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Wave Scattering by a Perfect Electromagnetic Conductor Wedge Residing between Isorefractive Media

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Abstract—In this study, plane wave diffraction by a perfect electromagnetic wedge which is lying between isorefractive media is investigated. The diffracted waves are constructed by using the relation between initial geometric optics waves and scattered waves at the transition boundaries. The uniform theory of diffraction method is used for derivation of the uniform wave expressions. Thus, obtained uniform expressions are analyzed numerically for different set of parameters.

1. INTRODUCTION

The diffraction is the bending process of waves from the discontinuous parts of the scatterers. The exact mathematical solution of this process for a perfect electric conductor (PEC) half-plane was first given by Sommerfeld [1]. Later on, the same problem was investigated by different authors for different surface boundary conditions [2–4]. The interaction of waves by a wedge is also a canonical problem and has a key role for the radar cross section applications [5, 6]. The aim of this paper is to obtain a solution for the diffraction of waves by a perfect electromagnetic conductor (PEMC) wedge which is lying at the boundary of isorefractive media. The PEMC surface can be defined by the combination of PEC and perfect magnetic conductor (PMC) surfaces. These kinds of surfaces satisfy both of the boundary conditions. The connection between PEC and PMC surfaces is satisfied by a special parameter M which is called as the admittance parameter [7]. Depending on the values of M, this structure acts as a PEC or PMC surface according to the limiting values of M. Another interesting property of such surfaces is the excitation of the cross-polarized field components in the reflected waves [8]. The first examination of the diffraction problem by a PEMC half-screen was given by Ahmed [9]. In order to obtain the diffracted waves, he used the transformation method, which was introduced by Lindell and Sihvola [10]. The scattering of waves by a PEMC cylinder was investigated by Ahmed and Naqvi [11]. Also, a PEMC half-screen, illuminated by a point source, was studied by Tiwana et al. [12]. In the literature, there are only two studies related with PEMC wedge. The first one was presented by Nayyeri et al. [13]. However, this study lacks explicit analytic expressions for the diffracted waves. The second one was put forth by Umul [14]. In his paper, z components of the electric and magnetic fields were expressed in terms of a combination of the scattered waves by soft and hard surfaces with the same geometry. Here, soft and hard refer to the surface boundary conditions of Dirichlet and Neumann, respectively. On the other hand, in the present study only the whole plane interpretation is considered for determination of unknown reflection coefficients. The solution has not been obtained in terms of the linear combination of scattered fields from the soft and hard surfaces.

In this work, contrary to the existing studies in the literature, a PEMC wedge was located at the boundary of isorefractive media. The existing studies predict the solution for single medium. However, a more general case is examined in the current paper because of the different electromagnetic properties of the different media. It should also be noted that analytical examinations are crucial for exhibiting

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the physical meanings of wave interactions with objects. On the other hand, numerical techniques do not lead to correct field expressions when the size of the scatterer is too large compared to the wavelength [15]. The two neighboring media are called isorefractive media when they have the same wave number but different intrinsic impedances. Also, the angle of reflection is equal to the angle of transmission at the interface of isorefractive media [16]. The wedge geometries, at the boundary of isorefractive media, for various boundary conditions were investigated in the literature [16, 17]. However, as far as we know, PEMC wedge at the boundary of isorefractive media has not been tackled before. Although the PEMC half-plane lying at the boundary of the isorefractive media has been investigated recently, the wedge problem, which is a more general case of a half-plane, is considered in this study [18]. The solution of this two dimensional boundary value problem is based on the method introduced in [19]. First of all, the geometry of the problem will be introduced.

The proper total geometric optics (GO) waves will be determined taking into account the introduced geometry. In addition, the initial wave expressions will be obtained by excluding the wedge geometry, which causes the diffraction. After that, scattered GO waves will be evaluated by subtracting the initial GO waves from the total GO waves. Later on, the unknown diffracted waves will be derived by establishing an analogy between the scattered GO and diffracted waves at the transition boundaries. The ultimate wave expressions will be expressed in terms of the Fresnel functions in order to obtain finite amplitude values at the transition boundaries. For this purpose, the well-known uniform theory of diffraction (UTD) method will be used. Finally, behaviors of the waves will be analyzed numerically for different sets of parameters.

The time factor of $\exp(j\omega t)$ is suppressed throughout the paper where ω represents the angular frequency.

2. DEFINITION OF THE PROBLEM

The geometry of the PEMC wedge is presented in Fig. 1. The upper half-space $y > 0$ is filled with a linear isotropic medium characterized by permittivity of ε_1 and permeability of μ_1. The lower half-space $y < 0$ is filled with a linear isotropic medium characterized by permittivity of ε_2 and permeability of μ_2. In this study, these quantities are assumed to be real and positive values. Also, the media satisfy the condition of $\varepsilon_1\mu_1 = \varepsilon_2\mu_2$. Thus, the media have mutual wavenumber k. However, the first medium has the intrinsic impedance of Z_1, and the second medium has the intrinsic impedance of Z_2. The edge of PEMC wedge coincides with the z axis and positioned at the boundary of isorefractive media. The incident plane wave in Medium I illuminates the upper surface of the PEMC wedge.

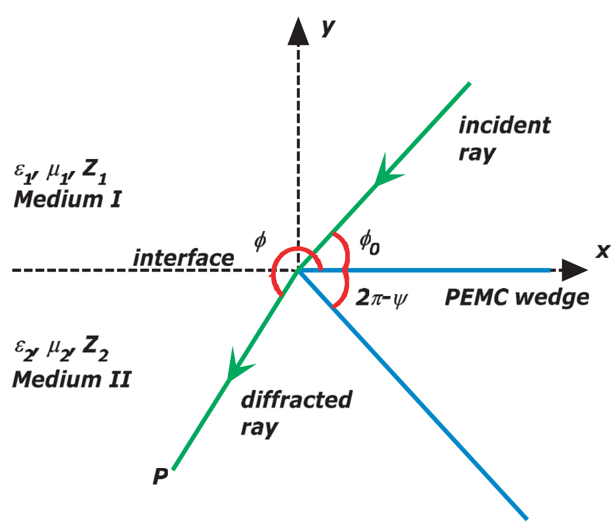

Figure 1. The geometry of the problem.

The electric field component of incident plane wave is

$$\vec{E}_i = \vec{e}_z E_0 e^{jk\rho\cos(\phi-\phi_0)} \tag{1}$$

where ϕ_0 is the angle of incidence. The cylindrical coordinates are represented with ρ, ϕ, and z. Also P represents the observation point (Fig. 1). The outer apex angle is represented with ψ. The boundary conditions are given as

$$\vec{n} \times \vec{E}_{T1}\Big|_{\phi=\pi} = \vec{n} \times \vec{E}_{T2}\Big|_{\phi=\pi}, \tag{2}$$

$$\vec{n} \times \vec{H}_{T1}\Big|_{\phi=\pi} = \vec{n} \times \vec{H}_{T2}\Big|_{\phi=\pi}, \tag{3}$$

for the isorefractive media since the field components are continuous across the boundary and there is not any current flow observed at the interface [16]. Also the boundary conditions on the surface of the wedge are defined by

$$\vec{n} \times \left(\vec{H}_{T1,2} + M\vec{E}_{T1,2}\right)\Big|_{\phi=0,\psi} = 0 \tag{4}$$

and

$$\vec{n} \cdot \left(\varepsilon_{1,2}\vec{E}_{T1,2} - \mu_{1,2}\vec{H}_{T1,2}\right)\Big|_{\phi=0,\psi} = 0, \tag{5}$$

where subindices T_1 and T_2 refer to the total fields in Medium I and Medium II, respectively [14]. In addition, $\vec{n}$ is the unit normal vector, and M represents the admittance parameter [7]. The problem under consideration is based on the determination of the total electric field intensity, and it is expressed as

$$\vec{E}_T = \vec{E}_{TGO} + \vec{E}_d, \tag{6}$$

where E_{TGO} refers to the total GO field, and E_d is the diffracted field. The determination of the diffracted field is essential to achieving the solution of the problem.

3. DETERMINATION OF GO WAVES

In this part, the total and scattered GO waves will be determined. In order to obtain the unknown components of wave expressions PEMC surface is taken into account as a whole surface. Also related boundary conditions given above will be used. The incident magnetic field intensity is obtained as

$$\vec{H}_i = -\frac{E_0}{Z_1}\left(\vec{e}_x \sin\phi_0 - \vec{e}_y \cos\phi_0\right) e^{jk(x\cos\phi_0 + y\sin\phi_0)} \tag{7}$$

by using the Maxwell-Faraday equation. The reflected electric and magnetic field intensities are defined by

$$\vec{E}_r = \left[-Z_1 H_{rz}\left(\vec{e}_x \sin\phi_0 + \vec{e}_y \cos\phi_0\right) + \vec{e}_z E_{rz}\right] e^{jk(x\cos\phi_0 + y\sin\phi_0)} \tag{8}$$

and

$$\vec{H}_r = \left[\frac{E_{rz}}{Z_1}\left(\vec{e}_x \sin\phi_0 + \vec{e}_y \cos\phi_0\right) + \vec{e}_z H_{rz}\right] e^{jk(x\cos\phi_0 + y\sin\phi_0)}, \tag{9}$$

respectively. The unknown z-components of reflected field intensities are derived by using the boundary conditions given in Eqs. (4) and (5) and the field expressions given in Eqs. (8)and (9) as

$$E_{rz} = R_z E_0 \tag{10}$$

and

$$H_{rz} = R_{xy} E_0 \tag{11}$$

where R_z and R_{xy} are the reflection coefficients of z and x-y components of electric field intensity, respectively. Due to the dependencies of electric and magnetic field intensities to each other, derived coefficients are also valid for magnetic field intensity. They are written as

$$R_z = \frac{1 - M^2 Z_1^2}{1 + M^2 Z_1^2} \tag{12}$$

and

$$R_{xy} = -\frac{2M}{1 + M^2 Z_1^2} \tag{13}$$

where M is the admittance of the surface. Thus, after the determination of unknown coefficients, it is possible to reach GO waves by using these coefficients. The total GO wave in terms of the electric field intensities is written as

$$\vec{E}_{TGO} = \left\{\vec{E}_i + \vec{E}_{r1} U\left[(\pi - \phi_0) - \phi\right] + \vec{E}_{r2} U\left[\phi - (\pi - \phi_0)\right]\right\} U(\pi - \phi) + \vec{E}_t U\left[(\pi + \phi_0) - \phi\right] U(\phi - \pi) \tag{14}$$

where E_{r1} and E_{r2} are the reflected fields from the surface of the wedge and the interface, respectively. E_t refers to the transmitted field intensity from Medium I to Medium II. $U(x)$ is the unit step function and shows the location of fields in the related region of space. The initial GO wave can be obtained when excluding the wedge geometry which causes the diffraction process. Hence, it is written as

$$\vec{E}_{in} = \left[\vec{E}_i + \vec{E}_{r2}\right] U(\pi - \phi) + \vec{E}_t U(\phi - \pi). \tag{15}$$

4. DERIVATION OF DIFFRACTED WAVES

In this part, we will make an analogy between scattered GO and diffracted waves. The problem under consideration is a 2D boundary value problem. Hence, the solution is based on the determination of total waves. The expression given in Eq. (6) can be rearranged as

$$\vec{E}_T = \vec{E}_{in} + \vec{E}_s, \tag{16}$$

where the sub-index S refers to the scattered wave. The diffracted wave only exists in the scattered wave since initial wave is composed of only GO waves. Thus, Eq. (16) can written as

$$\vec{E}_{TGO} + \vec{E}_d = \vec{E}_{in} + \vec{E}_{sGO} + \vec{E}_{sd} \tag{17}$$

where Eq. (17) reads to the relations as

$$\vec{E}_{TGO} = \vec{E}_{in} + \vec{E}_{sGO} \tag{18}$$

and

$$\vec{E}_d = \vec{E}_{sd}. \tag{19}$$

Equation (19) shows the diffracted field component of scattered wave directly related to the total diffracted wave. The general form of the high frequency asymptotic expression of the diffracted wave in the scalar form is defined as

$$E_{dz} = E_0 \frac{\sin\frac{\pi}{n}}{n} \frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}} \left[\frac{f_n(\phi, \phi_0)}{\left(\cos\frac{\pi}{n} - \cos\frac{\phi - \phi_0}{n}\right)\left(\cos\frac{\pi}{n} - \cos\frac{\phi + \phi_0}{n}\right)}\right] \times \frac{e^{jk\rho}}{\sqrt{k\rho}} \tag{20}$$

according to the geometrical theory of diffraction (GTD) [20]. The term n is equal to ψ/π, and f_n (ϕ, ϕ_0) is the unknown function which can be defined from the relation of

$$f_n(\pi \mp \phi_0, \phi_0) = -2\sin\frac{\pi \mp \phi_0}{n} \sin\frac{\phi_0}{n} \, A\left[E_{sGO}\right]\Big|_{\phi = \pi \mp \phi_0} \tag{21}$$

where $A[x]$ means only considering the amplitude of x by eliminating phase, and $\pi \pm \phi_0$ term represents the location of the transition boundaries [14]. In order to obtain the diffracted wave, f_n (ϕ, ϕ_0) function should be determined. It is seen from Eq. (21) that diffracted wave is directly related with the amplitude of scattered GO wave at the transition boundaries. Hence, the scattered GO waves can be determined from Eq. (18) by using Eqs. (14) and (15) as

$$\vec{E}_{sGO} = \vec{E}_{r1} U\left[(\pi - \phi_0) - \phi\right] - \vec{E}_{r2} U\left[(\pi - \phi_0) - \phi\right] - \vec{E}_t U\left[\phi - (\pi + \phi_0)\right] \tag{22}$$

by using Eqs. (15) and (16). The reflected electric field intensity from the interface is defined as

$$\vec{E}_{r2} = \vec{e}_z R_{int} E_0 e^{jk\rho\cos(\phi + \phi_0)} \tag{23}$$

and the transmitted field intensity is written as

$$\vec{E}_t = T_{int}\vec{E}_i \tag{24}$$

where R_{int} and T_{int} are the reflection and transmission coefficients of the interface and given as

$$R_{int} = \frac{1-\eta}{1+\eta} \tag{25}$$

and

$$T_{int} = \frac{2}{1+\eta} \tag{26}$$

where η is the ratio of the intrinsic impedances of Medium I to Medium II, i.e., Z_1/Z_2 [16].

It is possible to divide Eq. (22) to its subfields by addition and subtraction of incident electric field intensity as

$$\vec{E}_{sGO1} = \vec{E}_{r2}U\left[(\pi-\phi_0)-\phi\right] - R_{int}\vec{E}_iU\left[\phi-(\pi+\phi_0)\right] \tag{27}$$

and

$$\vec{E}_{sGO2} = \vec{E}_{r1}U\left[(\pi-\phi_0)-\phi\right] - \vec{E}_iU\left[\phi-(\pi+\phi_0)\right] \tag{28}$$

where the first scattered GO waves only includes the z-component, but the second one also has the other components. The unknown $f_n\,(\phi,\,\phi_0)$ function is determined for the z-component of Eq. (27) as

$$f_n^1\,(\phi,\phi_0) = 2\sin\frac{\phi}{n}\sin\frac{\phi_0}{n}R_{int} \tag{29}$$

and for Eq. (28) as

$$f_n^2\,(\phi,\phi_0) = 2\sin\frac{\phi}{n}\sin\frac{\phi_0}{n}\frac{M^2Z_1^2}{1+M^2Z_1^2} + 2\left(\cos\frac{\pi}{n}-\cos\frac{\phi}{n}\cos\frac{\phi_0}{n}\right)\frac{1}{1+M^2Z_1^2} \tag{30}$$

by the aid of Eq. (21). After the determination of unknown functions, z-components of diffracted wave for scattered GO waves given in Eqs. (27) and (28) can be written as

$$E_{dz}^1 = E_0\frac{\sin\frac{\pi}{n}}{n}\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\left[\frac{f_n^1\,(\phi,\phi_0)}{\left(\cos\frac{\pi}{n}-\cos\frac{\phi-\phi_0}{n}\right)\left(\cos\frac{\pi}{n}-\cos\frac{\phi+\phi_0}{n}\right)}\right]\times\frac{e^{jk\rho}}{\sqrt{k\rho}} \tag{31}$$

and

$$E_{dz}^2 = E_0\frac{\sin\frac{\pi}{n}}{n}\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\left[\frac{f_n^2\,(\phi,\phi_0)}{\left(\cos\frac{\pi}{n}-\cos\frac{\phi-\phi_0}{n}\right)\left(\cos\frac{\pi}{n}-\cos\frac{\phi+\phi_0}{n}\right)}\right]\times\frac{e^{jk\rho}}{\sqrt{k\rho}} \tag{32}$$

In order to obtain the finite amplitude values, diffracted waves must be expressed in terms of the Fresnel functions. In this process, diffracted wave expressions are multiplied and divided, and the terms are given by

$$\frac{2\sin\frac{\phi}{2}\sin\frac{\phi_0}{2}}{\cos\phi+\cos\phi_0} \tag{33}$$

and

$$\frac{2\cos\frac{\phi}{2}\cos\frac{\phi_0}{2}}{\cos\phi+\cos\phi_0} \tag{34}$$

for the representation of diffracted waves in terms of the uniform wave expressions of hard and soft surfaces. The uniform field expressions are defined by the relation of

$$sign\,(x)\,F\left[|x|\right] \cong \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{x}}\frac{e^{-jx^2}}{x} \tag{35}$$

where $sign(x)$ is the signum function and equal to 1 when $x > 0$ and equal to -1 when $x < 0$. The term $F[|x|]$ is the Fresnel function and can be defined by the integral of

$$F\left[x\right] = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \int_{x}^{\infty} e^{jt^2} dt \tag{36}$$

The argument of the Fresnel function is called as detour parameter and defined by the ray path difference of GO and diffracted waves [16]. The related detour parameters are defined as

$$\xi_{\pm} = -\sqrt{2k\rho} \cos \frac{\phi \pm \phi_0}{2} \tag{37}$$

by taking into account the locations of the roots of the diffracted waves. The detour parameters are inserted into Eq. (35), making the comparisons between obtained diffracted waves which are multiplied and divided by Eqs. (33) and (34), and the ultimate uniform diffracted waves are as

$$\begin{aligned} E_{dz}^{1} &= E_0 \frac{\sin \frac{\pi}{n}}{n} \left[\frac{f_n^1\left(\phi, \phi_0\right)}{\left(\cos \frac{\pi}{n} - \cos \frac{\phi - \phi_0}{n}\right)\left(\cos \frac{\pi}{n} - \cos \frac{\phi + \phi_0}{n}\right)} \right] \frac{\cos \phi + \cos \phi_0}{2 \sin \frac{\phi}{2} \sin \frac{\phi_0}{2}} \\ &\quad \times \left\{ e^{jk\rho\cos(\phi-\phi_0)} sign\left(\xi_-\right) F\left[|\xi_-|\right] - e^{jk\rho\cos(\phi+\phi_0)} sign\left(\xi_+\right) F\left[|\xi_+|\right] \right\}, \end{aligned} \tag{38}$$

$$\begin{aligned} E_{dz}^{21} &= E_0 \frac{\sin \frac{\pi}{n}}{n} \left[\frac{2 \sin \frac{\phi}{n} \sin \frac{\phi_0}{n} \frac{M^2 Z_1^2}{1 + M^2 Z_1^2}}{\left(\cos \frac{\pi}{n} - \cos \frac{\phi - \phi_0}{n}\right)\left(\cos \frac{\pi}{n} - \cos \frac{\phi + \phi_0}{n}\right)} \right] \frac{\cos \phi + \cos \phi_0}{2 \sin \frac{\phi}{2} \sin \frac{\phi_0}{2}} \\ &\quad \times \left\{ e^{jk\rho\cos(\phi-\phi_0)} sign\left(\xi_-\right) F\left[|\xi_-|\right] - e^{jk\rho\cos(\phi+\phi_0)} sign\left(\xi_+\right) F\left[|\xi_+|\right] \right\}, \end{aligned} \tag{39}$$

and

$$\begin{aligned} E_{dz}^{22} &= -E_0 \frac{\sin \frac{\pi}{n}}{n} \left[\frac{2\left(\cos \frac{\pi}{n} - \cos \frac{\phi}{n} \cos \frac{\phi_0}{n}\right) \frac{1}{1 + M^2 Z_1^2}}{\left(\cos \frac{\pi}{n} - \cos \frac{\phi - \phi_0}{n}\right)\left(\cos \frac{\pi}{n} - \cos \frac{\phi + \phi_0}{n}\right)} \right] \frac{\cos \phi + \cos \phi_0}{2 \cos \frac{\phi}{2} \cos \frac{\phi_0}{2}} \\ &\quad \times \left\{ e^{jk\rho\cos(\phi-\phi_0)} sign\left(\xi_-\right) F\left[|\xi_-|\right] + e^{jk\rho\cos(\phi+\phi_0)} sign\left(\xi_+\right) F\left[|\xi_+|\right] \right\}. \end{aligned} \tag{40}$$

The total diffracted wave is obtained as

$$E_{dz} = E_{dz}^{1} + E_{dz}^{21} + E_{dz}^{22} \tag{41}$$

and total wave is obtained as an addition of Eq. (14) to Eq. (41) as

$$E_{Tz} = E_{TGO} + E_{dz} \tag{42}$$

where E_{TGO} represents only the z-component of total GO wave.

5. NUMERICAL RESULTS

In this part, the amplitude variation of z-component of total wave intensity, total GO, and diffracted wave will be analyzed which were derived in the previous sections. The angle of incidence ϕ_0 is taken as 60°. The angle of observation varies between 0° and ψ where the outer apex angle is taken as 300°. The wavenumber k is equal to $2\pi/\lambda$ where λ is the wavelength. It is also noted that all the related parameters are normalized with respect to λ. The distance of observation ρ is taken as 6λ. The ratio of the impedances Z_1/Z_2 is taken as 0.5. The admittance parameter M is considered as 3.

In Fig. 2, it is seen that GO wave has two discontinuities at the transition regions of 120° and 240°. The amplitude of the diffracted wave reaches its maximum amplitude values at these regions.

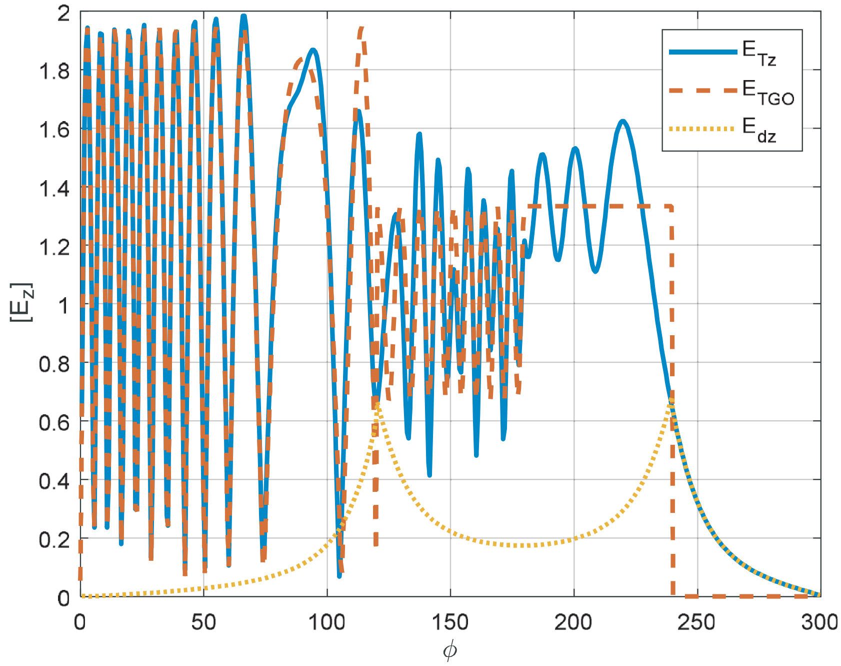

Figure 2. The z-components of total, GO and diffracted waves variations according to the angle of observation ϕ.

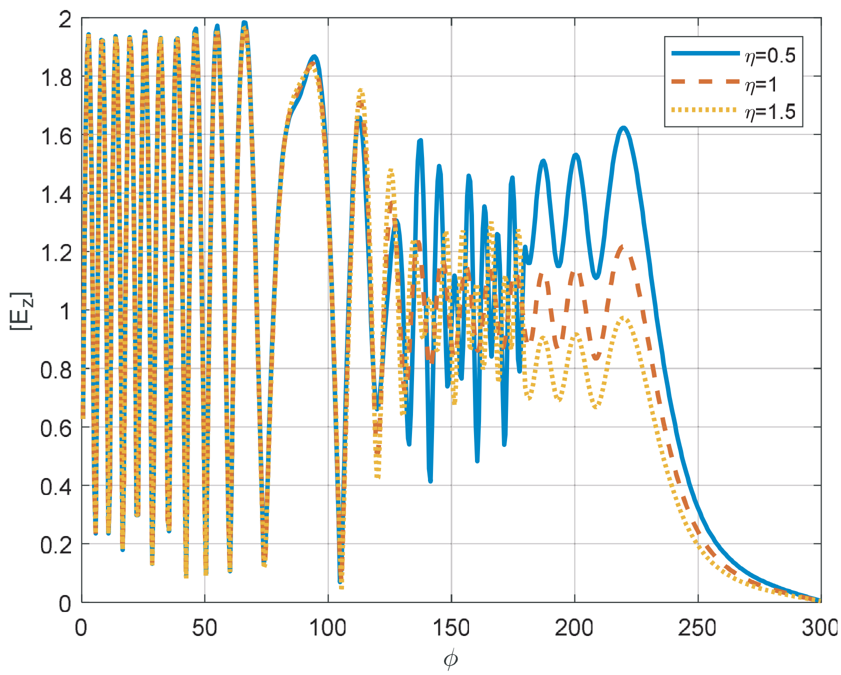

Figure 3. The z-components of total wave variations according to different values of η.

The diffracted wave compensates GO wave at these regions, thus the total wave is continuous in all directions of observation angle. The effect of the interface can be observed between the angles 120° and 180°. In addition, the transition from Medium I to Medium II is continuous at 180°.

Figure 3 depicts the variation of the total wave according to the observation angle with respect to the ratio of impedances η. A simple medium can be obtained when η is taken as 1. In other cases, the

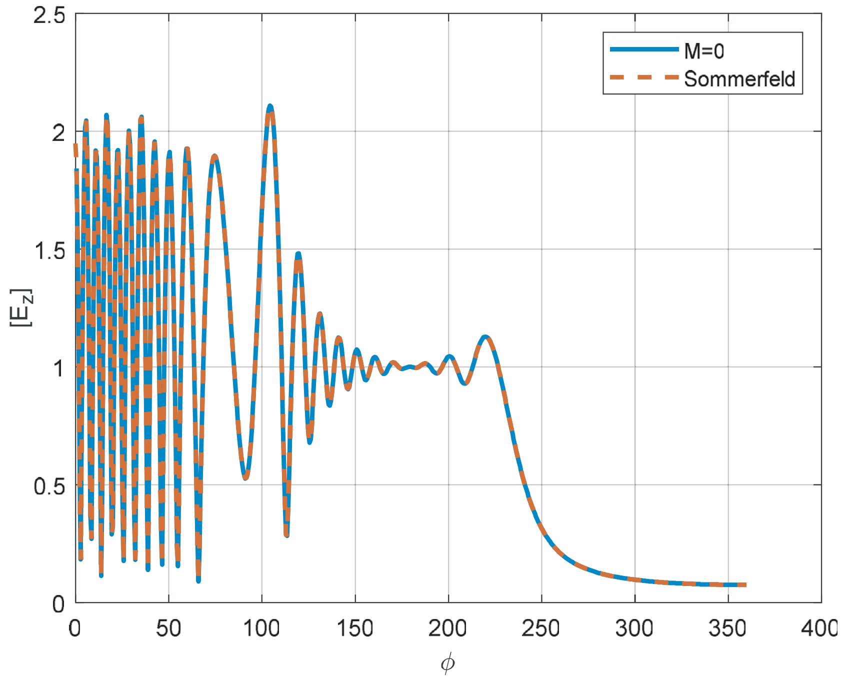

Figure 4. The comparison of z-components of total wave and exact solution of Sommerfeld.

effect of η on the total wave can be observed depending on the values of reflection and transmission coefficients of the interface. Especially depending on the transmission coefficient when η takes higher values, less amplitude of transmitted wave is observed. It is clear that this is due to the ambient density difference.

In Fig. 4, the wave behaviour is examined for the limiting cases. The admittance parameter M is taken as 0, hence the surface of the PEMC wedge acts as a hard-surface. The angle of observation varies between 0° and 360°. The term n is taken as 2. Thus the solution of the problem is reduced to the solution of the half-plane. In this way, it is possible to make a comparison between the exact solution of Sommerfeld's hard-half plane and our solution. As can be seen from Fig. 4 that our solution is very harmony with the exact solution of Sommerfeld [1]. Thus, our solution is validated at the limiting cases.

6. CONCLUSIONS

In this study, diffraction of waves by a PEMC wedge which is lying at the boundary of isorefractive media is examined. The relation between the scattered GO and diffracted waves at the transition boundaries is used in order to obtain the diffracted waves. Unlike the existing studies in the literature, a more general case for the scattering of waves by an object located between two different media which have the different electromagnetic properties are taken into account. The deficiencies of GO wave are compensated by the aid of the diffracted waves at the transition regions. The method of UTD is used to obtain finite amplitude values of waves in order to examine numerically. Thus, it is seen that all the mentioned waves are continuous in all directions of observation. In a future study, it is possible to expand the solutions for the line source illumination. For this purpose, the uniform field expressions should be expressed in terms of the cylinder Fresnel functions which are more suitable than the classical Fresnel functions [21, 22]. It is also possible to analyze the problem under consideration for the circular wave illumination [23].

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