.9\,\mathrm{m}, 0.011\,\mathrm{m})$, $(0.2\,\mathrm{m}, 0.1\,\mathrm{m}, 0.011\,\mathrm{m})$, $(0.2\,\mathrm{m}, 0.9\,\mathrm{m}, 0.011\,\mathrm{m})$, $(0.23\,\mathrm{m}, 0.1\,\mathrm{m}, 0.011\,\mathrm{m})$, and $(0.21\,\mathrm{m}, 0.9\,\mathrm{m}, 0.011\,\mathrm{m})$, respectively. The terminal loads of the lines are $Z_1 = Z_3 = Z_5 = 50\,\Omega$ and $Z_2 = Z_4 = Z_6 = 100\,\Omega$, respectively. A Gaussian pulse, expressed as $E_0\exp[-4\pi(t-t_0)^2/\tau^2]$, is perpendicular to the lines, where $E_0 = 1000\,\mathrm{V/m}$, $t_0 = 1.6\,\mathrm{ns}$, and $\tau = 2\,\mathrm{ns}$. To satisfy the stability condition of FDTD, the space and time steps selected by the hybrid method are $5\,\mathrm{mm}$ and $8.333\times 10^{-12}\,\mathrm{s}$, respectively.

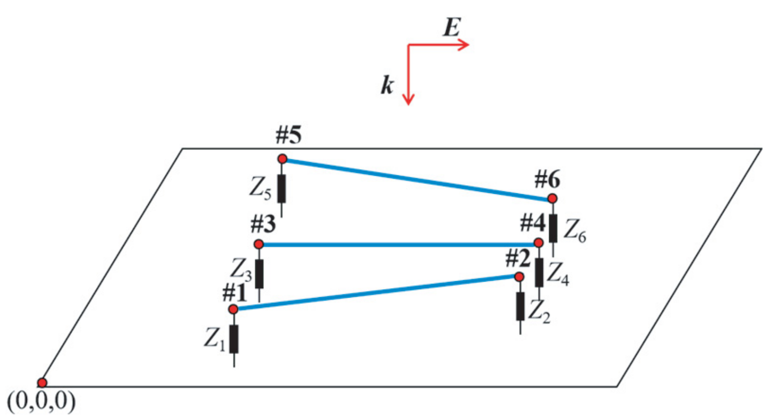

Figure 6. Coupling model of MTLs on the PEC ground excited by ambient wave.

Figure 7 shows the voltage responses on the loads Z_1 and Z_6 computed by the hybrid method, FDTD-SPICE, and CST. It can be seen clearly that the results of the three methods agree well to each other. Generally, the feature selective validation (FSV) method [19, 20] is a preferred tool to access the coincidence of two datasets, which is applied to compare the results obtained by this method and CST. Here, the feature difference measure (FDM) analysis of FSV method is employed, which uses six levels

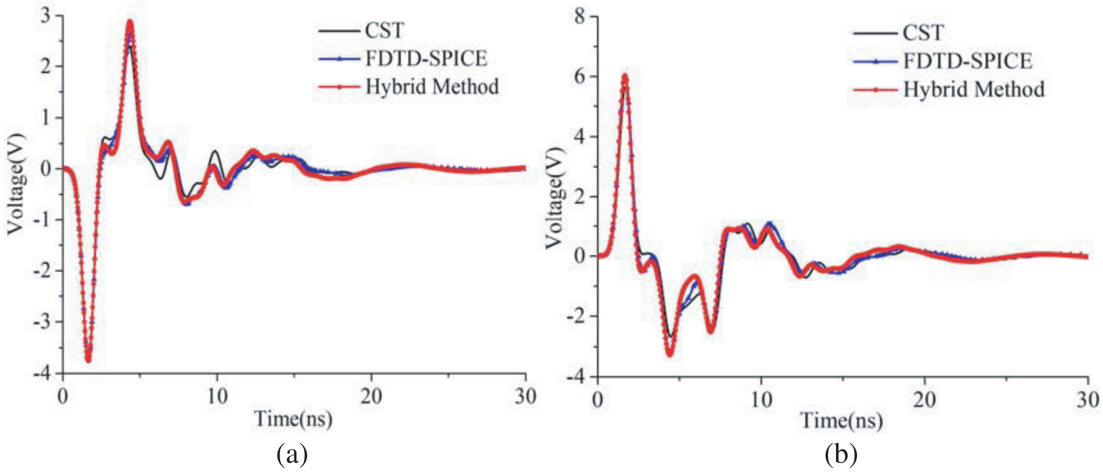

Figure 7. Voltage responses on the loads of NPTLs computed by the three methods for the first case. (a) Voltages on load Z_1. (b) Voltages on load Z_6.

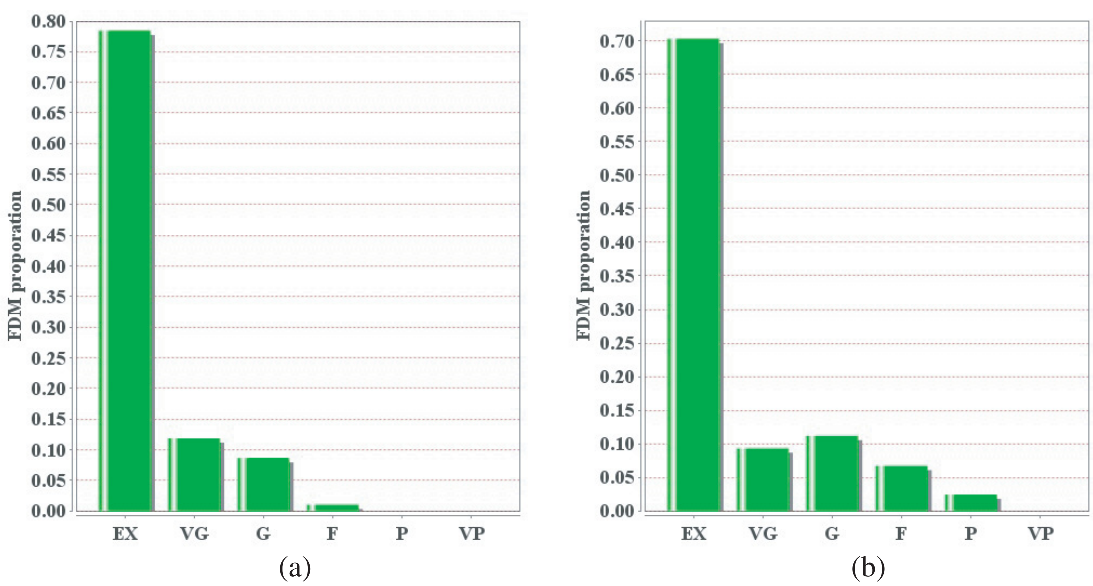

Figure 8. FDM analysis of the voltages for the first case. (a) Confidence histogram of FDM for the voltages on Z_1. (b) Confidence histogram of FDM for the voltages on Z_6.

named as "EX", "VG", "G", "F", "P", and "VP" to indicate the agreement degrees of the results. The six levels represent "Excellent", "Very Good", "Good", "Fair", "Poor", and "Very Poor", respectively. From Fig. 8 we can see that the good agreements of the oscillation periods of the voltages on Z_1 and Z_6 obtained by the two methods are more than 94%.

The second case is the coupling analysis of NPTLs in the shielded cavity excited by ambient wave, as shown in Fig. 9. The incident wave and the structures of NPTLs are the same with the first case. The dimension of the cavity is $L_c \times W_c \times H_c = 1\,\text{m} \times 0.3\,\text{m} \times 0.2\,\text{m}$. There are eight slots with size $l_s \times w_s = 0.1\,\text{m} \times 0.01\,\text{m}$ and distances $d_s = 0.15\,\text{m}$ and $d_m = 0.02\,\text{m}$ on the top surface of the cavity. The terminal loads of NPTLs are also $Z_1 = Z_3 = Z_5 = 50\,\Omega$ and $Z_2 = Z_4 = Z_6 = 100\,\Omega$.

In Fig. 10, the voltage responses on the loads Z_1 and Z_6 computed by the hybrid method, FDTD-SPICE, and CST are observed to verify the accuracy of this presented method applied to the coupling analysis of NPTLs in complex electromagnetic environment. Similarly, we can see that the results of the three methods can keep good agreement.

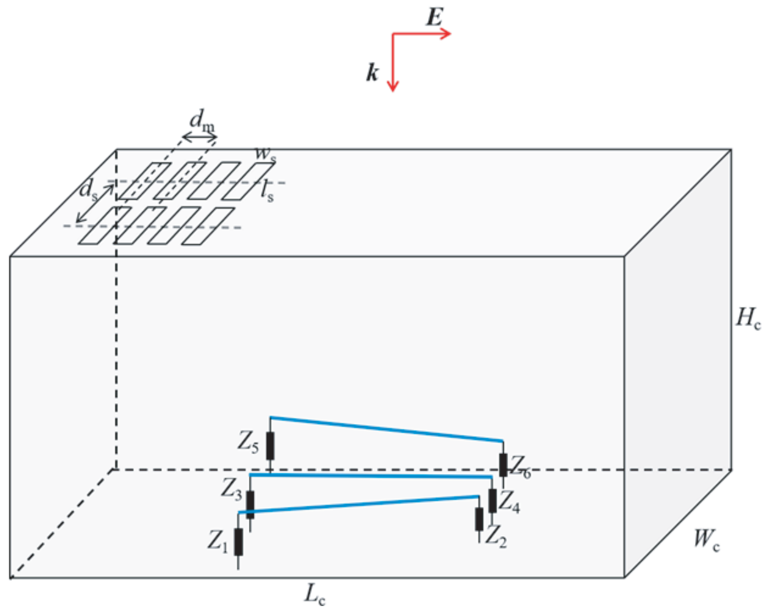

Figure 9. Coupling model of NPTLs in the shielded cavity.

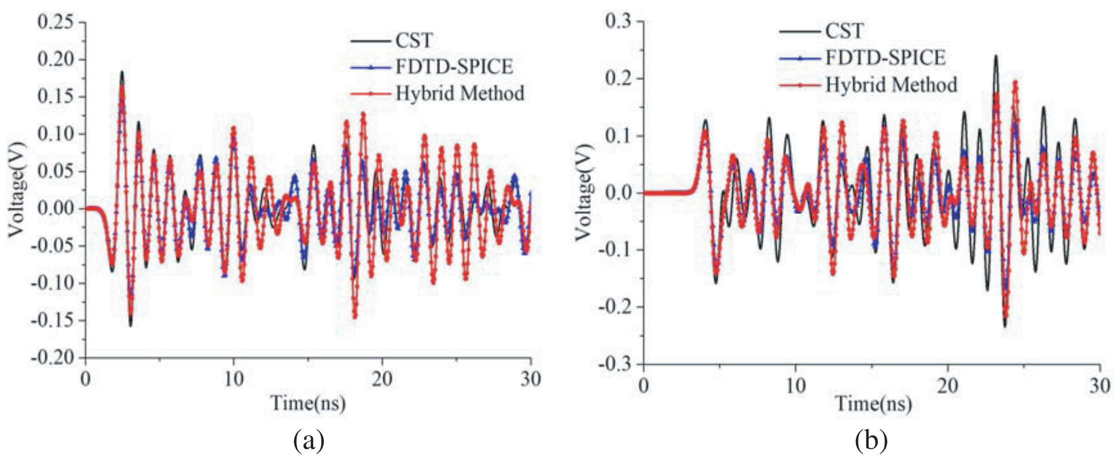

Figure 10. Voltage responses on the loads of NPTLs computed by the three methods for the second case. (a) Voltages on load Z_1. (b) Voltages on load Z_6.

In addition, it needs to explain that some values of the results computed by the hybrid method and FDTD-SPICE comparing with that of CST for the two cases have certain errors, because the scattering effects caused by the structures of NPTLs are ignored in this presented method and FDTD-SPICE method.

4. CONCLUSION

Finite-difference time-domain method, transmission line equations, and some interpolation schemes are combined to form a time domain hybrid method, which can solve the coupling problem of non-parallel transmission lines excited by space electromagnetic fields with high precision. The contributions of this method are represented in two aspects: (1) the calculation formulas suitable for the p.u.l distribution parameters and equivalent sources of NPTLs are derived; (2) the synchronous computations of space electromagnetic field radiation and transient responses on the NPTLs are realized.

Two cases about NPTLs, on the ground and in the shielded cavity respectively, excited by a plane wave are studied to verify the accuracy of this presented method. To evaluate the performance of this method, the results computed by the presented method are compared with that of FDTD-SPICE method and commercial software CST, which are in a good degree of agreement. Next, this hybrid method can be combined with the parallel technique to improve the efficiency of this method.

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