Progress In Electromagnetics Research C, Vol. 2, 217–232, 2008

COLD PLASMA INJECTION ON VLF WAVE MODE FOR RELATIVISTIC MAGNETOPLASMA WITH A.C. ELECTRIC FIELD

R. S. Pandey

Department of Applied Physics, Amity School of Applied Science
Amity University
Noida, Utter Pradesh, India

R. P. Pandey, K. M. Singh, and N. M. Mishra

Department of Physics
Veer Kunwar Singh University
Ara, Bihar, India

Abstract—The effect of cold plasma beam on electromagnetic whistler wave with perpendicular AC electric field has been studied by using the unperturbed Lorentzian (Kappa) distribution in the Earth's atmosphere for relativistic plasma. The cold plasma has been described by a simple Maxwellian distribution where as Lorentzian (Kappa) distribution function has been derived for relativistic plasma with temperature anisotropy in the presence of a perpendicular AC electric field to form a hot/warm background. The dispersion relation is obtained by using the method of characteristic solutions and kinetic approach. An expression for the growth rate of a system with added cold plasma injection has been calculated. Results for representative values of parameters suited to the Earth's magnetosphere has been obtained. It is inferred that in addition to the other factors, the relativistic plasma modifies the growth rate and it also shifts the wave band significantly. The relativistic electrons by increasing the growth rate and widening the bandwidth may explain a wide frequency range of whistler emissions in the Earth's magnetosphere.

1. INTRODUCTION

Whistler mode waves have been a common feature of spectrum wave observations at Earth's bow shock for many years [1, 2]. It has been shown that whistler wave can be excited through the application of electron beams gyrophase bunches ions [3] and wave steeping [4]. Whistler waves at space craft frequency between 1 and 7 Hz have been reported upstream of Earth [5]. While there are observations of whistler waves propagating parallel to the inter planetary magnetic field, most reported whistler wave observations and generation theories involve highly oblique propagation. Whistler mode waves have also been observed in commentary foreshocks and are thought to arise from the same mechanisms as above [6]. It is also reported that the whistler mode can be driven by an electron temperature anisotropy [7]. For the whistler instability the electrons are weakly resonant but the ions are non resonant with the waves.

Higher frequency whistler wave activity has also been observed upstream of the Earth's bow shock by plasma wave analysis. These waves possess spacecraft frame frequencies from approximately 10 to 100 Hz and are generally synchronous with plasma Oscillations at the electron plasma frequency [8–10]. Tokar and Gurnett [11] argued that these waves, when observed with the shock ramp result from electron beams with high thermal anisotropy and beam velocities directed towards the magnetosheath [12]. Similar high frequency whistler waves have been observed by ISEE 3 in the distant upstream plasma [13]. These waves are also coincident with electron plasma oscillations and they possibly result from streaming electrons with a solar wind, rather than a bow shok origin in accordance with the instability analysis of Gary and Feldman [14]. Orlowski et al. [15] suggest that whistler waves observed in planetary foreshocks may not be the result of in situ generation, but rather these observations may simply result from propagation away from the shock.

Many species of pickup ions, both of interstellar origin and from an inner distributed source have been discovered using data from the solar wind ion composition spectrometer (SWICS) on Ulysses. Velocity distribution function of these ions were measured over heliocentric distances between 1 and 5 AU, both at high low latitudes and in the disturbed slow solar wind as well as the steady fast wind of the polar coronal holes. This has given at the first glance plasma properties of superthermal ions in various solar wind flows and has enabled the authors to study the chemical and in case of He the isotropic composition of the local interstellar cloud. Among the new findings are (a) the surprisingly weak pitch angle scattering of low rigidity, superthermal ions tending to strongly anisotropic velocity distributions in radial magnetic fields (b) the efficient injection and consequent acceleration of pickup ions especially, He/Sup$^+$/ and H$^+$ in the turbulent solar wind and (c) the discovering of a new extended source releasing Carbon, Oxygen, nitrogen and possibly other atoms as well as acid molecules in the inner solar system. Pickup ion measurements are now used to study the characteristics of the local interstellar clouds (LIC) and in particular, to determine accurately the abundance of atomic H, He, N, O and Ne, the isotopes of He and Ne as well as the ionization fractions of H and He in the LIC.

Electric field measurements at magnetospheric heights and shock region have given values of AC field along and perpendicular to Earth's magnetic field [16–18]. Various authors have discussed the role of parallel DC and AC electric fields on the whistler mode instability in the magnetosphere generally adopting plasma dispersion function which is based on anisotropic Maxwellian distributions to describe the resonant population [19, 20].

Recently Pandey [21] have studied effect of cold plasma injection on whistler mode instability, and temporal evolution of whistler instability [22] describing cold plasma by a simple Maxwellian distribution and hot/warm background plasma in the presence of perpendicular AC field by a generalized distribution function reducible to bi-Maxwellian and loss-cone in the magnetosphere of Uranus.

However in the natural space environment, plasma are generally observed to possess a non Maxwellian high-energy tail that can be well modeled by a generalized Lorentzian (Kappa) distribution function containing a spectral index κ. The Maxwellian and kappa distributions differ substantially in the high energy tail but differences become less significant as Kappa increases.

The growth (or damping) rate of a particular wave mode in a generalized Lorentzian plasma can be significantly different from that in a Maxwellian plasma. Particularly when the resonant particles that give rise to the growth or damping of the wave may have speeds that greatly exceeds the thermal speed.

Tripathi and Misra [23] have discussed cold plasma injection on background hot anisotropic bi-Lorentzian Kappa in the presence of perpendicular A.C. electric field. In their treatment They have not considered velocity of background plasma in the order of velocity of light. In the present paper the velocity of background plasma has been considered in the order of velocity of light, so the relativistic approach of mass changing with velocity has been taken in account. Thus the mathematical treatment of Tripathi and Misra [23] has changed from velocity to momentum form. At the same time they have not in corporated the analytical study in detail. In the present paper giving the analytical treatment in detail the effect of cold plasma beam on electromagnetic whistler wave with perpendicular AC electric field has been studied by using the unperturbed Lorentzian (Kappa) distribution in the Earth's atmosphere for relativistic plasma. The cold plasma has been described by a simple Maxwellian distribution where as Lorentzian (Kappa) distribution function has been derived for relativistic plasma with temperature anisotropy in the presence of a perpendicular AC electric field to form a hot/warm background. The dispersion relation is obtained by using the method of characteristic solutions and kinetic approach. An expression for the growth rate of a system with added cold plasma injection has been calculated. Results for representative values of parameters suited to the Earth's magnetosphere has been obtained. The salient features of the Analysis and the results obtained have been discussed.

2. DISPERSION RELATION AND GROWTH RATE

A homogeneous anisotropic collision less plasma in the presence of an external magnetic field $B_0 = (B_0 e_z)$ and an electric field $E_{ox} = E_o \sin vte_x$ is assumed. In interaction zone in homogeneity is assumed to be small. In order to obtain the particle trajectories. Perturbed distribution function and dispersion relation, the linearised Vlasov-Maxwell equations are used. Separating the equilibrium and non equilibrium parts, neglecting the higher order terms and following the techniques of Pandey et al. [24] the linearized Vlasov equations are given as:

$$v.\left(\frac{\delta f_0}{\delta r}\right) + \left(\frac{e_s}{m_e}\right)\left[E_0 \sin vt + \frac{(v \times B_0)}{c}\right].\left(\frac{\delta f_0}{\delta r}\right) = 0 \tag{1}$$

$$\left(\frac{\delta f_1}{\delta r}\right) + v.\left(\frac{\delta f_1}{\delta r}\right) + \left(\frac{F}{M_e}\right).\left(\frac{\delta f_1}{\delta r}\right) = S\left(r, v, t\right) \tag{2}$$

where the force

$$F = e\left[E_0 \sin vt + \frac{(v \times B_0)}{c}\right] = m\frac{dv}{dt} \tag{3}$$

where ν is AC frequency, and the dispersion relation is defined as

$$S(r, v, t,) = -\left(\frac{e_s}{m_e}\right)\left[E_1 + \frac{(v \times B_0)}{c}\right].\left(\frac{\delta f_1}{\delta r}\right) \tag{4}$$

where s denotes the type of electrons. Subscript '0' denotes the equilibrium values. The perturbed distribution function f_1 is determined by using the method of characteristic, which is

$$f_1(r,v,t) = \int_0^{\infty} S\left\{r_0\left(r,v,t,\right), v_0\left(r,v,t\right)\right), t-t'\} \, dt$$

we have transformed the phase space coordinate system for $(r,\, v,\, t)$ to $(r_0,\, v_0 t - t')$.

The relativistic particle trajectories that have been obtained by solving Equation (3) for given external field configuration are

$$\begin{aligned} X_0 \;&=\; X + \left(\frac{P_\perp \sin\theta}{\omega_c m_c}\right) - \left[P_\perp \sin\left\{\frac{\theta + \left(\frac{\omega_c t}{\beta}\right)}{\omega_c m_c}\right\}\right] \\ &\quad + \left[\frac{\Gamma_x \sin vt}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - v^2\right\}}\right] - \left[\frac{v\Gamma_x \sin\left(\frac{\omega_c t}{\beta}\right)}{\omega_c\left\{\left(\frac{\omega_c t}{\beta}\right) - v^2\right\}}\right] \end{aligned}$$

$$\begin{aligned} Y_0 \;&=\; Y - \left(\frac{P_\perp \cos\theta}{\omega_c m_c}\right) - \left[P_\perp \cos\left\{\frac{\theta + \left(\frac{\omega_c t}{\beta}\right)}{\omega_c m_c}\right\}\right] + \left(\frac{\Gamma_x}{v\omega_c}\right) \\ &\quad - \frac{\left\{1 + v2\beta2\cos\left(\frac{\omega_c t}{\beta}\right) - \omega_c^2 \cos vt\right\}}{\beta^2\left\{\left(\frac{\omega_c}{\beta}\right)^2 - v^2\right\}} \end{aligned}$$

$$z_0 = z - \frac{P_z}{\beta m_e} \tag{5}$$

and the velocities are

$$\begin{aligned} v_{xo} \;&=\; P_\perp \cos\left\{\theta + \frac{\left(\frac{\omega_c t}{\beta}\right)}{\beta m_e}\right\} \\ &\quad + \left[\frac{v\Gamma_x}{p\left\{\left(\frac{\omega_c}{\beta}\right)^2 - v^2\right\}}\right]\left\{\cos vt - \cos\left(\frac{\omega_c t}{\beta}\right)\right\} \end{aligned}$$

$$v_{yo} \;=\; P_\perp \sin\left\{\theta + \frac{\left(\frac{\omega_c t}{\beta}\right)}{\beta m_e}\right\}$$

$$+\left[\frac{\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2-\upsilon^2\right\}}\right]\left\{\left(\frac{\omega_c}{\beta}\right)\sin\upsilon t-\upsilon\sin\left(\frac{\omega_c t}{\beta}\right)\right\}$$

$$v_{zo}=\frac{P_z}{\beta m_e} \tag{6}$$

$$v_x=\frac{P_\perp\cos\theta}{\beta m_e},\quad v_y=\frac{P_\perp\sin\theta}{\beta m_e},\quad v_z=\frac{P_z}{\beta m_e}$$

$$\Gamma_x=\frac{eE_o}{m_e},\quad m_e=\frac{m_s}{\beta},\quad \beta=\sqrt{1-\frac{v^2}{c^2}},\quad \omega_c=\frac{eB_0}{m_e}$$

$P_\perp$ and P_z denote momenta perpendicular and parallel to the magnetic field. Using Equations (5), (6) and the Bessel identity and performing the time integration, following the technique and method of Misra and Pandey [13], the perturbed distribution function is found after some lengthy algebraic simplifications as:

$$\begin{aligned} f_1 \;=\; & -\left(\frac{ie_s}{me\beta\omega}\right)\sum J_s(\lambda_3)\exp i(m-n)\theta \\ & \left[\frac{J_mJ_nJ_pU*E_{1x}-iJmV*E_1+J_mJ_nJ_pW*}{\omega-\left(\frac{k_{||}P_z}{\beta m_e}+p\upsilon-\frac{(n+g)\omega_c}{\beta}\right)}\right] \end{aligned} \tag{7}$$

Due to the phase factor the solution is possible when $m=n$. Here.

$$\begin{aligned}
U^* \;&=\; \left(\frac{c_1P_\perp n}{\beta\lambda_1 m_e}\right)-\left(\frac{n\upsilon c_1 D}{\lambda_1}\right)+\left(\frac{p\upsilon c_1 D}{\lambda_2}\right)\\
V* \;&=\; \left(\frac{c_1P_\perp J_nJ_p}{\beta\lambda_1 m_e}\right)+c_1DJ_pJ_n\omega_c\\
W^* \;&=\; \left(\frac{n\omega_cFm_e}{k_\perp P_\perp}\right)+\left(\frac{\beta m_eP_\perp\omega\partial f_0}{\partial P_z}\right)+G\left\{\left(\frac{p}{\lambda_2}\right)-\left(\frac{n}{\lambda_1}\right)\right\}\\
C_1 \;&=\; \left\{\frac{(\beta m_e)}{P_\perp}\right\}\left(\frac{\partial f_0}{\partial P_\perp}\right)\left(\omega-\frac{k_{||}p_z}{\beta m_e}\right)+k_{||}\beta m_e\left(\frac{\partial f_0}{\partial P_\perp}\right)\\
D \;&=\; \left[\frac{\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2-\upsilon^2\right\}}\right]\\
F \;&=\; \frac{Hk_\perp P_\perp}{\beta m_e}\\
H \;&=\; \left\{\frac{(\beta m_e)^2}{P_\perp}\right\}\left(\frac{\partial f_0}{\partial P_\perp}\right)\left(\frac{P_z}{\beta m_e}\right)+\beta m_e\left(\frac{\partial f_0}{\partial P_z}\right)
\end{aligned}$$

$$G = \frac{Hk_{\perp}\upsilon\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}} \tag{8}$$

$$J_n(\lambda_1) = \frac{dJ_n(\lambda_1)}{d\lambda_1}$$

$$J_n(\lambda_2) = \frac{dJ_p(\lambda_2)}{d\lambda_2}$$

and the Bessel function arguments are defined as

$$\lambda_1 = \frac{k_{\perp}P_{\perp}}{\omega_c m_e}\frac{k_{\perp}\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}}$$

$$\lambda_3 = \frac{k_{\perp}\upsilon\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}}$$

The conductivity tensor $||\sigma||$ is found to be

$$\|\sigma\| = \frac{-i\sum\left(e^2/\beta m_e\right)^2\omega\ \int d^3PJ_g\left(\lambda_3\right)||s||}{\left[\omega - \left(\frac{k_{||}P_z}{\beta m_e}\right) - \left((n+g)\frac{\omega_c}{\beta}\right) + pv\right]}$$

where

$$||S|| = \begin{vmatrix} P_{\perp}J_n^2J_p\left(\frac{n}{\lambda_1}\right)U* & iP_{\perp}J_nV* & P_{\perp}J_n^2J_p\left(\frac{n}{\lambda_1}\right)W* \\ P_{\perp}J_nJ_nJ_p\left(\frac{n}{\lambda_1}\right)U* & iP_{\perp}J_nV* & P_{\perp}J_nJ_p\left(\frac{n}{\lambda_1}\right)W* \\ P_zJ_n^2J_p\left(\frac{n}{\lambda_1}\right)U* & iP_zJ_nV* & P_zJ_n^2J_p\left(\frac{n}{\lambda_1}\right)W* \end{vmatrix}$$

By using these in the Maxwell's equations we get the dielectric tensor,

$$\varepsilon_{ij} = 1 + \sum\left\{\frac{4\pi e_s^2}{\left(\beta m_e\right)^2\omega}\right\}\int\frac{d^3PJg\left(\lambda_3\right)||S||}{\left(\omega - \frac{k_{||}P_z}{\beta m_e}\right) - \left\{\frac{(n+g)\omega_c}{\beta}\right\} + pv}$$

For parallel propagating whistler mode instability, the general dispersion relation reduces to

$$\varepsilon_{11} \pm E_{12} = N^2$$

$$N^2 = \frac{k^2c^2}{\omega^2}$$

The dispersion relation for relativistic case with perpendicular AC electric field for $g = o$, $p = 1$, $n = 1$ is written as:

$$\frac{k^2c^2}{\omega^2} = 1 + \frac{4\pi e_s^2}{(\beta m_e)^2 \omega^2} \int \frac{d^3P}{\beta} \left[\frac{P_\perp}{2} - \frac{\upsilon \Gamma_x m_e}{2\left(\frac{\omega_c^2}{\beta^2} - \upsilon^2\right)}\right]$$
$$\left[\left(\beta\omega - \frac{k_{||}P_{||}}{m_e}\right)^2 \frac{\partial f_0}{\partial P_\perp} + \frac{P_\perp k_{||}}{m_e} \frac{\partial f_0}{\partial P_\perp}\right] \frac{1}{\beta\omega - \frac{k_{||}P_{||}}{m_e} - \omega_c + \beta\upsilon} \quad (9)$$

The bi-Lorentzian Kappa distribution function is given as

$$f_{ok} = \frac{N}{\pi^{3/2}k^{1/2}\theta_\perp^2\theta_{||}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa+1/2)} \left(1 + \frac{P_{||}^2}{\kappa\theta_{||}^2} \frac{P_\perp^2}{\kappa\theta_\perp^2}\right)^{-(\kappa+1)} \quad (10)$$

where $\theta_\perp$ and $\theta_{||}$ are perpendicular and parallel moment a for a temperature T. The plasma frequency

$$\omega_{pw}^2 = \frac{4\pi e^2 n_w}{m}$$

Substituting and using Equations (9), (10) and doing integration by parts the dispersion relation is found as,

$$\frac{k^2c^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2} \int \frac{d^3P}{\beta} \left[1 - \frac{\upsilon \Gamma_x m}{P_\perp \left(\frac{\omega_c^2}{\beta^2} - \upsilon^2\right)}\right]$$
$$\left[\frac{\left(\beta\omega - \frac{k_{||}P_{||}}{m}\right)}{\left(\beta\omega - \frac{k_{||}P_{||}}{m}\right)} - \frac{P_\perp^2}{2m^2c^2} \frac{\left(\omega^2 - k_{||}^2c^2\right)}{\left(\beta\omega - \frac{k_{||}P_{||}}{m} - \omega c + \beta\upsilon\right)^2}\right] f_{ok} \quad (11)$$

Doing some lengthy integrals the general dispersion relation becomes

$$\frac{k^2c^2}{\omega^2} = 1 + \frac{\omega_p^2}{\omega^2\beta\theta_\perp^2} \left[X_i \frac{\beta m\omega}{k_{||}\theta_{||}} \left(\frac{\kappa-1}{\kappa-3/2}\right) Z_{k-1} \left\{\left(\frac{\kappa-1}{\kappa}\right)^{1/2} \xi\right\}\right.$$
$$+ \left(X_2 - X_5 \frac{\omega^2}{k^2c^2}\right) \left\{1 + \xi \left(\frac{\kappa-1}{\kappa}\right)^{1/2} \left(\frac{\kappa-1}{\kappa-3/2}\right)\right\}$$
$$\left. Z_{k-1} \left\{\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\right\} \xi\right] \quad (12)$$

where,

$$X_1 = \theta_\perp^2 - \frac{\upsilon\Gamma_x m}{\left(\frac{\omega_c}{\beta}\right) - \upsilon^2} \sqrt{\pi\theta_\perp}$$

$$
\begin{aligned}
X_2 &= \theta_\perp^2 A_{kT} - \frac{\upsilon \Gamma_x m}{\left(\frac{\omega_c}{\beta}\right) - \upsilon^2} \sqrt{\pi \theta_\perp} \left(\frac{\theta_\perp^2}{\theta_{||}^2} \frac{1}{2} - 1 \right) \\
X_3 &= \frac{\theta_\perp^2}{\theta_{||}^2} \left[\theta_\perp^2 - \frac{\upsilon \Gamma_x m}{\left(\frac{\omega_c}{\beta}\right) - \upsilon^2} \sqrt{\pi \theta_\perp} \left(\frac{\theta_\perp^2}{\theta_{||}^2} \frac{1}{2} - 1 \right) \right] \\
A_{kT} &= \frac{\theta_\perp^2}{\theta_\perp^2} - 1 \\
\theta_\perp &= \left(\frac{2k-3}{k\theta_\perp^2} \right)^{1/2} \left(\frac{k_B T_\perp}{m} \right)^{1/2} \\
\theta_{||} &= \left(\frac{2k-3}{k} \right)^{1/2} \left(\frac{k_B T_{||}}{m} \right)^{1/2}
\end{aligned}
$$

$\theta_{||}$ and $\theta_\perp$ are respective thermal speeds parallel and perpendicular to the background magnetic field.

$$
\xi = \frac{\beta m \omega - m \omega c + \beta m \upsilon}{k_v \theta_{||}} \tag{13}
$$

Using the dispersion relation for cold electron plasma, the total dispersion relation becomes.

$$
\begin{aligned}
\frac{k^2 c^2}{\omega^2} &= 1 + \frac{\omega_p^2}{\omega^2 \beta \theta_\perp^2} \left[X_i \frac{\beta m \omega}{k_{||} \theta_{||}} \left(\frac{\kappa - 1}{\kappa - 3/2} \right) Z_{k-1}^* \left\{ \left(\frac{\kappa - 1}{\kappa} \right)^{1/2} \xi \right\} \right. \\
&\quad + \left(X_2 - X_5 \frac{\omega^2}{k^2 c^2} \right) \left\{ 1 + \xi \left(\frac{\kappa - 1}{\kappa} \right)^{1/2} \left(\frac{\kappa - 1}{\kappa - 3/2} \right) \right\} \\
&\quad \left. Z_{k-1}^* \left\{ \left(\frac{\kappa - 1}{\kappa} \right)^{1/2} \right\} \xi \right] \frac{\omega_{pc}^2 \omega}{\omega - \omega_c}
\end{aligned}
$$

For real k and substituting

$$
\frac{k^2 c^2}{\omega^2} \gg 1
$$

And using the expression of $Z_k^*(\xi)$ in the limit

$$
\begin{aligned}
Z_k^*(\xi) &= \frac{1 k! k^{\left(k - \frac{1}{2}\right)} \sqrt{\pi}}{\Gamma_{(k-1/2)} \xi^2 (k+1)} \left\{ 1 - \frac{k(k+1)}{\xi^2} + \ldots \right\} \\
&\quad - \left(\frac{2k-1}{2k} \right) \frac{1}{\xi} \left(1 + \frac{k}{2k - 1\xi} + \ldots \right)
\end{aligned} \tag{14}
$$

For $|\xi| \to \infty$.

The expression for growth rate for real frequency ω_r in dimensionless form is found to be

$$\frac{\Gamma}{\omega_c} = \frac{\frac{\sqrt{\pi}(k-1)\kappa^{\kappa-1/2}}{\beta(k-3/2)!k}\left(\frac{X_2}{X_1} - k_3\right) k_4^3 \left\{-\left(\frac{k_4}{k}\right)\right\}^{-2k}}{1 + \beta X_4 + \frac{\kappa}{\kappa-3/2}\frac{k^2}{2}\left(\frac{1+\beta X_4}{k_4^2} + \frac{2X_4 X_3 \theta_{||}^2}{X_1 k^2 c^2}\right)} - \frac{k^2}{k_4}\left(\frac{x_2}{x_1} - k_3\right) \quad (15)$$

$$X_3 = \frac{k^2}{\delta\beta_1}\left[(1 + \beta X_4) + \frac{X_2}{X_1}\frac{\beta_1}{1 + \beta X_4}\right] \quad (16)$$

where

$$k_3 = \frac{\beta X_3}{k_4} + \frac{X_4}{X_1}\frac{X_3^2}{k^2}\frac{\theta_{||}^2}{c^2}, \quad k = \frac{k_{||}\theta_{||}}{m_e \omega_c}, \quad k_4 = 1 - \beta X_3 + \beta X_4,$$

$$X_3 = \frac{\omega_r}{\omega_c}, \quad X_4 = \frac{-\beta \upsilon}{\omega_c}, \quad \beta_1 = \frac{k_B T_{||}\mu_0 n_0}{B_0^2}$$

$$\delta = \left(1 + \frac{\omega_{pc}^3}{\omega_{pw}^2}\right) = \left(1 - \frac{n_c}{n_w}\right) \quad (17)$$

when the relativistic factor is not considered, that is when the velocity of plasma does not approach velocity of light, Then $m_s = m_e$ and the expressions for Growth rate and real frequency reduce to Tripathi and Misra [23].

3. RESULTS AND DISCUSSION

For numerical evaluation of normalized growth rate and real frequency of relativistic whistler mode in the presence of perpendicular AC electric field has been analyzed for Kappa distribution function of electron density in the magnetosphere for a system with added cold plasma injection using Equations (15) and (16) respectively. Following plasma parameters have been considered. $n_c/n_w = 10$, 20, 30, $B_0 = 8 \times 10^{-9}T$, $A_T = [(T_\perp/T_{||}) - 1] = 0.25$, 0.5, 0.75, $\kappa = 2$, 3, 4, relativistic factor $b_1 = v/c = 0.3$, 0.6, 0.9, $E_0 = 20\,\text{mV/m}$. AC field frequency v varies from zero to 400 Hz. According to this choice of plasmaparameters, the explanations and details of results are given as follows.

Figure 1 depicts the variation of normalized growth rate and real frequency with respect to normalized $\overline{k}$ for various values of temperature anisotropy for kappa distribution index $\kappa = 2$. At this location the growth rate as well as the bandwidth increases with the increase of the temperature anisotropy and maxima is shifted towards the higher $\overline{k}$ values. It is clear from the figure that the temperature anisotropy is the main source of energy to drive the excitation of the wave. Lorentzian (Kappa) plasma series expansion brings change in perpendicular thermal velocity $\theta_\perp$. Therefore any change in $\theta_\perp$ shall affect marginally $T_\perp$, affecting temperature anisotropy terms. Temperature anisotropy being the primary source of instability gets further modified by Lorentzian (Kappa) distribution function, giving rise to further increase in growth rate. recently it was found that superthermal electron in Kappa distribution modifies the intensity and Doppler frequency of electron plasma lines. The inclusion of temperature anisotropy in Lorentzian (Kappa) plasma can explain the observed higher frequencies spectrum of whistler waves [25].

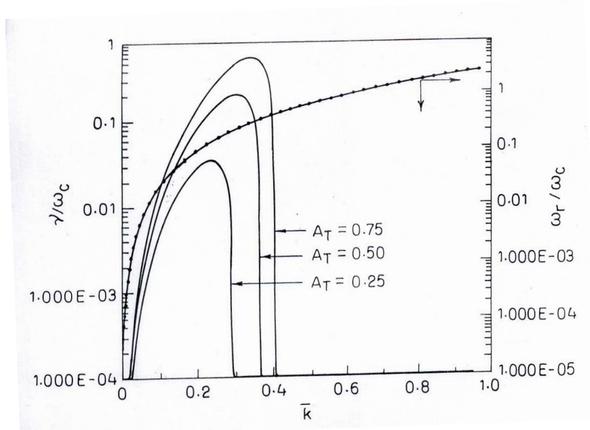

Figure 1. Variation of growth rate and real frequency with respect to $\overline{k}$ for various values of temperature anisotropy AT for Lorentzian Kappa distribution at spectral index $\kappa = 2$ at other fixed plasma parameters.

Figure 2 shows variation of normalized growth rate and real frequency versus $\overline{k}$ for various values of AC electric field frequency for other fixed plasma parameters. The growth rate increases with increase of the value of a.c. frequency, maxima shifts to lower values of $\overline{k}$. it means that the a.c. frequency modifies resonance frequency. The increase of AC frequency increases the growth rate due to the negative exponential of Landau damping. The perpendicular electric field which modifies the perpendicular velocity contributing to the energy exchange contributes significantly to the emission of VLF signals and can explain the low frequency side of the spectrum. The energy exchange between electrons, the components of the wave electric field and the impressed AC field perpendicular to the magnetic field mainly contributes to the cyclotron growth or the damping of the waves.

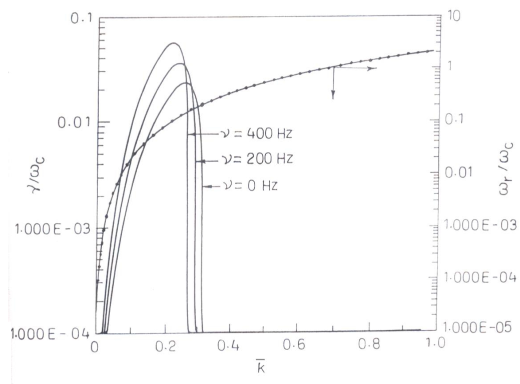

Figure 2. Variation of growth rate and real frequency with respect to $\overline{k}$ for various values of AC electric field frequency for other fixed plasma parameters.

Thus the frequency of the perpendicular AC electric field brings the maxima to different $\overline{k}$ as if the resonant charged particles were oscillating at different cyclotron frequencies and absorbing energy and thus growing.

Figure 3 exhibits variation of normalized growth rate and real frequency versus $\overline{k}$ for various values of the ratio n_c/n_w number density of cold and warm electrons at other fixed plasma parameters. The dispersive properties of the whistler waves are known to depend sensitively on this ratio of plasma density. It is clear as the growth rate as well as the band width increases with n_c/n_w. The maxima also shifts towards the higher value of $\overline{k}$ with increase of this ratio. In the absence of an AC field the growth rate increases with the increase of n_c/n_w [20]. The result is the same for the lower values of the AC frequency. This shows that at lower AC frequencies, as the density of the cold plasma increases, more and more energy is transmitted by the cold plasma particles to the wave during their interaction as such the wave grows.

Figure 4 shows the variation of normalized growth rate and real frequency with $\overline{k}$ for variation of the relativistic factor $b_1 = (v/c)$. With the increase of the relativistic factor the growth rate increases and the band width widens. This shows that the velocity of the energetic electrons have triggering effect on the growth of the wave.

Figure 5 shows the variation of normalized growth rate and real frequency with $\overline{k}$ for various values of spectral index κ. As κ increases the value of maximum growth rate increases and band width shrinks towards higher wave number. For $\kappa \rightarrow \infty$ the value of normalized growth rate approaches the value of growth

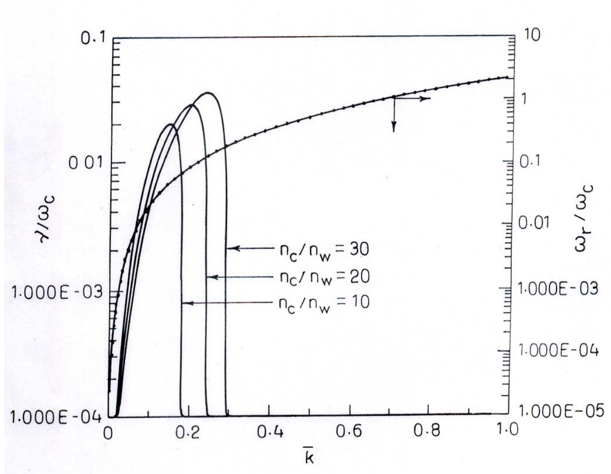

Figure 3. Variation of growth rate and real frequency with respect to $\overline{k}$ for various values of the ratio n_c/n_w at other fixed plasma parameters.

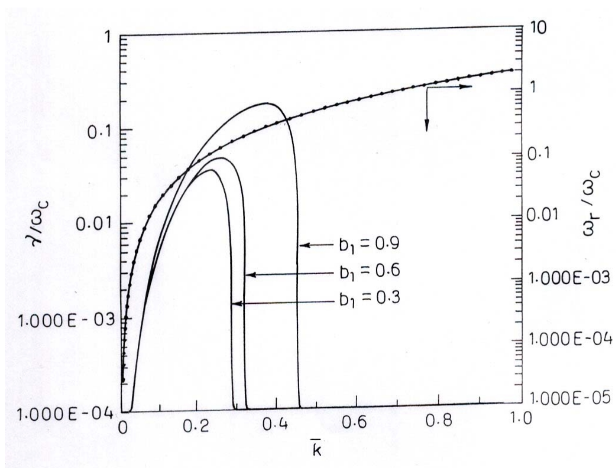

Figure 4. Variation of growth rate and real frequency with respect to $\overline{k}$ for various values of relativistic factor b_1 at other fixed plasma parameters.

rate for Maxwellian distribution function. Although the temperature anisotropy is the main source to drive instability. This effect remains basically applicable to the Lorentzian (Kappa) plasma also, except that the limit of temperature anisotropy in this case is little higher because of series solution involving κ.

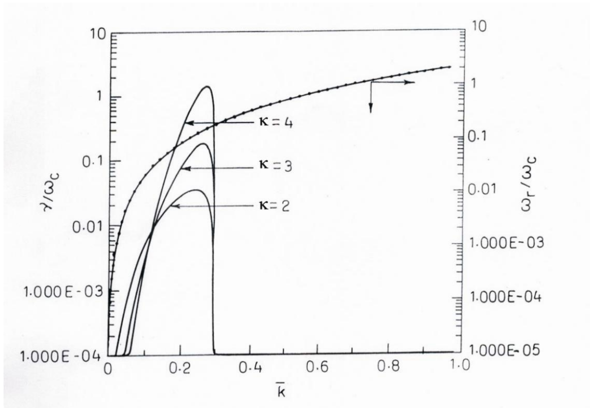

Figure 5. Variation of growth rate and real frequency with respect to $\bar{k}$ for various values of spectral index Kappa at other fixed plasma parameters.

4. CONCLUSIONS

The velocity of background plasma has been considered in the order of velocity of light, so the relativistic approach of mass changing with velocity has been taken in account. Thus changing the mathematical treatment from velocity to momentum form in detail, an expression for the growth rate of the system has been calculated and the results for representative values of the parameters suited to Earth's magnetosphere has been obtained. It is inferred that A.C. field frequency modifies the resonance criteria which influences the growth rate. Also the growth rate increases by increasing the number density of cold plasma and Temperature anisotropy. Plasma particles having higher Kappa spectral index provide additional source of energy. In addition to the other factors the relativistic plasma modifies the growth rate and also shifts the wave band significantly. The relativistic electrons by increasing the growth rate and widening the band width may explain a wide frequency range of whistler emissions in the Earth's magnetosphere.

REFERENCES

1. Heppner, J. P., M. Sugiura, T. L. Skillman, B. G. Ledley, and M. Campbell, "OGO A magnetic field observations," *J. Geophys Res.*, Vol. 72, 5417, 1967.

2. Fairfield, D. H., "Whistler waves observed upstream from collision less shocks," *J. Geophys. Res.*, Vol. 79, 1368, 1974.
3. Gurgiolo, C., K. K. Wong, and D. Winske, "Low and high frequency waves generated by gyrophase bunched ions at oblique shocks," *Geophys. Res. Lett.*, Vol. 20, 783, 1993.
4. Hoppe, M. M. and C. T. Russell, "Whistler mode wave packets in the Earth's foreshock region," *Nature*, Vol. 287, 417, 1980.
5. Hoppe, M. M., C. T. Russell, T. E. Eastman, and E. W. Greenstadt, "Characteristics of the VLF waves associated with upstream ion beams," *J. Geophys. Res.*, Vol. 87, 643, 1982.
6. Tsurutani, B. T., R. M. Thorne, E. J. Smith, J. T. Gosling, and H. Matsumoto, "Steepend magnetospheric waves at comet Giacolaini-Zinner," *J. Geophys Res.*, Vol. 92, 11,074, 1987.
7. Kennel, C. F. and H. E. Petschek, "Limit on Stably trapped particle fluxes," *J. Geophys. Res.*, Vol. 71, 1, 1966.
8. Anderson, R. R., G. K. Parks, T. E. Eastman, D. A. Gurnett, and L. A. Frank, "Plasma waves associated with energetic particles streaming into the solar wind from the Earth's bow shock," *J. Geophys. Res.*, Vol. 86, 4493, 1981.
9. Greenstadt, E. W., R. W. Fredericks, C. T. Russel, F. L. Scarf, R. R. Anderson, and D. A. Gurnett, "Whistler mode wave propagation in the solar wind near the bow shock," *J. Geophys, Res.*, Vol. 86, 4511, 1981.
10. Toker, R. L., D. A. Gurnett, and W. C. Feldman, "Whistler mode turbulence generated by electron beams in Earth's bow shock," *J. Geophys. Res.*, Vol. 89, 105, 1984.
11. Tokar, R. L. and D. A. Gurnett, "The propagation and growth of whistler mode wave generated by electron beam in Earth's bow shock," *J. Geophys. Res.*, Vol. 90, 105, 1985.
12. Feldman, W. C., R. C. Anderson, S. J. Bame, S. P. Gary, J. T. Gosling, D. J. McComas, M. F. Thomson, G. Paschmann, and M. M. Hoppe, "Electron velocity distributions near the Earth's bow shock," *J. Geophys. Res.*, Vol. 88, 96, 1983.
13. Kennel, C. F., F. L. Scarf, F. V. Coroniti, R. W. Fredericks, D. A. Gurnnett, and E. J. Smith, "Correlated whistler and electron plasma oscillation burst detected on ISEE 3," *Geophys. Res. Lett.*, Vol. 7, 129, 1980.
14. Gary, S. P. and W. C. Feldman, "Solar wind heat flux regulation by the whistler instability," *J. Geophys Res.*, Vol. 82, 1087, 1977.
15. Orlowski, D. S., G. K. Crawford, and C. T. Russel, "Upstream waves at mercury, venus and earth, comparisons of the properties

of one Hertz waves," *Geophys Res. Lett.*, Vol. 17, 2293, 1990.

16. Mozer, F. S., R. B. Torbert, U. V. Fahleson, C. Falthammar, A. Gonfalone, A. Pedersen, and C. T. Russel, "Electric field measurements in the solar wind bow shock, magnetosheath, magnetopause and magnetosphere," *Space Sci. Rev.*, Vol. 22, 791, 1978.
17. Wygant, J. R., M. Bensadoun, and F. S. Mozer, "Electric field measurements at sub critical, oblique bow shock crossings," *J. Geophys. Res.*, Vol. 92, 11, 109, 1987.
18. Lindquist, P. A. and F. S. Mozer, "The average tangential electric field at the noon magnetopause," *J. Geophys. Res.*, Vol. 95, 17, 137, 1990.
19. Misra, K. D. and R. S. Pandey, "Generation of whistler emissions by injection of hot electrons in the presence of a perpendicular A.C. electric field," *J. Geophys. Res.*, Vol. 100, 19405, 1995.
20. Misra, K. D. and B. D. Singh, "On the modification of whistler mode instability in the magnetosphere in the presence of parallel electric field by cold plasma injection," *J. Geophys. Res.*, Vol. 85, 5138, 1980.
21. Pandey, R. P., S. M. Karim, K. M. Singh, and R. S. Pandey, "Effect of cold plasma injection on whistler mode instability triggered by perpendicular AC electric field at Uranus," *Earth Moon and Planets*, Vol. 91, 195, 2002.
22. Pandey, R. P., R. S. Pandey, and K. D. Misra, "Temporal evolution of whistler instability due to cold plasma injection in the presence of perpendicular AC electric field in the Magnetosphere of Uranus," *Earth Moon and Planets*, Vol. 91, 209, 2002.
23. Tripathi, A. K. and K. D. Misra, "Whistler mode instability in a Lorentzian (κ) magnetoplasma in the presence of Perpendicular A.C. electric field and cold plasma injection," *Earth, Moon and Planets*, Vol. 88, 131, 2002.
24. Pandey, R. P., K. M. Singh, and R. S. Pandey, "A theoretical study of the whistler mode instability at the Uranian Bow Shock," *Earth, Moon and Planets*, Vol. 87, 59, 2001.
25. Tripathi, A. K. and K. D. Misra, "Computer analysis of whistler mode instability in the presence of perpendicular AC electric field or a Lorentzian (Kappa) magnetoplasma," *Ind. J. Radio & Space Phys.*, Vol. 30, 279, 2001.