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Parameter Design of Invisible Anti-Cloak Based on Nonlinear Transformation

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Abstract—In this paper, we propose a new methodology to design an electromagnetic invisibility anti-cloak, which is based on nonlinear coordinate transformation. Cylindrical and elliptical shapes are presented to show the validation of the proposed methodology. We verify and analyze the above model with nonlinear transformation respectively. Full-wave simulations are given to illustrate the ability of the nonlinear transformation, which is advantageous for reducing the design difficulty of the anti-cloak. The cloak shielding is broken, and the electromagnetic waves can go through the cloak. It is of particular importance in microwave communication applications.

1. INTRODUCTION

In recent years, the electromagnetic cloaks based on transformation optics (TO) have attracted great attention since the introduction of electromagnetic metamaterials. In order to control the spatial electromagnetic fields, the method of transformation optics (TO) was independently presented by Pendry et al. in 2006 [1]. After that, the transformation media for electromagnetic (EM) waves were proposed by Cummer et al. [2]. In addition, to demonstrate the feasibility of such invisibility cloaks [1–10, 18], many theoretical studies and experiments have been done. At the same time, electromagnetic (EM) wave bending [11, 19], concentrators [12–15], rotators [16], and polarization splitters [17] have been proposed for more applications. Although researchers have made great progress in the designs of various invisibility cloaks [20–23], so far, currently available technologies to convert the cloaks into practical applications are still facing numerous bottlenecks. For the traditional cloak [24], the target object kept inside the cloaked region is blinded, and the communication between the object and outside space is impossible since no electromagnetic wave can reach the object from outside medium and vice versa. Therefore, Chen et al. proposed a method to defeat the perfect stealth effect when adding another conversion medium into the cloak [25]. The concept is elaborated on a tunneling between cloak and anti-cloak [26]. Information can be transmitted freely without being affected by the invisibility cloak.

Although great progress has been made in this area, the anisotropy and inhomogeneity of the constitutive parameters make it difficult to implement the ideal cloak. There have been a lot of researchers discussing the concept of parameter simplification due to easier implementation [27]. The scattering is inevitable in the reduced cloak; however, the scattering field can be reduced by the proper design of material parameters. In order to solve these defects, nonlinear transformation is proposed in this paper. Compared with linear transformation, quadratic transformation is an advanced method to reduce scattering electromagnetic field [28, 29]. It is believed that nonlinear transformation [30] has an advantage of invisible anti-cloak design and implementation compared to linear transformation. In addition, the nonlinear mapping can change the variations of the constitutive parameters of the anti-cloak and reduce the difficulty of the realization of artificial metamaterials by selecting high quality parameter values, so as to accelerate the realization of the anti-cloak.

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In this paper, according to the theory of anti-cloak, a new method based on nonlinear transformation is proposed to obtain the formulae of the permittivity and the permeability. Based the new formulae obtained above, the electromagnetic invisibility anti-cloak is designed. We focus on investigating the effect of nonlinear transformation on the constitutive parameter of invisible anti-cloak. The nonlinear mapping relationship used here is the function of power. The simulated results show that the change of constitutive parameter induced by nonlinear transformation could make the cloak fabricated much easier than previous method, which demonstrates the feasibility of proposed method.

2. NONLINEAR TRANSFORMATION FOR INVISIBILITY ANTI-CLOAK AND FULL-WAVE SIMULATION

Maxwell's equations are form-invariant under coordinate transformation. Based on the theory of coordinate transformation, a volume in the original space p can be compressed into a virtual space p'. The specific permittivity ε' and permeability μ' of the electromagnetic invisibility cloak can be derived from [31]

$$\varepsilon' = \frac{\Lambda\varepsilon\Lambda^{\mathrm{T}}}{\det\Lambda}, \qquad \mu' = \frac{\Lambda\mu\Lambda^{\mathrm{T}}}{\det\Lambda} \tag{1}$$

where Λ is the Jacobi matrix tensor of the transformation from the original space to the transformed space with components defined by $\Lambda_{ij} = \partial p'_{\mathrm{i}}/\partial p_j$. In the original space, ε is the permittivity, and μ is the permeability. How to determine matrix Λ is very important for the design of transformation mediums.

The details of a two-dimensional stealth cloak with an anti-cloak is shown in Figure 1. It consists of three parts, namely invisibility region, anti-cloak shell, and cloak shell. The anti-cloak layer in the middle allows electromagnetic waves to penetrate from the outside into the inside. ε_r and μ_r are the relative permittivity and permeability of the inner cylinder, respectively. The vacuum is in the outside of cloak ($\varepsilon_r = 1, \mu_r = 1$).

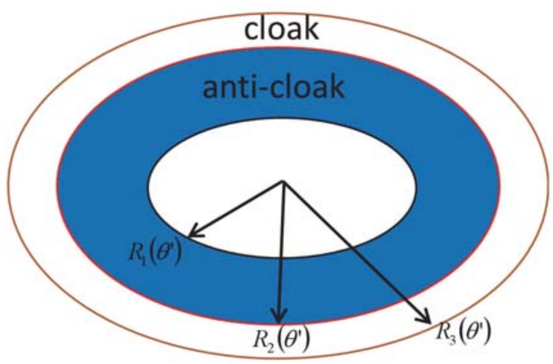

Figure 1. Schematic diagram of stealth cloak with anti-cloak.

Although linear transformation is intuitionistic and straightforward, it has the same effect on the radial points, which makes the space near the inner boundary deform more than the space near the external boundary. This means that the variation of constitutive parameters near the internal boundary is different. In this paper, in order to overcome these limitation, the nonlinear transformation is applied to change the situation. The power function is the nonlinear transformation for simplicity. The nonlinear transformation can compress the cylindrical region with $0 \le r \le R_3(\theta)$ into an annular region with $R_2(\theta') \le r' \le R_3(\theta')$ so that a cloak shown in Figure 1 is created. The equation is shown as Eq. (2)

$$\begin{cases} r' = [R_3(\theta) - R_2(\theta)]\left[\dfrac{r}{R_3(\theta)}\right]^n + R_2(\theta), \quad 0 \le r \le R_3(\theta) \\ \theta' = \theta \\ z' = z \end{cases} \tag{2}$$

For the anti-cloak, the coordinate transformation is given as Eq. (3)

$$\begin{cases} r' = \left[R_1(\theta) - R_2(\theta)\right] \left[\dfrac{r}{R_1(\theta)}\right]^n + R_2(\theta), \quad 0 \le r \le R_1(\theta) \\ \theta' = \theta \\ z' = z \end{cases} \tag{3}$$

The region with $0 \le r \le R_1(\theta)$ is the virtual space, which can be compressed into the region with $R_1(\theta') \le r' \le R_2(\theta')$ in the real space. The anti-cloak designed in this paper has the same shape of internal and external boundary as shown in Figure 1.

As mentioned above, by substituting Eqs. (2) and (3) into Eq. (1), the permittivity and permeability of the cylindrical cloak and cylindrical anti-cloak can be obtained. The radii from the inside to the outside are a, b, and c. Because ε' and μ' are consistent, the components of the relative permittivity tensor can be expressed as

$$\begin{cases} \varepsilon'_{rr} = n\dfrac{r'-b}{r'}\varepsilon_r \\ \varepsilon'_{\theta\theta} = \dfrac{1}{n}\dfrac{r'}{r'-b}\varepsilon_r \\ \varepsilon'_{zz} = \dfrac{1}{n}\left(\dfrac{c}{c-b}\right)^2 \dfrac{r'-b}{r'}\left(\dfrac{r'-b}{c-b}\right)^{2-\frac{2}{n}}\varepsilon_r \end{cases} \tag{4a}$$

$$\begin{cases} \varepsilon'_{rr} = n\dfrac{r'-b}{r'}\varepsilon_r \\ \varepsilon'_{\theta\theta} = \dfrac{1}{n}\dfrac{r'}{r'-b}\varepsilon_r \\ \varepsilon'_{zz} = \dfrac{1}{n}\left(\dfrac{a}{a-b}\right)^2 \dfrac{r'-b}{r'}\left(\dfrac{r'-b}{a-b}\right)^{2-\frac{2}{n}}\varepsilon_r \end{cases} \tag{4b}$$

From Eqs. (4a) and (4b), the electromagnetic parameters are double negative (DNG) in the region of the anti-cloak, and it can be realized by using metamaterials. Equation (4a) is the parameter of the cloak shell ($R_2(\theta') \le r' \le R_3(\theta')$), and Equation (4b) is the anti-cloak shell ($R_1(\theta') \le r' \le R_2(\theta')$). The object in the cloak cannot be seen because it can control the electromagnetic waves bypassing the concealed region and restore the waveform outside the cloak. The Comsol Multiphysics Software [32], which is a commercial simulation tool based on the finite-element methods, is used in this paper to validate the effect of the nonlinear transformation. To verify the above formulae, a cylindrical anti-cloak is set up. The cylinder is put along z-direction in Figure 2. The inner radius of the anti-cloak is $R_1(\theta') = a = 0.1\,\text{m}$; the outer radius is $R_2(\theta') = b = 0.2\,\text{m}$; $R_3(\theta') = c = 0.4\,\text{m}$; the anti-cloak is filled with an inner cylinder ($\varepsilon_r = 2, \mu_r = 1$). In this paper, the incident excitation is a TE-polarized plane wave spread vertical z direction, so the effective electromagnetic parameters are ε'_{zz}, μ'_{rr} and $\mu'_{\theta\theta}$. A TE-polarized plane wave illuminates from the left to the right, and the anti-cloak works at the frequency of 0.7 GHz.

It can be seen from Figure 2 that the electromagnetic wave bends smoothly around the cloaked area and can recover to the waveform after leaving the cloak. The cloaking performances are perfect. They can both be invisible and receive external information at the same time. The nonlinear transformation not only has a very ideal effect, but also can make the anti-cloak easier to be realized in practice with metamaterials. The circular rings in the transformation regions are parameter layers. The parameters of anti-cloak are inhomogeneous and anisotropic. In order to reduce the difficulty of the anti-cloak implementation, the discrete method is adopted to design the parameters of the anti-cloak, and they are homogeneous.

This nonlinear mapping can change the variation rate, and the curves of constitutive parameter vary with n, shown in Figure 3. In order to avoid the singularity at the boundary $b = 0.2\,\text{m}$ in the computation, select $r' = 0.2002\,\text{m}$ to calculate the parameter value. With the value of n increasing, the value of μ_θ near the middle boundary does not change smoothly. The variation ranges of μ_r are enlarged, and ε_z is also changed, as shown in Figures 3(a), (b), and (c), respectively. When $n = 1$,

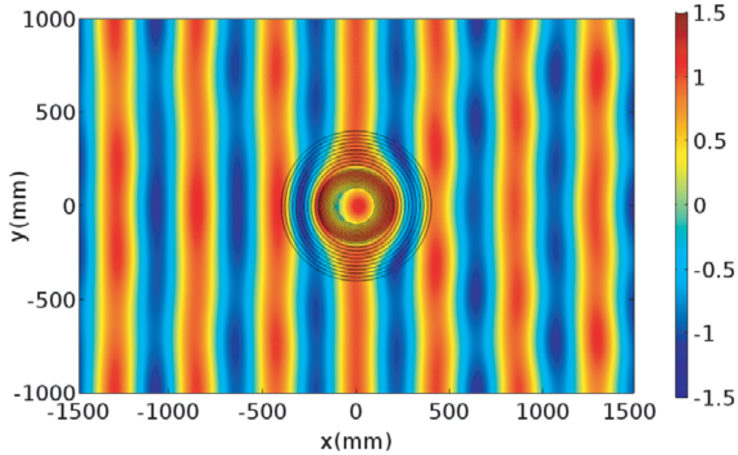

Figure 2. Total electric field distribution near the anti-cloak under an incident TE plane-wave from left to right for a dielectric cylindrical object ($\varepsilon_r = 2, \mu_r = 1$) with the nonlinear transformation when $n = 2$.

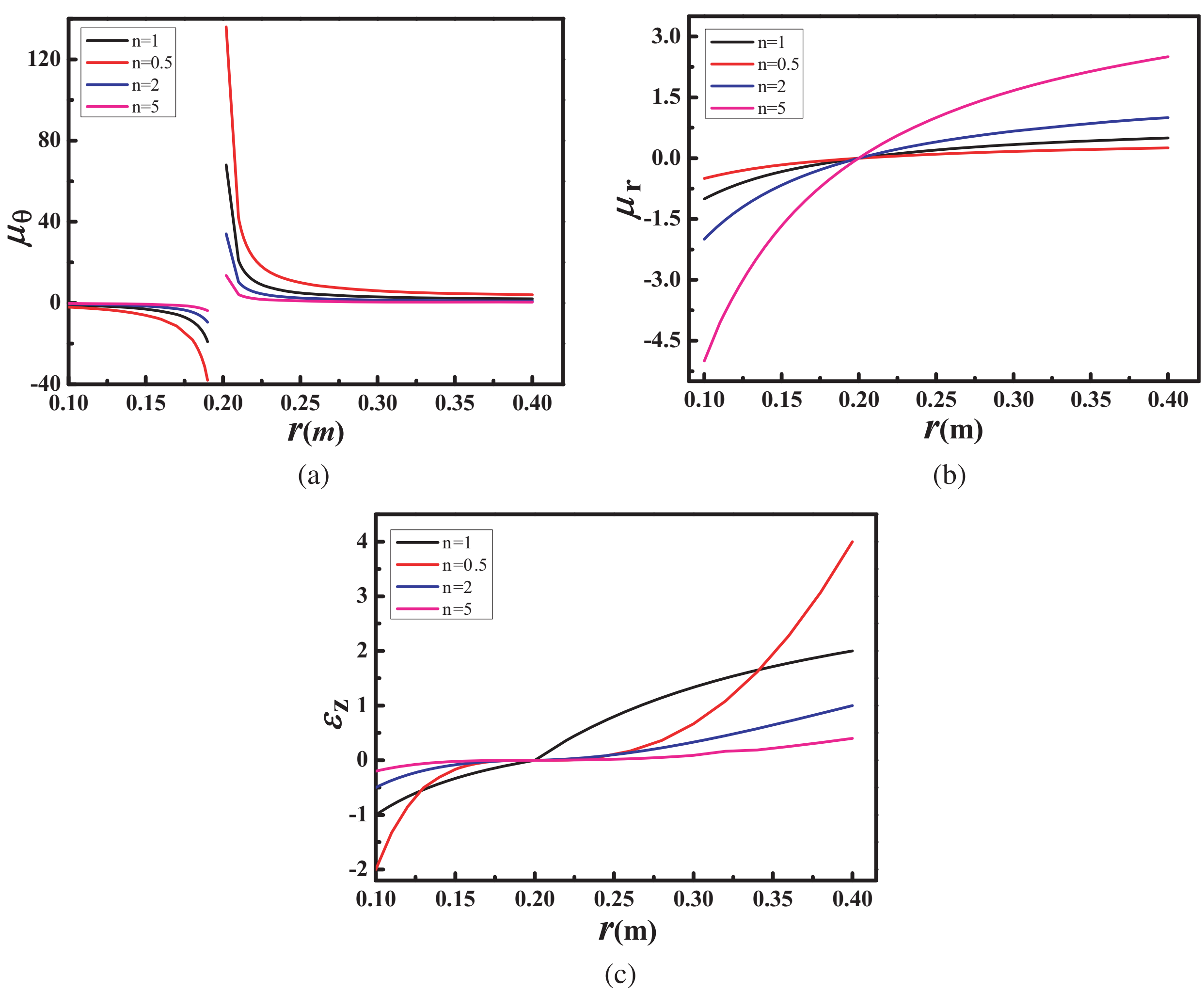

Figure 3. Constitutive parameter vary with n of example one. (a) μ_θ varies with the r. (b) μ_r varies with the r. (c) ε_z varies with the r.

the nonlinear transformation can be the linear transformation, and the differences of the constitutive parameters between them are shown in Figure 3. Therefore, there will be an appropriate value of n to satisfy the material parameters of the current design and effectively improve the realization of the cloak.

To better validate the superiority of the nonlinear transformation, we also apply nonlinear transformation to an ellipse anti-cloak whose equation in polar coordinate is $R_2(\theta) = b\sqrt{1+3\cos^2(\theta)/(3\sin^2(\theta)+1)}$, and substitute this into the equations mentioned above. We can get the parameter expression of the elliptical anti-cloak as follows

$$\begin{cases} \varepsilon'_{rr} = \left\{ n\dfrac{r'-R_2(\theta')}{r'} + \left(\dfrac{-n+1}{n}\right)^2 \dfrac{r'-R_2(\theta')}{r'} R_3^2(\theta') \left[\dfrac{\partial R_3(\theta')}{\partial\theta'}\right]^2 \right. \\ \qquad \left. + \dfrac{2(-n+1)R_3(\theta')}{nr'} \dfrac{\partial R_3(\theta')}{\partial\theta'} \dfrac{\partial R_2(\theta')}{\partial\theta'} + \dfrac{1}{n[r'-R_2(\theta')]r'} \left[\dfrac{\partial R_2(\theta')}{\partial\theta'}\right]^2 \right\} \varepsilon_r \\ \varepsilon'_{\theta\theta} = \dfrac{1}{n} \dfrac{r'}{r'-R_2(\theta')} \varepsilon_r \\ \varepsilon'_{zz} = \dfrac{1}{n} \left(\dfrac{R_3(\theta')}{R_3(\theta')-R_2(\theta')}\right)^2 \dfrac{r'-R_2(\theta')}{r'} \left(\dfrac{r'-R_2(\theta')}{R_3(\theta')-R_2(\theta')}\right)^{2-\frac{2}{n}} \varepsilon_r \end{cases} \tag{5a}$$

$$\begin{cases} \varepsilon'_{rr} = \left\{ n\dfrac{r'-R_2(\theta')}{r'} + \left(\dfrac{-n+1}{n}\right)^2 \dfrac{r'-R_2(\theta')}{r'} R_1^2(\theta') \left[\dfrac{\partial R_1(\theta')}{\partial\theta'}\right]^2 \right. \\ \qquad \left. + \dfrac{2(-n+1)R_1(\theta')}{nr'} \dfrac{\partial R_1(\theta')}{\partial\theta'} \dfrac{\partial R_2(\theta')}{\partial\theta'} + \dfrac{1}{n[r'-R_2(\theta')]r'} \left[\dfrac{\partial R_2(\theta')}{\partial\theta'}\right]^2 \right\} \varepsilon_r \\ \varepsilon'_{\theta\theta} = \dfrac{1}{n} \dfrac{r'}{r'-R_2(\theta')} \varepsilon_r \\ \varepsilon'_{zz} = \dfrac{1}{n} \left(\dfrac{R_1(\theta')}{R_1(\theta')-R_2(\theta')}\right)^2 \dfrac{r'-R_2(\theta')}{r'} \left(\dfrac{r'-R_2(\theta')}{R_1(\theta')-R_2(\theta')}\right)^{2-\frac{2}{n}} \varepsilon_r \end{cases} \tag{5b}$$

where Eq. (5a) is the cloak shell ($R_2(\theta') \leq r' \leq R_3(\theta')$), and Eq. (5b) is the anti-cloak shell ($R_1(\theta') \leq r' \leq R_2(\theta')$). They also obey the phenomenon mentioned above. Figure 4 gives the simulated results, which verifies the effectiveness and correctness of the proposed anti-cloak design.

Although the nonlinear transformation that we proposed is somewhat complicated from

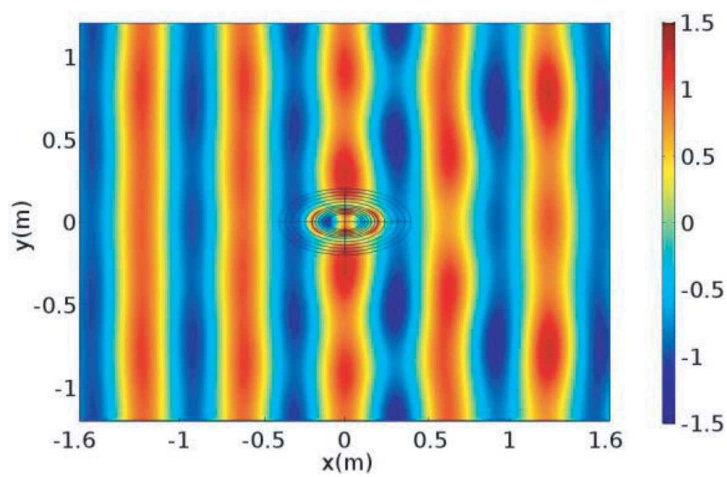

Figure 4. Total electric field distribution near the anti-cloak under an incident TE plane-wave from left to right for a dielectric elliptical object ($\varepsilon_r = 2, \mu_r = 1$) with the nonlinear transformation when $n = 2$.

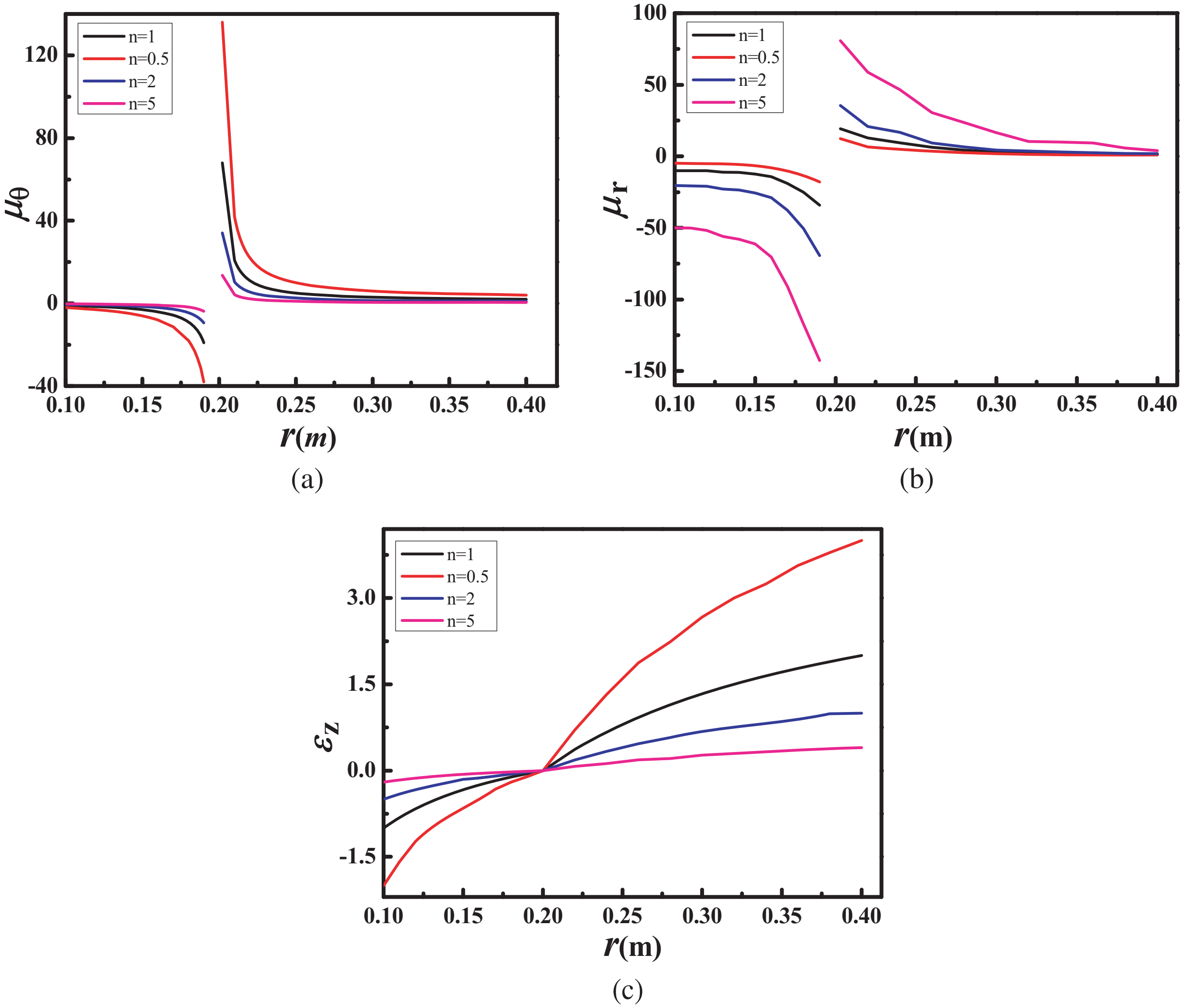

Figure 5. Constitutive parameter vary with n of example 2. (a) μ_θ varies with the r. (b) μ_r varies with the r. (c) ε_z varies with the r.

Equations (4a), (4b), (5a), and (5b), this method can greatly reduce the rate of change of constitutive parameters. As shown in Figure 3 and Figure 5, the value of n represents the parameter change curves obtained by different nonlinear mapping formulas. The ranges of values for the vertical axis are different, and some may shrink obviously. This greatly reduces the difficulty of the realization of artificial metamaterials and accelerates the realization of the anti-cloak. As mentioned above, the same curves of the constitutive parameter of the ellipse anti-cloak are shown in Figures 5(a), (b), and (c). We can also find the appropriate value of n which can determine the difficulty of the implementation of the ideal cloak.

The nonlinear transformation used in the design of invisibility anti-cloak has perfect performances. The anisotropy and singularity of electromagnetic parameters are always the problem of the cloak implementation, and it is hoped to make some progress in future research. But from Figure 3 or Figure 5 in this paper, the advantage is how fast the parameters change decided by the nonlinear transformation which is the difference of n value. The smaller change range may accelerate the cloak implementation. In this paper, the best value of n is 2, and the performance is perfect. The observer in the cloaked area can receive the information from the outside, simultaneously invisibility for the outside. In other words, an anti-cloak is equivalent to an invisibility cloak with a "window", and information can be transmitted between the outside and inside freely through the "window".

3. DISCUSSION

From the comparison of the above results, we can know that the nonlinear transformation is feasible. Although the transmitted field has some perturbations visible in Figures 2 and 4, it does not affect its practical function. Of course, it also reduces the difficulty of the implementation of artificial metamaterials, which may result from the discrete method adopted in this paper. It is hoped to break through this point in future studies to improve the deficiencies in the results and accelerate the realization of the cloak. The anisotropy and singularity of electromagnetic parameters are always the problem of the cloak implementation. From the formula, we first deduced and demonstrated a cylinder invisibility anti-cloak. On this basis, we extended the formula to the ellipse to make it more universal. We hope that the formula of nonlinear transformation can be extended to the invisibility cloak of any two-dimensional figure in the future work.

4. CONCLUSION

The nonlinear transformation of the electromagnetic invisibility anti-cloak is presented in this paper. The constitutive tensor parameters with nonlinear transformation are obtained by setting the anti-invisibility cloak conformal with the invisibility cloak and the masking body. The parameter of the layer of the anti-cloak is also anisotropic and inhomogeneous, which can be realized with electromagnetic metamaterials. The anti-cloak plays an important role in the transmission of information from outside to the inside of the cloak, while the objects inside the cloak remain invisible to the outside world. Compared with the linear transformation, the nonlinear mapping adopted in this paper is more flexible and intuitionistic. It may make the invisible anti-cloak more easy to be implemented because of reducing the difficulty of design.

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