

APPLICATION OF GENERALIZED MULTIPOLE TECHNIQUE TO THE ANALYSIS OF DISCONTINUITIES IN SUBSTRATE INTEGRATED WAVEGUIDES

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Abstract—In this work, complex propagation constant of substrate integrated waveguide (SIW) with lossy dielectric is determined with the help of the generalized multipole technique (GMT). We then apply the GMT to compute scattering parameters of some discontinuities in SIW. The obtained results are compared with the results generated by a commercial finite-element solver.

1. INTRODUCTION

GMT is a flexible and accurate semi-analytic method for computational electromagnetic problems, from static to dynamic, and from scattering to waveguide problems [1]. It has been successfully applied to complicated structures such as photonic-crystal waveguides [2, 3], and optical microring resonators [4]. Due to their relatively small radiation losses, supercell approach has been successfully utilized to compute the propagation constants. However, the SIW, which is a promising substitution for rectangular waveguides in planar circuit realization [5], and a special case of electromagnetic bandgap materials [7] is shown to have radiation loss, so that the application of the supercell approach to the SIW may lead to error in computation of propagation constant. Evidently, search for the propagation constants of the SIW should be carried out in the complex plane, which ends up in a very time consuming numerical effort. As will be described in the next section, we replace this search by an efficient iterative procedure.

The second goal of this work is the modeling of some SIW discontinuities using the GMT. It should be mentioned that to this date

SIW discontinuities have been investigated using FDTD [5, 8] and a commercial software [9]. In contrast to FDTD, GMT only requires the discretization of the boundaries, which exhibits therefore no problems with open space and provides exponential convergence for the smooth geometry of the vias. Since GMT is a frequency domain method, it can easily take the dielectric loss into account in any data format such as experimental diagrams or Drude or Lorentz model.

Here, we benefit from an approach based on a port solver which has already been applied to the analysis of photonic-crystal waveguide discontinuities [3]. This has been shown to be very efficient in extraction of the intrinsic parameters of the discontinuities while avoiding spurious reflections which naturally occur in other methods due to the impedance mismatch at the waveguide terminations [10].

2. ANALYSIS USING THE GENERALIZED MULTIPOLE TECHNIQUE

2.1. Modal Analysis of the SIW

GMT is based on the expansion of the electromagnetic fields in terms of appropriate basis functions which are solutions to the Helmholtz equation. The commonly used basis functions are the multipolar functions. For an introduction to the GMT, one can refer to [1]. Here, we apply the GMT to the modal analysis of the SIW.

Figure 1 shows a typical SIW structure, which is comprised of two periodic perfect conducting via-walls with period L and separated by a distance W . We assume that the substrate is lossy. Since the substrate thickness h is much smaller than W , the waveguide dominant mode under investigation is a y -invariant TM_y mode. Now, the unit cell of the SIW can be subdivided into the finite region D_1 , $|x| < W'/2$, and two semi-infinite regions D_2 determined by $|x| > W'/2$. Close to the boundaries of the region D_1 but outside it, we place P multipoles of order $n = 1, \dots, N_p$ at the locations \vec{r}_p indicated in Figure 1 to generate the solution to Maxwell's equations within this region. Accordingly, the E_y component at the field point \vec{r} is given by

$$E_y(\vec{r}) = \sum_{p=1}^P \sum_{n=1}^{N_p} (A_n \cos n\phi_p + B_n \sin n\phi_p) H_n^{(2)}(k_d |\vec{r} - \vec{r}_p|) \quad (1)$$

in which ϕ_p represents the azimuth angle at which the field point \vec{r} is seen by the p -th multipole. Here, $k_d = \omega \sqrt{\mu_o \varepsilon_o \varepsilon_r}$ where ε_r is the complex relative permittivity of the substrate. Typical locations of the multipoles can be seen in Figure 1.

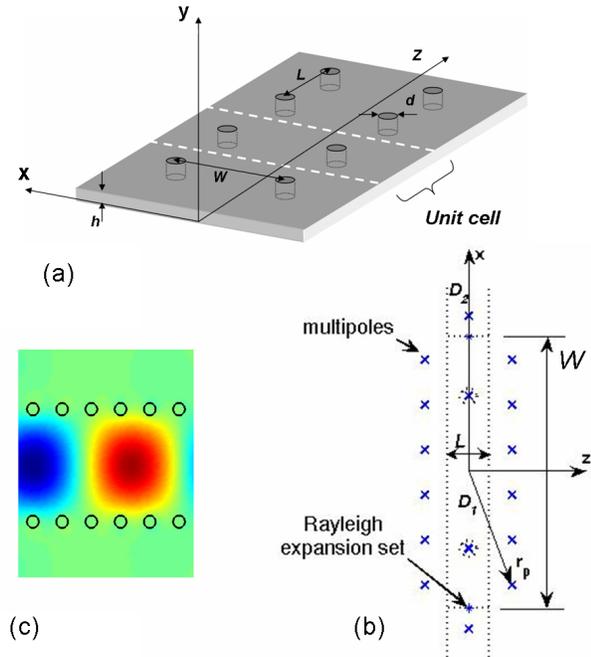


Figure 1. (a) Topology of an SIW structure and its unit cell. (b) Insertion of multipoles for GMT analysis. The location of multipoles is indicated by \times . (c) Electric field at $t = 0$.

In view of the periodicity of the structure, the E_y component in region D_2 is expressed by a Rayleigh expansion as

$$E_y = \sum_{m=-M}^M C_m \exp(-jk_{xm}|x|) \exp\left(-j\left(k_z + \frac{2m\pi}{L}\right)z\right) \quad (2)$$

where k_z denotes the unknown propagation constant of the dominant mode and k_{xm} represents the wave-number in the x -direction which satisfies $k_{xm}^2 + (k_z + 2m\pi/L)^2 = k_d^2$. In (2), the number of spatial harmonics is limited to $2M + 1$. Note that the unknown coefficients A_n , B_n , and C_n in (1) and (2) will be determined when the boundary conditions on the interface of D_1 and D_2 are applied using a generalized point matching technique. The resulting over-determined system of equations is then solved using the QR factorization technique.

To compute the complex propagation constant $k_z = \beta - j\alpha$, we use an iterative procedure. First, at the initial frequency f_1 , β_1 and α_1 are computed with a search in the complex plane. For $f_2 = f_1 + \delta f$

with $\delta f/f_1 \ll 1$, we assume that $\alpha_2 = \alpha_1$ and search for β_2 on the real axis, then we correct α_2 in an iterative procedure with assuming $\beta = \beta_1$. We continue this procedure to achieve the desired accuracy.

2.2. Modeling SIW Discontinuities

To model the discontinuity, two reference planes are assumed in the input and output waveguides at a distance from the discontinuity. The modes of these waveguides are computed using the method described in the previous section. In each reference plane there are a finite number of propagating modes and an infinite number of evanescent modes. We model the fields of the discontinuity region with multipole expansions. These fields should be expressed in terms of the incident and reflected modal fields of the waveguides on the reference planes. With this approach the unknown amplitudes of the reflected and transmitted modes and so the scattering parameters can be obtained. The main difficulty here is the size of the model. Decreasing the size of the model increases the number of the evanescent modes to be considered at the reference planes. To reduce the simulation time while retaining the accuracy, we have considered a sufficient number of evanescent modes in each reference plane.

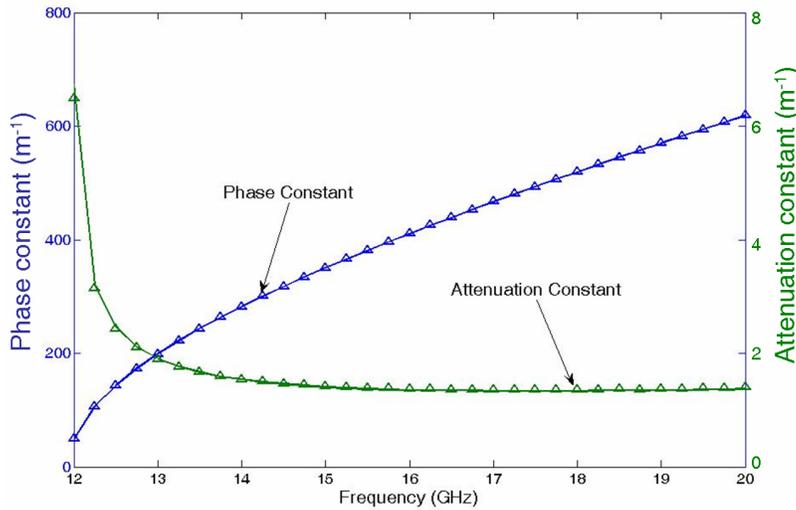


Figure 2. Propagation constant of an SIW structure versus frequency with $W = 7.2$ mm, $L = 2$ mm, and $d = 0.8$ mm. Substrate is RO4003C.

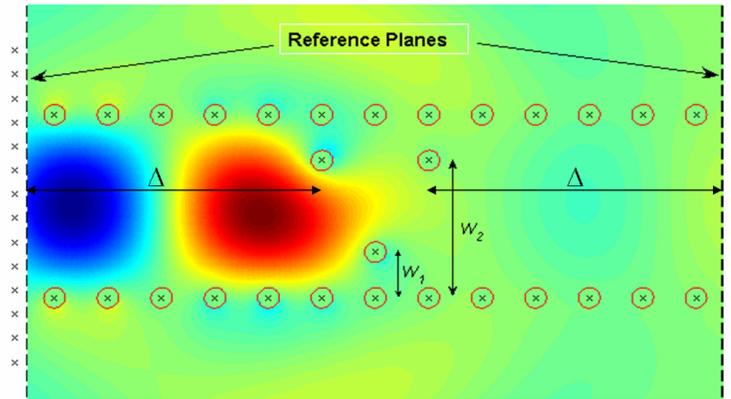


Figure 3. Triple post discontinuity. Location of multipoles is indicated by \times . The plotted field is the electric field at $t = 0$ and the frequency of 20 GHz.

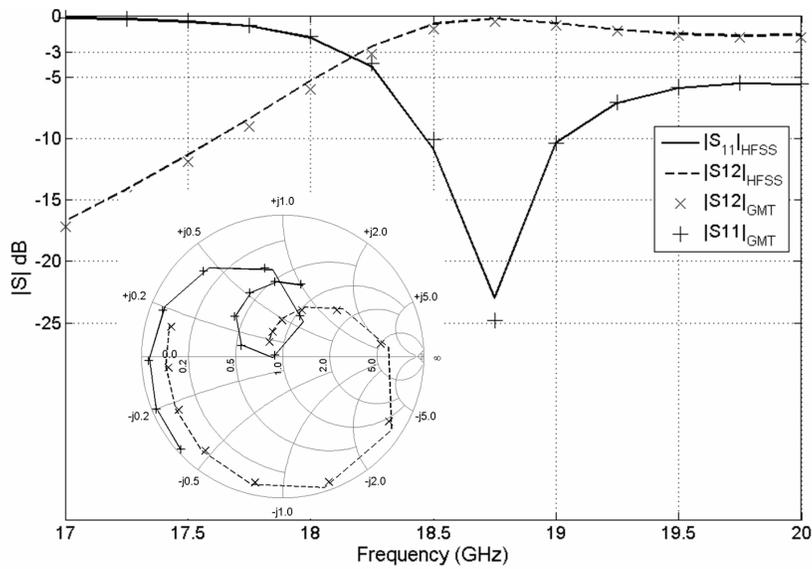


Figure 4. Scattering parameters of a triple post discontinuity. The inset figure shows the Smith diagram with the scattering parameters.

2.3. Numerical Results

To compute the propagation constants, we need to model a unit cell of the SIW. Figure 1 shows the model and the electric field at 20 GHz, for an SIW with $d = 0.8$ mm, $W = 7.2$ mm, and $L = 2$ mm. The substrate is RO4003C, with $\varepsilon_r = 3.38$ and $\tan(\delta) = 0.0027$. If N denotes the order of the multipoles, then it would be as follows: for multipoles to the side of each periodic boundary $N = 6$, for multipoles located in the center of the vias $N = 5$, and for multipoles located close to $x = \pm W/2$, $N = 6$ was required. We also used 11 Rayleigh terms for simulation of the fields in domain D_2 . The chosen locations of the multipoles led to convergent results and low average error of less than 0.1 percent. Figure 2, shows the propagation constants of the mentioned SIW within a frequency band of 8 GHz. The results were compared with the HFSS simulator using multimode calibration technique [11]. The perfect agreement shows the accuracy of the GMT.

For the discontinuity shown in Figure 3, the input and output waveguides have the same dimensions with $W = 7.2$ mm, $L = 2$ mm, $d = 0.8$ mm and $\varepsilon_r = 2.23$ and $\tan(\delta) = 0.0009$. This discontinuity is a triple post each of a diameter of 0.8 mm, $W_1 = 1.8$ mm, $W_2 = 5.4$ mm, and $\Delta = 5.5L$. We excited the discontinuity with the dominant mode at the input reference plane. The discontinuity has been modelled with some multipolar expansions with the orders as follows: for multipoles located in the center of the vias near the reference planes $N = 8$, for

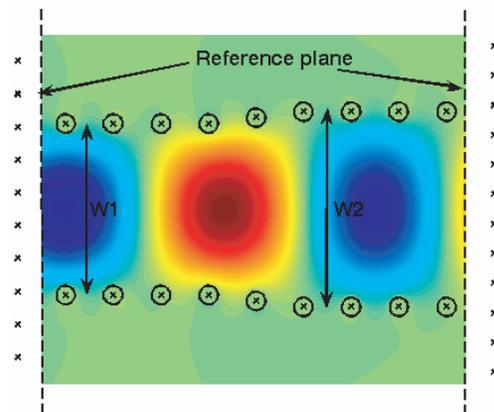


Figure 5. Modeling a step discontinuity with GMT. $W_1 = 7.2$ mm and $W_2 = 8.2$ mm. The plotted field is the electric field at $t = 0$ and $f = 20$ GHz.

the multipoles located in the other vias $N = 7$, and for the multipoles beside each reference plane $N = 3$ were required. The maximum relative error on the boundaries was 2 percent. We considered the dominant mode along with three evanescent modes at each reference plane. The computed S_{11} and S_{12} are shown in Figure 4 and compared with the results obtained using the HFSS simulator.

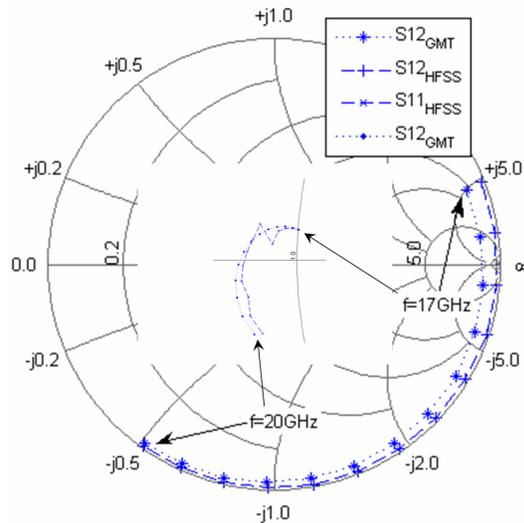


Figure 6. Scattering parameters of the step discontinuity with the topology indicated in Figure 5. In the middle of the diagram, S_{11} parameter has been shown on a different scale.

As the last example, we considered a step discontinuity, with the transition from an SIW with $W = 7.2\text{mm}$ to another one with $W = 8.2\text{mm}$. Other specifications of the input and output waveguides are the same as the SIW analyzed in the previous example. Just the first mode is propagating in both the input and output waveguide. The computed results are shown in the Smith diagram of Figure 5. We can also see the computed electric field with GMT in Figure 5.

3. CONCLUSION

The propagation constants of the SIW has been computed using the GMT and by means of an iterative technique. The computed attenuation constant takes into account both the dielectric losses and leakage through via-walls. To show the main advantage of the GMT,

the scattering parameters of some SIW discontinuities have also been computed using this technique. The results were compared with those obtained with the HFSS. The GMT has been proved to be a suitable method for the analysis of SIW structures due to its flexibility to almost arbitrary discontinuities with lossy dielectrics, its high accuracy and reduced computation time.

ACKNOWLEDGMENT

The authors would like to thank Professor C. Hafner of the Swiss Federal Institute of Technology Zurich for his generous help with regard to the method of GMT. They would also like to thank Iran Telecommunications Research Centre (ITRC) for their support of this project.

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