

## NEAR-INFRARED LIGHT SCATTERING BY ICE-WATER MIXED CLOUDS

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**Abstract**—Based on the Mie theory, the light scattering properties of clouds consisting of pure water, pure ice spheres and concentric water-ice spheres are studied in the near-infrared regions, respectively. We computed the single scattering albedo, phase function, and asymmetry parameters of water clouds, ice clouds, and ice-water mixed clouds. The near infrared reflectivity of the ice-water mixed clouds is computed by using the adding-doubling method and compared to the other two types of clouds. It is shown that it is possible to use the near infrared reflectivity to derive the microphysical characteristics of the clouds.

### 1. INTRODUCTION

Light scattering is an important technique in characterization of clouds. Spectral and angular variations of intensity and polarization of scattering light beams depend on the microstructure, such as mean particle size, concentration and optical depth, of the particles. Hansen [1], Van de Hulst [2] and Liou [3] have pointed out that some of the microphysical properties of clouds, can be inferred from analysis of the clouds' near-infrared reflectivity. Theoretical computations of the reflectivity of clouds are usually reduced to solving the radiative transfer equation (RTE) owing to its complexity, however, remote sensing of the clouds rely on accurate knowledge of the reflectivity. This makes the use of numerically exact solutions of RTE popular. The adding-doubling method was first used by van de Hulst, and Hansen extended it to consider the polarization. The method has been used extensively for illumination by unidirectional light at the top [1–3, 7, 11, 12].

The radar bright band, caused by melting ice particles within the clouds, has been observed and explained by earlier work. In these papers, a melting particle is modeled by a spherical particle which has an average dielectric constant of water and ice. In millimeter wavelengths, this model is tolerable owing to the Rayleigh scattering. However, the Mie scattering is dominate in the near-infrared regions, this model is not appropriate and practical. In this paper, an ice-water spherical particle is modeled by a water-coated ice sphere.

An important cloud characteristic which can be derived by remote sensing is the phase of the cloud particles. This can be gained by the accurate computation of the reflectivity of clouds. In this paper, we made such calculations by using the adding-doubling method in the near-infrared wavelength region. The plane albedo, angular distributions of intensities, and spherical albedo are presented. From our results we conclude that the light reflected by clouds is sensitive to the particle phase at the wavelength  $\lambda \approx 3.3\mu\text{m}$ .

## 2. SINGLE SCATTERING

Major shapes of ice crystals are plates, columns, needles, dendrites, and bullets. The Magano-Lee classification of natural crystals includes 80 shapes, ranging from elementary needle to the irregular germ [4]. Though there are several methods to calculate the scattering properties of nonspherical particles, microphysical properties of clouds cannot be characterized by only one of these shapes. In this paper, we replace the ice particles with equal-volume spherical shaped particles, the melting ice particles are replaced by water-coated ice spheres [5].

Due to the variability of physical properties of clouds both in space and time domains, the size of clouds particles is polydisperse. Thus one can consider a radius of a droplet,  $r$ , as a random value, which is characterized by the distribution function  $f(r)$ . In most cases, the function  $f(r)$  can be represented by gamma distribution, modified gamma distribution, log normal distribution, power law distribution, etc. [6]. Hansen and Travis [1] found that the effective radius and variance

$$r_{ef} = \frac{\int_0^\infty r \pi r^2 f(r) dr}{\int_0^\infty \pi r^2 f(r) dr} \quad (1)$$

$$v_{ef} = \frac{\int_0^\infty (r - r_{ef})^2 \pi r^2 f(r) dr}{r_{ef}^2 \int_0^\infty \pi r^2 f(r) dr} \quad (2)$$

are important parameters for any particle-size distribution. Hansen found that the size distribution for different cloud with the same values of  $r_{ef}$  and  $v_{ef}$  will have similar scattering properties. Therefore we can simply choose the gamma distribution defined as equation (3),

$$f(r) = \text{const} \times r^{(1-3b)/b} \exp\left(\frac{-r}{ab}\right) \quad (3)$$

where  $a = r_{ef}$ ,  $b = v_{ef}$ . In particular, we use  $b = 0.11111$  in this paper.

When electromagnetic waves propagate in clouds, it will be scattered and absorbed, the probability of surviving photons is defined as the single-scattering albedo,

$$\omega_0 = \frac{\sigma_{sca}}{\sigma_{ext}} \quad (4)$$

where  $\sigma_{sca}$  and  $\sigma_{ext}$ , which are defined below, are scattering coefficient and extinction coefficient, respectively.

$$\sigma_{sca} = N \int_0^\infty C_{sca} f(r) dr \quad (5)$$

$$\sigma_{ext} = N \int_0^\infty C_{ext} f(r) dr \quad (6)$$

where  $N$  is the concentration of particles,  $C_{sca}$  is the scattering cross section, and  $C_{ext}$  is the extinction cross section. These cross sections can be obtained from the well-known Mie theory [7].

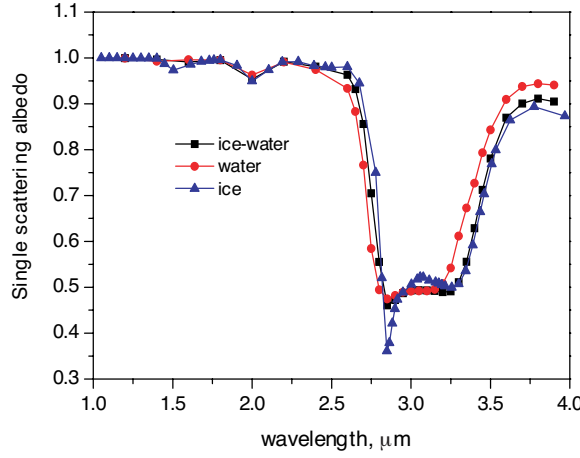
Using Mie theory, the spectral dependence of  $\omega_0$  for the gamma distribution at  $a_{ef} = 6 \mu\text{m}$  for the spectral range of  $1.0\text{--}4.0 \mu\text{m}$  is calculated and plotted in Fig. 1. We can see from Fig. 1 that the single scattering albedos of the three types of clouds are very close, and they decrease with the increase of the imaginary part of refractive index.

Another important function is the single scattering phase function, which characterizes the probability of photon scattering in a given direction, specified by the scattering angle  $\theta$ . For the spherical shape, light scattering by cloud particles is azimuthally symmetric, thus

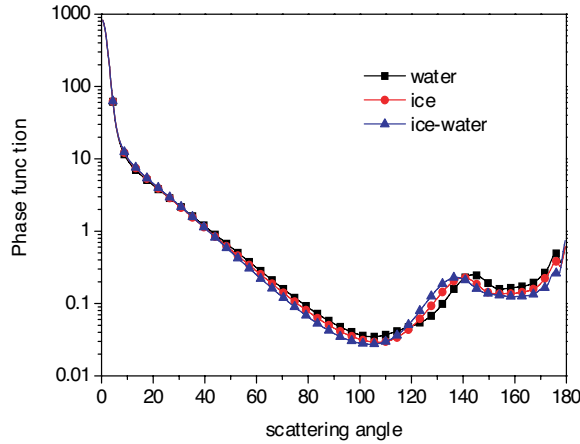
$$\frac{1}{2} \int_0^\pi p(\theta) \sin \theta d\theta = 1 \quad (7)$$

The phase function  $p(\theta)$  does not depend on the concentration of particles, but on the refractive index, shape and size. The function  $p(\theta)$ , obtained from Mie theory, is presented in Fig. 2 at  $\lambda = 1.0 \mu\text{m}$  for three types of clouds with  $r_{ef} = 6 \mu\text{m}$ .

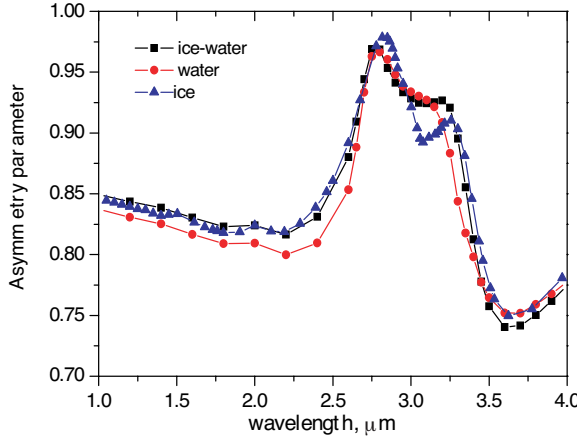
From Fig. 2 we can see that the main features of this phase function is the strong diffraction peak at  $\theta \approx 0^\circ$ . There are small differences among the three types of clouds in the range of scattering angles from  $20^\circ$  to  $60^\circ$ , while the differences in the backward scattering angles are relatively larger.



**Figure 1.** Single scattering albedo of three types of clouds characterized by gamma distribution at  $r_{ef} = 6 \mu\text{m}$ .



**Figure 2.** Phase functions of three types of clouds obtained from Mie theory characterized by gamma distribution at  $r_{ef} = 6 \mu\text{m}$  and  $\lambda = 1.0 \mu\text{m}$ .



**Figure 3.** Asymmetry parameters of three types of clouds characterized by gamma distribution at  $r_{ef} = 6 \mu\text{m}$ .

The asymmetry parameter of the phase function is defined as

$$g = \frac{1}{2} \int_0^\pi p(\theta) \sin \theta \cos \theta d\theta \quad (8)$$

From (8), we can see that the so-called asymmetry parameter is the average cosine of the scattering angle. In Fig. 3, we can see that the asymmetry parameter of water clouds is slightly less than that of the ice clouds and ice-water mixed clouds.

### 3. MULTIPLE SCATTERING

In this paper, the cloud is assumed to be vertically homogeneous plane-parallel layer. The radiative transfer equation can be written as [8, 9]:

$$\begin{aligned} & \cos \theta \frac{dI(\tau, \vartheta, \vartheta_0, \phi)}{d\tau} + I(\tau, \vartheta, \vartheta_0, \phi) \\ &= \frac{\omega_0}{4\pi} \int_0^{2\pi} d\phi' \int_0^\pi I(\tau, \vartheta', \vartheta_0, \phi') P(\gamma') \sin \vartheta' d\vartheta' + \frac{\omega_0}{4\pi} SP(\gamma) e^{-\tau \sec \vartheta_0} \end{aligned} \quad (9)$$

where  $P(\gamma)$  is phase function,  $\gamma$  is scattering angle. Note that the following equations is used for (9)

$$\cos \gamma' = \cos \vartheta \cos \vartheta' + \sin \vartheta \sin \vartheta' \cos(\phi - \phi') \quad (10)$$

$$\cos \gamma = \cos \vartheta \cos \vartheta_0 + \sin \vartheta \sin \vartheta_0 \cos \phi \quad (11)$$

where,  $\vartheta_0$  is the angle of incidence,  $\vartheta$  is the angle of observation, and  $\phi - \phi'$  is the difference of incidence and observation angle.

We are desired to find the intensity for the light diffusely reflected and transmitted by the atmosphere, i.e.,  $I_r(0, \mu, \phi)$  and  $I_t(\tau, \mu, \phi)$ , respectively. It is convenient to define reflection and transmission functions as follows

$$I_r(0, \mu, \phi) = \mu_0 R(\tau, \mu, \mu_0, \phi - \phi_0) F \quad (12)$$

$$I_t(\tau, \mu, \phi) = \mu_0 T(\tau, \mu, \mu_0, \phi - \phi_0) F \quad (13)$$

where the quantity  $R(\tau, \mu, \mu_0, \phi - \phi_0)$  is called the reflection function,  $T(\tau, \mu, \mu_0, \phi - \phi_0)$  is the transmission function, and the incident solar flux through the upper boundary is  $\pi F \mu_0$ .

Van de Hulst [10] showed that if the solution is known for multiple scattering from a plane-parallel atmosphere with thickness  $\tau_0$ , then the solution may be used to obtain the solution for layers of thickness  $2\tau_0, 4\tau_0$ , etc., by an adding-doubling procedure. So, we must know the reflection and transmission functions for the initial layer. One convenient way is to start the computation from a layer with small optical thickness where the multiple scattering may be neglected. The reflection and transmission function for single scattering are give by [11, 12]:

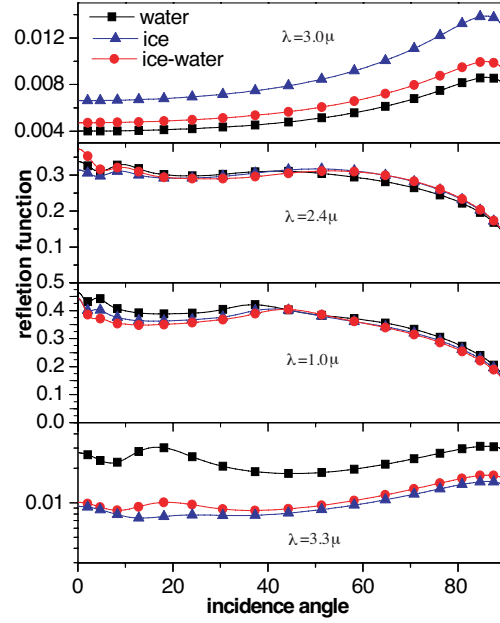
$$R_1(\tau_0; \mu, \mu_0, \phi - \phi_0) = \frac{\omega_0}{4(\mu + \mu_0)} \left\{ 1 - \exp \left[ -\tau_0 \left( \frac{1}{\mu} + \frac{1}{\mu_0} \right) \right] \right\} \times P(\mu, \mu_0, \phi - \phi_0) \quad (14)$$

$$T_1(\tau_0; \mu, \mu_0, \phi - \phi_0) = \frac{\omega_0}{4(\mu - \mu_0)} \left[ \exp \left( \frac{-\tau_0}{\mu_0} \right) - \exp \left( \frac{-\tau_0}{\mu} \right) \right] \times P(\mu, \mu_0, \phi - \phi_0) \quad \text{if } \mu \neq \mu_0$$

$$T_1(\tau_0; \mu, \mu_0, \phi - \phi_0) = \frac{\omega_0 \tau_0}{4\mu_0^2} \exp \left( \frac{-\tau_0}{\mu_0} \right) \times P(\mu, \mu_0, \phi - \phi_0) \quad \text{if } \mu = \mu_0 \quad (15)$$

Hansen [11] found that the scattering quantities of the combined layer for light incident from above can be introduced by following equations:

$$\begin{aligned} Q_1 &= R_a R_b \\ Q_n &= Q_1 Q_{n-1} \\ S &= \sum_{n=1}^{\infty} Q_n \\ D &= T_a + S \exp(-\tau_a/\mu_0) + S T_a \end{aligned} \quad (16)$$



**Figure 4.** Reflection functions of three types of clouds obtained from adding-doubling method at the wavelength  $\lambda = 1.0, 2.4, 3.0, 3.3 \mu\text{m}$  and the effective radius  $r_{ef} = 6 \mu\text{m}$ . The particle in clouds are characterized by the gamma particle-size distribution with  $b = 0.11111$ .

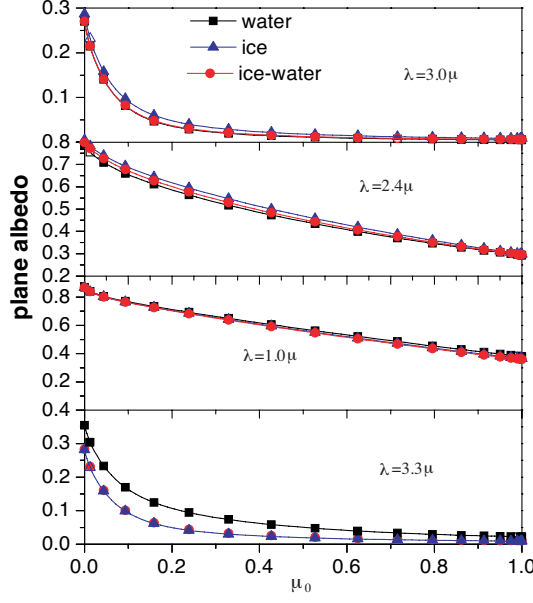
$$\begin{aligned}
 U &= R_b \exp(-\tau_a/\mu_0) + R_b D \\
 R(\tau_a + \tau_b) &= R_a + \exp(-\tau_a/\mu) U + T_a U \\
 T(\tau_a + \tau_b) &= \exp(-\tau_b/\mu) D + T \exp(-\tau_a/\mu_0) + T_b D
 \end{aligned}$$

In (16), the product of two functions implies the integration over the adjoining angles. For arbitrary functions,  $X$  and  $Y$ , and the production  $Z$  is defined as

$$Z(\mu, \mu_0, \phi - \phi_0) = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} X(\mu, \mu', \phi - \phi') Y(\mu, \mu_0, \phi - \phi'_0) \mu' d\mu' d\phi' \quad (17)$$

It is advantageous to expand the  $R(\mu, \mu_0, \phi)$  in a Fourier series in azimuth as shown below. Each term in the Fourier series can be treated independently, thus large saving in computer storage can be achieved.

$$R(\mu, \mu_0, \phi) = R^0(\mu, \mu_0) + 2 \sum_{m=1}^{\max} R^m(\mu, \mu_0) \cos m\phi \quad (18)$$



**Figure 5.** Plane albedos of three types of clouds obtained from adding-doubling method at wavelength  $\lambda = 1.0, 2.4, 3.0, 3.3 \mu\text{m}$  effective radius  $r_{ef} = 6 \mu\text{m}$ , and optical thickness  $\tau = 8$ . The particle in clouds are characterized by the gamma particle-size distribution with  $b = 0.11111$ .

After equation (16) is solved for each  $m$ , the reflection function for any directions of wave propagation can be obtained from (18), and then we can find the plane and the spherical albedos,  $A_p(\mu_0)$  and  $A_s$ , respectively, as follows,

$$A_p(\mu_0) = \frac{1}{\pi} \int_0^1 d\mu \mu \int_0^{2\pi} d\phi R(\mu, \mu_0, \phi) = 2 \int_0^1 R^0(\mu, \mu_0) \mu d\mu \quad (19)$$

$$A_s = 2 \int_0^1 A_p(\mu_0) \mu_0 d\mu_0 \quad (20)$$

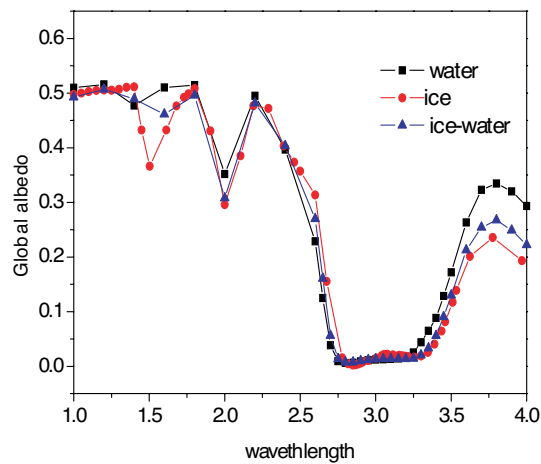
This case when the sun is at the zenith is special since the results are azimuth-independent, i.e.,  $R(\mu, \mu_0, \phi) = R^0(\mu, \mu_0)$ , and the computational burden is greatly reduced. Fig. 4 illustrates the intensity of the reflected light at,  $\lambda = 1.0, 2.4, 3.0$ , and  $3.3 \mu\text{m}$  for three different types of clouds. The results are obtained for the optical thickness  $\tau = 8$ , the effective radius  $r_{ef} = 6 \mu\text{m}$ , and the effective variable  $v_{ef} = 0.11111$ . From the plots in Fig. 4, we can see that the intensity is more sensitive to particle component at  $\lambda = 3.3 \mu\text{m}$ .



This suggests that it is possible to derive some information about the particle, e.g., the particle size or component, at this wavelength.

Fig. 5 depicts the plane albedo  $A_p$  as a function of cosine of the illumination zenith  $\mu_0$  for the three types of clouds. It is obvious that plane albedo decreases with the increase of the imaginary part of refractive index.

Fig. 6 shows the spectral distribution of spherical albedo of the three types of clouds with the effective radius  $r_{ef} = 6 \mu\text{m}$ . In Fig. 6, we can see that the spherical albedos have a rapid decrease in the  $2.2\text{--}2.8 \mu\text{m}$  region, and they are approximately constant from  $2.8 \mu\text{m}$  to  $3.4 \mu\text{m}$ . This is because the absorption features become stronger in these regions.



**Figure 6.** Spherical albedo of three types of clouds obtained from adding-doubling method at the optical thickness  $\tau = 8$ . The particle in clouds are characterized by the gamma particle-size distribution with  $b = 0.11111$ .

#### 4. CONCLUSION

In this paper, we calculated the reflection function, plane and spherical albedos of water clouds, ice clouds and ice-water mixed clouds. The results illustrate that the intensity in the near infrared is sensitive to particle phase, and the sensitivity to particle phase is especially high at the wavelength  $\lambda \approx 3.3 \mu\text{m}$ , indicating the candidate wavelength to be used for measuring the intensity of reflected light in the remote sensing of clouds.

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