

# Comparative Study of Circular Flat Spiral Coils Structure Effect on Magnetic Resonance Wireless Power Transfer Performance

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**Abstract**—Wireless power transfer (WPT) via coupled magnetic resonance is an encouraging technology to be applied in many fields. In this paper, a method using a circular coil spiral inductor structure to wirelessly transfer energy is proposed. It represents the characteristic of six parallel air core inductors mutually coupled in the free space for wireless power transfer system. Based on the analytical model and circuit theory, the relationship between the coil design parameters and the system performance is deduced, and the effects of the outer radius, inner radius, channel width, and coil turns are thoroughly studied to improve the system performance at different axial distances and in lateral misalignment. Also, an elimination method for transmission efficiency dead-zone (TEDZ) is proposed. The proposed method utilizes angular rotation of the receiver ( $P_x$ ) to eliminate the zero-coupling point which causes TEDZ and boosts the coupling coefficient such that the TEDZ is eliminated, and the high efficiency region is extended.

## 1. INTRODUCTION

Power supply and charging cables of consumer electronics and battery powered portable devices have become a major cause of electronic waste issue. Wireless power transfer (WPT) technology has been proposed to replace the power cable of consumer electronics and charging of the battery powered portable devices. There have been a number of studies reported in literature which aim to apply WPT technology to consumer electronics applications. Stationary WPT for EV charging has lower implementation cost and better market acceptance than dynamic WPT. Inductive power transfer (IPT) system is limited to a few centimeters whereas magnetic resonance can be used with larger range [1, 2]. As a medium-range wireless power transfer technology with higher power and greater transmission efficiency, magnetic coupling resonant wireless power transfer has come to be a research hotspot. The resonance electromagnetic field generated in a transmitting resonator transfers power to the receiving resonator. The received power at the secondary resonator is rectified to charge the battery pack [3].

Modeling of a WPT system is crucial for performance evaluation and optimization. Simplicity and the accuracy of the model are important. Based on the circuit theory and analytical model, this paper models a WPT system and analyzes different coils to comprehensively study their effect on transmission efficiency. The modeling methodology needs to provide guidelines in the selection of system performance indices and design parameters. Concerning the study on the coil structure, circular flat spiral coils are widely adopted in WPT studies as this structure has been widely adopted to help improve power transfer performance and gain higher tolerance to misalignment than other coil structures [4, 5]. In [6], the authors eliminated the frequency splitting phenomenon and improved the efficiency of the system by enlarging the coils. In this paper, the effect of lateral misalignment on the transmission efficiency and output power was analyzed, and the paper could comprehensively analyzes the effect of spiral inductor structures on the transmission efficiency.

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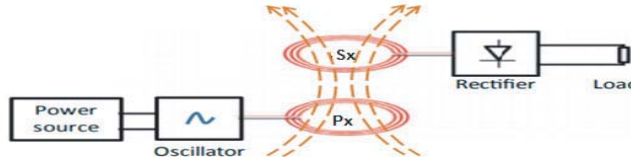
In practical applications such as wireless EV charging systems or wireless mobile device charging system,  $Sx$  may be laterally misaligned with  $Px$  in all directions [7]. Placing the sending and receiving coils in a perfect position is a challenging task, which is not realistic in every application scenario. In fact, lateral misalignment of coils is persistent, and an important decrease of the energy transmission efficiency of the system occurs [8–10].

The transmission efficiency dead-zone, a region where the transmission efficiency first sharply drops from high efficiency down to zero and then recovers to a low efficiency value, is identified in this work. The TEDZ, an annular area, is located between the high efficiency region and low efficiency region. A section in this paper focuses on how to eliminate the TEDZ in laterally misaligned WPT systems. The identification of transmission efficiency dead-zone is verified by simulation results obtained from a physical model.

## 2. PROPOSED SYSTEM DESIGN

The working principle of WPT is similar to the power transformer, but with some differences; for example, transformer has two coils placed very close to each other, and usually each coil is wound on a ferrite material to increase the magnetic coupling, but in case of WPT the core is the open air.

The basic elements to achieve WPT are two, transmitter ( $Px$ ) and receiver ( $Sx$ ). The high frequency AC current in the transmitting coil makes a time varying magnetic field passing through the closed loop of the receiving coil (Fig. 1), and the field induces electromotive force (emf) in the receiving coil. The current in the receiving coil that is generated by the induced emf flows into the power receiving part. The power receiving part transforms the AC current in the receiving coil to DC current, which can be used to operate electric devices.



**Figure 1.** Basic elements of wireless power transfer.

### 2.1. Relative Positions

In a WPT system, the relative positions of  $Sx$  and  $Px$  can be categorized into three types:

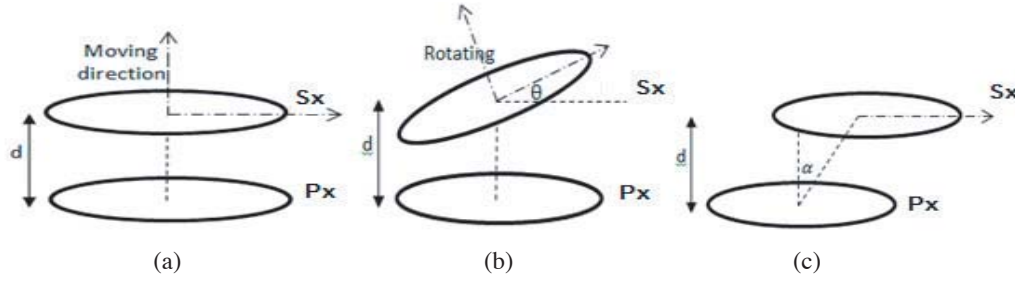
- Perfectly aligned
- Angularly misaligned
- Laterally misaligned.

In a perfectly aligned WPT system (Fig. 2(a)), the axes of  $Px$  and  $Sx$  are perfectly aligned, while the  $Sx$  plane is in parallel with the  $Px$  plane. In an angularly misaligned WPT system (Fig. 2(b)), there is an angle ( $\theta$ ) between the axis of  $Px$  and the axis of  $Sx$ . In a laterally misaligned WPT system (Fig. 2(c)), the plane of  $Sx$  is in parallel with the plane of  $Px$ .  $Sx$  is movable in its plane, leading to the lateral misalignment ‘ $\Delta$ ’ between the axes of  $Sx$  and  $Px$ . In Fig. 2(c), the angle  $\alpha$  represents the extent that the center of  $Sx$  deviates angularly from the axis of  $Px$ .

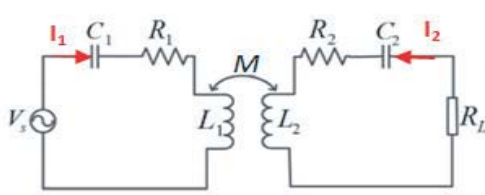
## 3. ANALYSIS OF WPT VIA COUPLED MAGNETIC RESONANCES

### 3.1. The WPT System with Two Coils

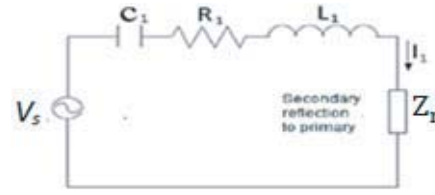
The basic circuit model of inductive coupling using two coil equivalent model of WPT is presented in Fig. 3. It consists of  $Px$ ,  $Sx$  coil inductance.  $L_1$  and  $L_2$  are the primary and secondary inductors, respectively.  $C_1$  and  $C_2$  are the primary and secondary tuning capacitors.  $R_1$  and  $R_2$  are the internal



**Figure 2.** Relative positions of the  $Sx$  and  $Px$  in WPT systems: (a) perfectly aligned; (b) angularly misaligned and (c) laterally misaligned.



**Figure 3.** Series-series circuit model for WPT system.



**Figure 4.** WPT equivalent RLC circuit as seen by the primary side.

resistances of the primary and secondary coils, and  $R_L$  is the load resistance. The  $Px$  coil is connected with high frequency AC source, and  $Sx$  coil is with the load circuit. Besides,  $M$  is the mutual inductance between the primary and secondary inductors.  $V_{in}$  and  $I_{in}$  are the input voltage and current, and  $I_1$  and  $I_2$  are the primary and secondary resonant currents. Fig. 4 illustrates the equivalent circuit when all the quantities are referred to the primary side. The peak value of AC voltage source is 120 V, and the frequency is 40 kHz. The reactance of the primary and secondary circuits are respectively  $Z_1$ ,  $Z_2$ .

When the WPT system is operating at  $\omega_0$ , voltage law equations for the system can be expressed in phasor form as:

$$Vs = Z_1 I_1 + Z_m I_2 \tag{1}$$

$$0 = (Z_2 + R_L) I_2 + Z_m I_1 \tag{2}$$

And

$$Z_1 = R_1 + j\omega_0 L_1 + \frac{1}{j\omega_0 C_1} \tag{3}$$

$$Z_2 = R_2 + j\omega_0 L_2 + \frac{1}{j\omega_0 C_2} + R_L \tag{4}$$

$$Z_m = j\omega_0 M \tag{5}$$

The dependent voltage of the transmitting coil due to the receiving coil is given by:  $V_{12} = -Z_m I_2 = I_1 \frac{Z_m^2}{(Z_2 + R_L)}$ . By dividing  $V_{12}$  by  $I_1$ , we obtain an expression for the reflected impedance as:  $Z_r = \frac{Z_m^2}{Z_2 + R_L}$ . The input impedance of the series-series IPT is:

$$Z_{in} = Z_1 + Z_r = Z_1 + \frac{Z_m^2}{(Z_2 + R_L)} \tag{6}$$

The power (input power  $P_{in}$ ) delivered to the input of the terminated two-port network is computed as:

$$P_{in} = |I_1|^2 \text{Re}(Z_{in}) \tag{7}$$

On the other hand, the power (output power  $P_{out}$ ) actually delivered to the load can be expressed as:

$$P_{out} = |I_2|^2 R_L \tag{8}$$

We divide Eq. (8) by Eq. (7) to obtain transmission efficiency ( $\eta$ ) in terms of system impedances:

$$\eta = \frac{P_{out}}{P_{in}} = \left| \frac{Z_m}{(Z_2 + R_L)} \right|^2 \frac{R_L}{\text{Re}(Z_{in})} \quad (9)$$

The next step is to determine for which load resistance the efficiency is at its maximum. In order to do this, the derivative of Eq. (9) with respect to  $R_L$  needs to be equal to zero [3–6]:

$$\frac{d\eta(R_L)}{dR_L} = 0 \quad (10)$$

Wang et al. [10] propose a formula which was derived with circuit theory to get the optimal load resistance for a certain mutual induction between two resonators, and because the mutual induction is dependent on the distance this formula also indicates what the optimal load resistance is for a certain distance between two resonators. The formula for the optimal load resistance is [3–6]:

$$R_{L,matched} = \sqrt{\frac{\omega^2 M^2 R_2}{R_1} + R_2^2} \quad (11)$$

As the distance between the  $Px$  and  $Sx$  coils increases, magnetic flux decreases. This decrease in magnetic flux decreases  $M$  between  $Sx$  and  $Px$ . With all other parameters held constant, the decrease of  $M$  causes a decrease in  $R_{L,matched}$ .

System efficiency can also be expressed in terms of the magnetic coupling coefficient  $k$  and quality factor  $Q$ . It is shown that the maximum achievable transfer efficiency is dependent on the coils composition and can be given as [2–4]:

$$\eta_{max} = \frac{(kQ)^2}{(1 + \sqrt{(kQ)^2 + 1})^2} \quad (12)$$

where  $Q$  is the combined quality factor of both coils, and  $k$  is a unitless value between 0 and 1. The coupling factor ‘ $k$ ’ is defined as the amount of magnetic flux penetrating the receiving coil compared to the whole generated flux. It could be measured by the following expression:

$$k = \frac{M}{\sqrt{L_2 L_1}} \quad (13)$$

Here  $M$  is the mutual inductance between two coils having respectively self-inductances  $L_1$  and  $L_2$ . Systems are classified as loosely coupled systems if  $k < 0.5$  and closely or tightly coupled system if  $k > 0.5$ .

Quality factor is a measure of how well a single coil is operating. An overall combined system quality factor is defined as:

$$Q = \sqrt{Q_2 Q_1} \quad (14)$$

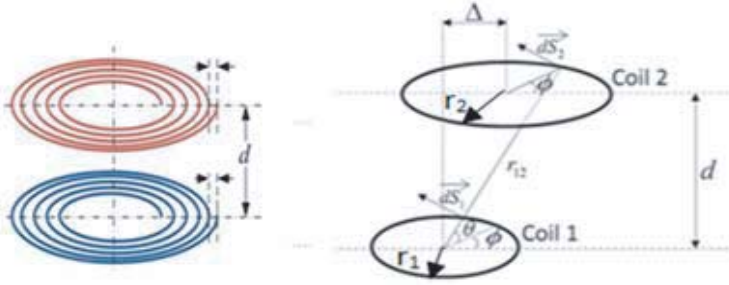
with:

$$Q_{1,2} \approx \frac{\omega_0 L_{1,2}}{R_{1,2}} \quad (15)$$

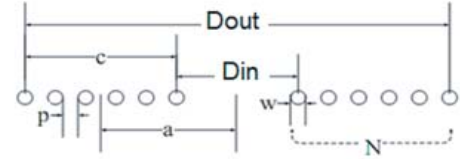
where  $R$  is the resistance of the coil at the frequency of operation.

#### 4. MATHEMATICAL MODELS FOR MUTUAL INDUCTANCE COMPUTATION

The most important expression to compute the mutual inductance of circular coils is given by Neumann’s formula. Consider the two circular spiral coils as shown in Fig. 5 with currents  $I_1$ ,  $I_2$  flowing in primary and secondary coils. The geometry for two circular coils of  $N_1$  and  $N_2$  turns are depicted in Fig. 6. It is assumed that the coil pitch values,  $p_1$  and  $p_2$ , are much less than the coil radii values,  $r_1$  and  $r_2$ . From Fig. 6 it is obvious that  $D_{out}$  is the outer diameter,  $D_{in}$  the inner diameter,  $w$  the diameter of the wire used for making the coil, and  $p$  the spacing between turns.



**Figure 5.** Inductive system with misaligned coils.



**Figure 6.** Cross-sectional view of six turn circular spiral coil.

Power transmission efficiency of inductive systems is highly dependent on the lateral distance ‘ $\Delta$ ’ and the vertical distance ‘ $d$ ’ separating the sending side and the receiving side. Another possible scenario for the misalignment is illustrated in Fig. 5. For circular coils shown in Fig. 5, here Neumann’s equation is used:

$$M = \frac{\mu_0}{4\pi} \int_{c1} \int_{c2} \frac{dS' \cdot dS}{r_{12}} \quad (16)$$

where  $dS'$  and  $dS$  are the wire elements, and  $r_{12}$  is the distance between the two wire elements. The expression for mutual inductance is:

$$M = \frac{\mu_0}{\pi} \sqrt{r_2 r_1} \int_0^\pi \frac{\left( \cos(\theta) - \frac{d}{r_2} \cos(\phi) \right) \psi(k)}{\sqrt{V^3}} d\phi \quad (17)$$

$$V = \sqrt{1 - \cos^2(\phi) \sin^2(\theta) - 2 \frac{d}{r_2} \cos(\phi) \cos(\theta) + \frac{d}{r_2^2}} \quad (18)$$

$$k = \frac{4\alpha V}{(1 + \alpha V)^2 + \xi^2} \quad (19)$$

$$\xi = \beta - \alpha \cos(\phi) \sin(\theta), \quad \alpha = \frac{R_2}{r_1}, \quad \beta = \frac{c}{r_1} \quad (20)$$

$$\psi(k) = \left( \frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) = Q_{\frac{1}{2}}(x), \quad x = \frac{2 - k^2}{k^2} \quad (21)$$

In the above equations,  $\mu_o$  is the magnetic permeability of vacuum, and  $r_1, r_2$  are transmitting and receiving coil turns radii. Its value is  $4\pi \times 10^{-7}$  H/m.  $K(k)$  and  $E(k)$  are the complete elliptic integrals of the first kind and second kind, respectively.  $Q_{1/2}(x)$  is the Legendre function of the second kind with half integral degree.

The original Wheeler expression for calculating the inductance of spiral coils is calculated [9]:

$$L = \frac{a^2 N^2}{8a + 11c} \quad (22)$$

where ‘ $N$ ’ is the number of turns. The parameters  $c$  and  $a$  can be given [9]:

$$c = \frac{D_{out} - D_{in}}{2} \quad (23)$$

$$a = \frac{D_{out} - c}{2} = \frac{D_{out} + D_{in}}{4} \quad (24)$$

$$D_{out} = D_{in} - (p + w)N \quad (25)$$

The Wheeler formula can be derived as shown:

$$L = \frac{(D_{out} - D_{in})^2 N^2}{8(15D_{out} - 7D_{in}) 2.54} \quad (26)$$

## 5. SIMULATION RESULTS

Power transmission efficiency of inductive systems is highly dependent on the lateral distance ‘ $\Delta$ ’ and the vertical distance ‘ $d$ ’ separating the sending and receiving coils. To analyze the influence of mutual inductances on the performance of the WPT systems, the simulation model is set up using MATLAB. To find the coil-pair which is the least sensitive to misalignment, a design example has been presented in this section. In the exemplified prototype, the working frequency is chosen as 40 kHz. The corresponding key parameters are listed in Table 1, where the coil inductance and resistance are directly measured utilizing the LCR meter. The electrical parameters are calculated for 48 V output at 500 W, and input voltage is considered to be 120 V.

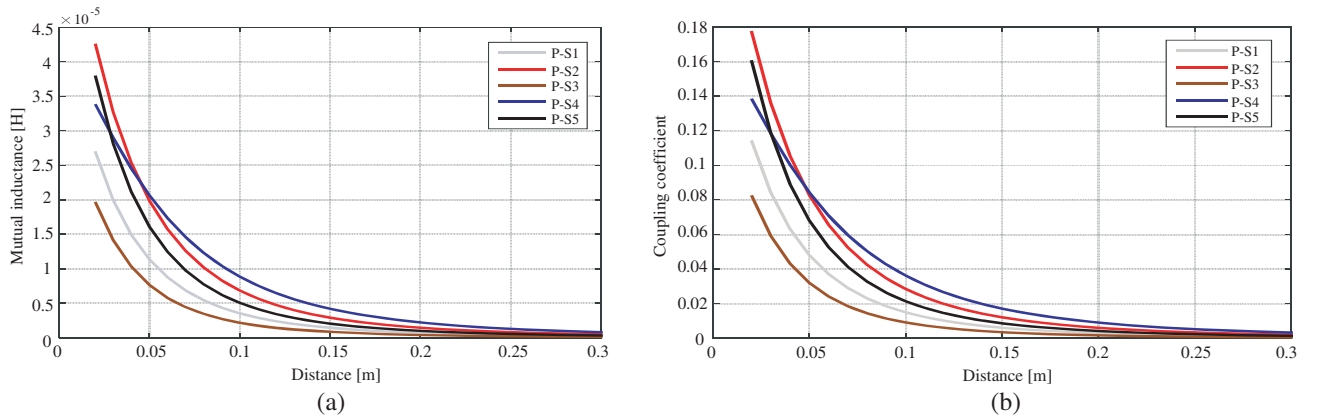
**Table 1.** Parameters of designed coils.

Type	$N$	$D_{out}$ (mm)	$D_{in}$ (mm)	$p$ (mm)	$w$ (mm)	$R$ (Ohm) (Measured)	$L$ ( $\mu\text{H}$ ) (Measured)	$L$ ( $\mu\text{H}$ ) (Analytical)
Coil-P	52	280	140	0	1.244	1	722.56	728.64
Coil-S1	16.4	280	140	0.03	1.244	$32.43 \times 10^{-2}$	75.91	72.51
Coil-S2	15	240	166	1.244	1.244	$27.90 \times 10^{-2}$	75.50	74.8
Coil-S3	20	196	96	1.244	1.244	0.29	74.24	73.99
Coil-S4	13	280	214	1.244	1.244	0.28	75.44	75.11
Coil-S5	16.4	220	140	1.244	1.244	$28.65 \times 10^{-2}$	74.61	73.33

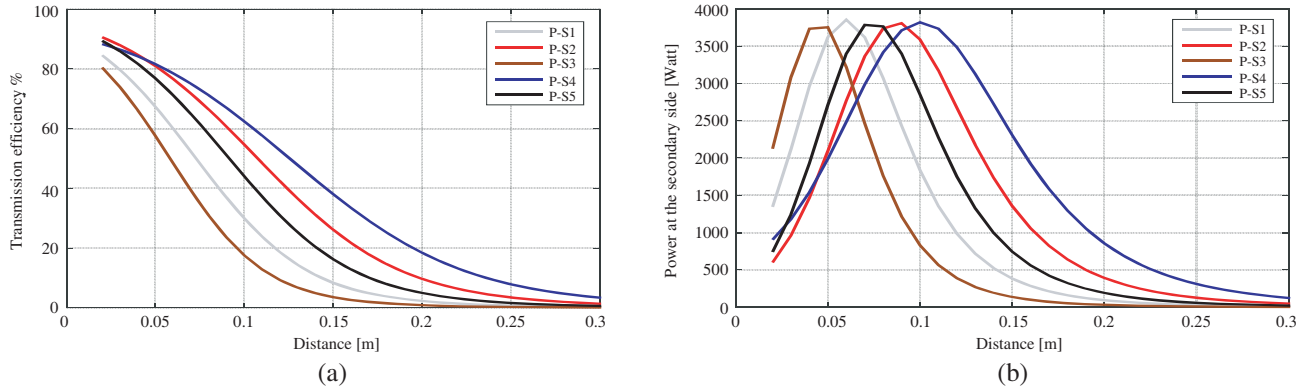
Four coil-pair combinations are considered for finding the topology least sensitive to misalignment and distance air gap. For example, in all the coil pairs, the primary is kept the same in all five coil pairs. However, the secondary coil is of a different size with coil 3 having a minimum inner diameter.

### 5.1. Effect of Vertical Distances on System Performance

Curves in Fig. 7(a) and Fig. 7(b) present, respectively, the coupling factor and the mutual inductance between coils having a variable vertical distance from 0 mm to 300 mm. Out of all the geometric characteristics, the inner diameter most affects the accuracy of the analytical expression. If two coils have the same inner diameter, then the coil having the smaller outer diameter will give a more accurate result from the analytical expression. Along with the increases in vertical distance, the mutual inductance  $M$  shows an exponential decreasing trend. By comparing the coils, P-S4 has the highest mutual inductance.



**Figure 7.** Comparison between results from analytical approach for different distance values: (a) Mutual inductance; (b) Coupling coefficient.



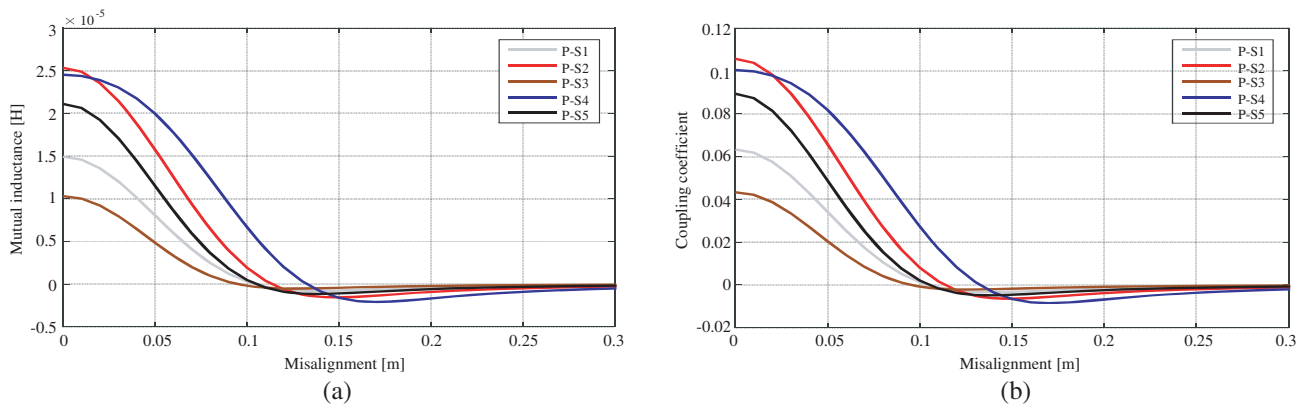
**Figure 8.** Transfer efficiency and power of the five different coupling coils versus offset distance: (a) Transfer efficiency; (b) Power.

Meanwhile, P-S2 comes to the second, and P-S3 is the last. Different coil geometries can produce different types of mutual inductance.

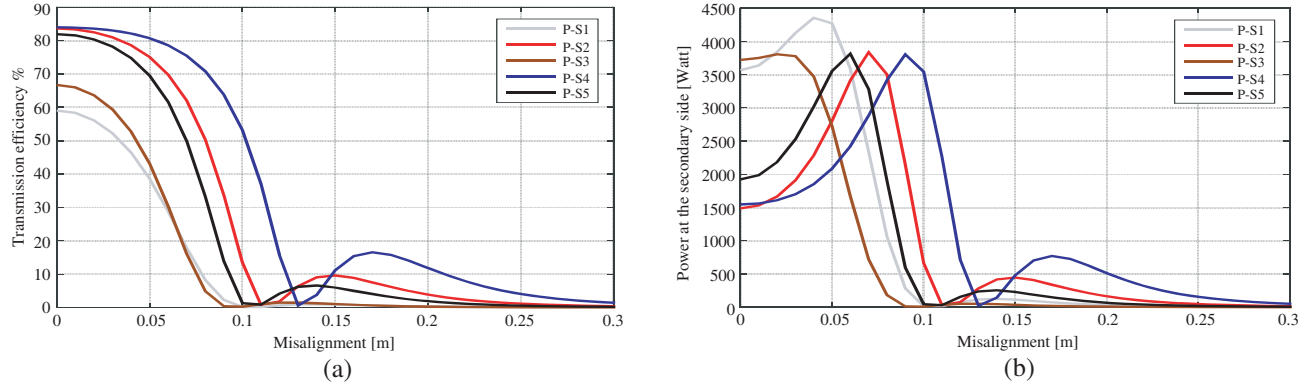
The graphical presentation of output power and transmission efficiency is shown in Fig. 8. Power efficiency drops when the vertical distance increases. Besides, we can say that P-S4 has the best performances among all coils. It produces the best power efficiency and is much higher than the power efficiency produced by other coils. The results show that it is not necessary to keep a minimum distance to get maximum output power, and the maximum power is obtained at a distance of 17 cm rather than minimum distance, which is 8 cm. This proves that in WPT, certain distance is necessary to get maximum output power at which our system becomes completely resonant. According to Fig. 7 and Fig. 8, the performance trends are: P-S4 > P-S2 > P-S5 > P-S1 > P-S3.

### 5.2. Effect of Lateral Misalignments on System Performance

In practical operations of WPT devices, some lateral misalignments should be considered to achieve a high-efficiency power transfer. From the plots shown in Fig. 9, one can observe that coil-pair P-S4 gives the best performance under a misalignment situation at the fixed air-gaps. Coil-pair P-S3 gives the worst performance under misalignment variations. Among the mutual inductance profiles at different air-gaps, the coil-pair P-S3 is superior. Only at a perfectly aligned condition, i.e., 0–15 mm misalignment does P-S2 give mutual inductance greater than P-S4. Additionally, in the case of a large lateral misalignment, not all magnetic field lines produced by the sending coils go through the receiving coil.



**Figure 9.** Performance of coil-pairs under misalignment at fixed air-gap: (a) Mutual inductance; (b) Coupling coefficient.

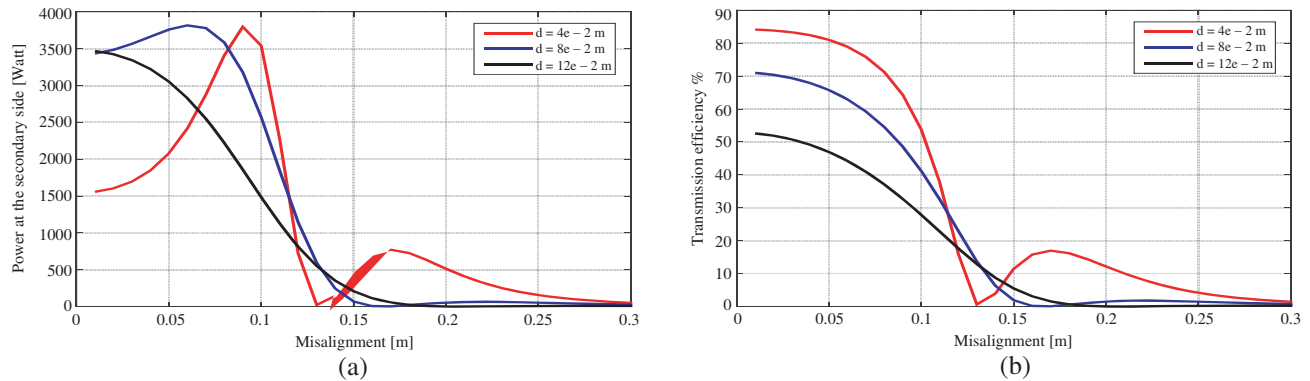


**Figure 10.** Transfer efficiency of the above four different coupling coils versus the misalignment at 4 cm air-gap: (a) Power; (b) Transfer efficiency.

As shown, coupling coil with P-S4 presents the best performance with the highest coupling coefficient  $k$  and transfer efficiency versus coil offsets. As for load power characteristics against misalignment shown in Fig. 10, these coupling coils have a non-identical trend, but the difference of the offset positions when power reaches a maximum. In addition, the power is more different, which means that this coil structure is more sensitive to the offset variation. From Fig. 10(a), it can be observed that the efficiency peak does vary significantly with P-S4, and the efficiency is quite uniform across the range of misalignment. It can also be noted that for P-S4, a null occurs in the transfer efficiency, and coupling coefficient at horizontal offset is about 140 mm, as shown in Fig. 9 and Fig. 10. This is caused by flux cancelation at this point, where magnetic flux of equal magnitude but with opposite directions passes through the receiving coil.

In P-S4, the efficiency drops sharply when the misalignment is 140 mm, because the magnetic flux in the receiving coil is decreased sharply. There is an increase of the efficiency when misalignment is 150 mm. By increasing the vertical distance ' $d$ ' from 4 mm to 12 mm of the sending coil diameter, the transmitted power decreases by more than 19% and 60% of its initial value. System efficiency decreases because as the vertical distance between the coils increases, the quantity of magnetic flux linkage between the coils decreases. This causes less power transfer and lower efficiency.

From Fig. 11, we can conclude that for maximum power transfer to occur we need to minimize the distance between the coils. Based on the results shown in Fig. 11 and Fig. 10, we conclude that WPT systems are highly dependent on both lateral misalignment and vertical distances. They show a sharp decrease of mutual inductance, coupling coefficient, and transmitted power. Consequently, the efficiency of the WPT system drops dramatically. This is justified by the high magnetic field losses occurring during the power transmission phase.



**Figure 11.** Transfer efficiency and power versus the misalignment at different air-gap: (a) Power; (b) Transfer efficiency.



### 5.3. Transmission Efficiency Dead-Zone

In Fig. 9(a), when the receiving coil is positioned at 100 mm from the origin for  $d = 40$  mm, the combined magnetic flux is high, so efficiency is high, but when it becomes 130 mm, the combined magnetic flux decreases, resulting in an efficiency close to 0%. When it is located at 117 mm from the origin, the combined magnetic flux increases again, increasing efficiency. At this position, a small number of fluxes transmitted from  $S4$  first pass through  $P$ , leading to zero coupling between  $P$  and  $S4$ .

In order to solve the problem that the coupling coefficient decreases when the receiving coil is located at 130 mm from the origin, this section proposes a method of removing the transmission efficiency shadow area by increasing the magnetic flux coupled by rotating the transmitting coil. It can be observed from Fig. 11 that the transmission efficiency dead-zone (TEDZ) significantly reduces the high efficiency range of the WPT system and its overall transmission efficiency for applications where lateral misalignment exists.

In order to obtain maximum transmission efficiency after the elimination of the TEDZ, the rotation angle of the  $Sx$  has to be tuned to an optimum value such that the maximum amount of flux is able to pass through  $Sx$ . The optimum rotation angle ( $\beta_o$ ) is obtained when the axis of  $Sx$  aligns to the line of  $Sx$ -to- $Px$  as shown in Fig. 12. The values of  $\beta_o$  when misalignment ranges from 115 mm to 155 mm are calculated in Table 2. It can be shown that the maximum transmission efficiency exists when  $\beta$  is equal to  $\alpha$ , which can be calculated by using  $\beta_o = tg^{-1} \left( \frac{Misalignment}{distance} \right)$ .

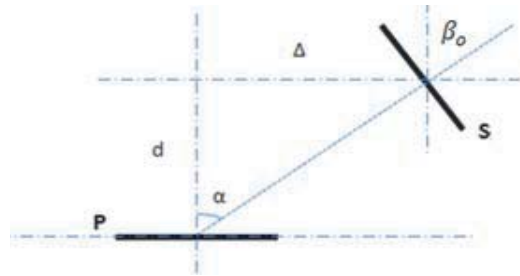


Figure 12. TEDZ elimination method by using the optimum rotation angle of  $Sx$ .

The obtained maximum transmission efficiency curve is shown in Fig. 13. The black curve is the original maximum transmission efficiency curve without the presented TEDZ elimination method. The red curve represents the maximum transmission efficiency obtained by using the angular rotation of the  $Sx$  based on the presented method, which results in the elimination of the TEDZ.

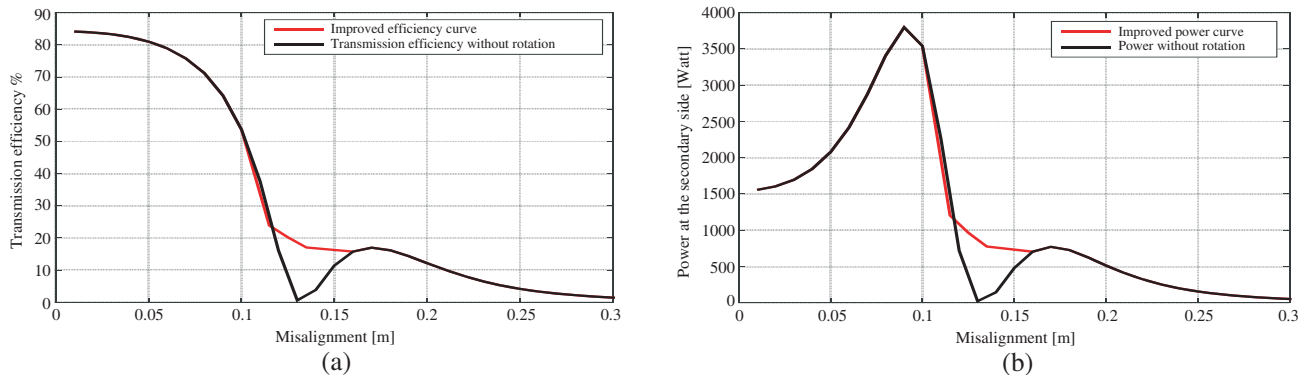


Figure 13. Improved transmission efficiency curve for the misaligned WPT system: (a) Improved transmission efficiency with rotation; (b) Improved efficiency curve.

**Table 2.** The optimal rotation angles ( $\beta_o$ ) of  $P$  in the TEVD.

Distance $d$ (m)	Misalignment $\Delta$ (m)	Optimum rotation angle (degree) = $\arctan (\Delta/d)$
0.040	0.1150	70.8210
0.040	0.1250	72.2553
0.040	0.1350	73.4956
0.040	0.1450	74.5778
0.040	0.1550	75.5297

## 6. CONCLUSION

Through analytical method, this paper has built the models of self-inductance and mutual inductance of circular flat spiral coils used in WPT system. By this model it is found that the mutual inductance decreases along with the increase of air gap distance and misalignment. This paper analyzes the correlation between the circular spiral coil design parameters and the WPT system performance based on the coupled circuit theory and the analytical model of the system. In order to improve the performance regarding the coil-system efficiency and power transfer amount, design parameters of the coil including outer radius, inner radius, channel width, and coils turns are studied. Through model simulation, this paper identifies the transmission efficiency dead-zone cause and nature in laterally misaligned WPT systems. Then the paper proposes a method to eliminate the TEDZ based on an angular rotation of the wireless power receiver  $Sx$  by a given angle that is found equal to the misalignment angle between the transmitter and receiver axes. The study can facilitate the optimization for maximum efficiency at the desired operating distance or misalignment between coils for any given WPT application.

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