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COHERENT FIELD APPROXIMATION OF PLANE WAVE SCATTERING FROM 1D-ROUGH MIRRORS

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Abstract—For an harmonic plane wave impinging on a perfectly reflecting smooth plane the total field, incident and reflected, satisfying on this plane a Dirichlet or Neumann boundaray condition, has an integral representation that we extend to the specular reflection from a perfectly reflecting rough plane. To make this generalization possible, some constraints must be imposed on the wavelength of the incident field and on the rough amplitude to make the diffuse field negligible so that only the coherent field is important and we may use the fact that the coherent power is identical to that of a smooth surface. This generalized integral representation supplies an approximation of the coherent field valid far from the rough plane. We limit the discussion to acoustic, TE, TM electromagnetic wave incident on 1D-perfectly reflecting rough planes with roughness described by zig-zag functions piecewise linear with opposite slop on adjacent intervals.

1. INTRODUCTION

Wave scattering from rough surfaces has been of interest in physics and engineering for many years because of its large number of applications in optics, acoustics, radiowave propagation and radar techniques. Theoretical investigations in such domains involve a trade off between rigorous mathematical and physical, analytical and computational treatments. So, a variety of approximate and numerical methods are employed to compute solutions of these scattering problems, discussed at length in published works among which [1, 8] are some of the most recent ones with an important bibliography in [1].

Now, the simplest and one of the most important boundary value problem in acoustics and electromagnetism is the scattering of

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a uniform plane wave incident on a plane boundary S between two different media, because its solutions can be used to analyze scattering problems on solids large compared with wavelength. We are interested here in scalar and TE, TM electromagnetic wave scattering from 1D-rough surfaces when the roughness amplitude is small enough to make negligible the diffuse structured field so that we have to deal with coherent reflection in the specular direction: a kind of situation that would expect a far observer eager to optimize, by playing upon wavelength and direction of observation, the signal-to-noise ratio in the reception of the backward radiation coming from an illuminated surface.

So, we first consider an harmonic plane wave impinging on a perfectly reflecting 1D-smooth plane $z = 0$ and we prove that the total field incident plus reflected satisfying a Dirichlet or Neumann boundary condition on the smooth plane has an integral representation generalizing the usual angular spectrum representation [2]. The same problem is investigated when the smooth plane is changed into a rough plane and we prove that the far total field in the specular direction has also an integral representation when the roughness amplitude h is small and when the wavelength λ of the incident field is such as $\lambda^{-1}h \ll 1$ so that the diffracted field may be neglected. This integral representation is a coherent field approximation which can be called geometrical theory of rough reflection. Roughness is characterized by a function generalizing the zig-zag function recently introduced by Murakami [9] in a different context and which is a description well suited to the integral representation approach.

This paper is organized as follows: Section 2 is devoted to the integral representation of the total field for harmonic plane waves impinging on perfectly reflecting, smooth 1D-planes and to its extension to the total field in the specular direction for fields incident on a rough plane. The constraints to impose on roughness amplitude and wavelength to justify this approach are carefully discussed. We give in Section 3 some generalizations of the Murakami zig-zag functions to describe roughness and we analyse their properties useful in the integral representation of the total field. We present in Section 4 an application to the total far field coming from the specular reflection of harmonic plane waves on perfectly reflecting, weakly structured Murakami 1D-planes. Conclusive comments are given in Section 5.

2. INTEGRAL REPRESENTATION APPROACH

Suppose that the harmonic plane wave $\psi^i(x, z) = \exp(ik_x x + ik_z z)$ impinges on a perfectly reflecting smooth plane $z = 0$ on which the total field, incident and reflected

$$\psi(x,\,z) = \psi^i(x,\,z) + \psi^r(x,\,z) = \exp(ik_x x)\sin(k_z z) \tag{1}$$

satisfies the Dirichlet boundary condition $[\psi(x,\,z)]_{z=0} = 0$. Then, with a Green's function g ful-filling the boundary condition $[g(x,\,z;\,x',\,z')]_{z=0} = 0$ the total field has the integral representation

$$\psi(x,\,z) = \int_{-\infty}^{\infty} dx'[g(x,\,z;\,x',\,z')\partial_{z'}\psi(x',\,z')]_{z'=0} \tag{2}$$

in which [10]

$$\begin{aligned} g(x,\,z;\,x',\,z') \;=\; (1/4\pi) \int_{-\infty}^{\infty} d\beta k_z^{-1} \exp(i\beta x - i\beta x') \\ [\exp(i\kappa_z|z-z'|) - \exp(i\kappa_z|z^\dagger - z'|)] \end{aligned} \tag{2a}$$

where $z^\dagger = -z$ belongs to the image point and $\kappa_z^2 = k^2 - \beta^2$ with $k^2 = k_x^2 + k_z^2$.

The substitution of (2a) into (2) gives

$$\begin{aligned} \psi(x,\,z) &= \int_{-\infty}^{\infty} d\beta \exp(i\beta x) F(\beta), \\ F(\beta) &= (1/4\pi) \int_{-\infty}^{\infty} dx' \exp(-i\beta x')[A(x',\,z')]_{z'=0} \end{aligned} \tag{3}$$

in which the kernel $A(x',\,z')$ is

$$A(x',\,z') = \kappa_z^{-1}\partial_{z'}\psi(x',\,z')[\exp(i\kappa_z|z-z'|) - \exp(i\kappa_z|z^\dagger - z'|)] \tag{4}$$

and since according to (1)

$$\begin{aligned} \partial_{z'}\psi(x',\,z') =& k_z \exp(ik_x x')\cos(k_z z') \\ A(x',\,z') =& k_z/\chi_z \exp(ik_x x')\cos(k_z z') \\ & [\exp(i\chi_z|z-z'|) - \exp(i\chi_z|z^\dagger - z'|)] \end{aligned} \tag{4a}$$

Assuming the incident field in the half space $z > 0$ so that $z^\dagger < 0$ and $|z - z'|_{z'=0} = -|z^\dagger - z'|_{z'=0} = z$, we get

$$\begin{aligned} [A(x',\,z')]_{z'=0} &= 2k_z/\chi_z \exp(ik_x x')\sin(k_z z), \\ F(\beta) &= k_z/\chi_z \sin(\chi_z z)\delta(k_x - \beta) \end{aligned} \tag{5}$$

where δ is the Dirac distribution and since $\kappa_z^2 = k^2 - \beta^2$ the form factor $F(\beta)$ reduces to

$$F(\beta) = \sin(k_z z)\delta(k_x - \beta) \tag{5a}$$

and substituting (5a) into (3) gives the total field (1) justifying the integral representation (2).

We now suppose that $\psi^i(x,\, z)$ impinges on a weakly rough, perfectly reflecting 1D-plane endowed with the roughness function $\zeta(x)$ and we are interested in the total field $\phi(x,\, z)$ far from the rough plane and in the direction of the specular reflection. We want also to get $\phi(x,\, z)$ from an integral representation similar to (2). To reach this result we impose the following constraints:

1. The mean roughness amplitude $\bar{\zeta}$ is zero making possible to use the Green's function (2a) with the image point $z^\dagger$ taken with respect to $\bar{\zeta}$. In addition, the mean square amplitude $\bar{\zeta}^2$ is very small justifying $0(\bar{\zeta}^2)$ approximations in which 0 is the Landau symbol.
2. The wavelength λ of the incident field satisfies the inequality $\lambda^{-1}h \ll 1$ in which $h = \max|z(x)|$ so that the diffuse field may be neglected reducing scattering to reflection.
3. For an integral representation approach similar to (2) of the specular reflection when the reflected harmonic plane wave is $\exp(ik_x x - ik_z z)$, an approximation of the total field in the integrand is given by the expression (1) of the total field for a smooth plane, in agreement with the fact that the coherent power is identical to that of a smooth surface [11]. Of course, $\phi(x,\, z)$ does not satisfy the Dirichlet boundary condition on the rough surface but this is not an obstacle to the use of the integral relation in the far field.

Assuming these conditions fulfilled, the integral representation

$$\phi(x,\, z) = \int_{-\infty}^{\infty} dx' [g(x,\, z;\, x',\, z')\partial_{z'}\psi(x',\, z')]_{z'=z(x')} \tag{6}$$

in which ψ, g are the expressions (1), (2a) gives a coherent field approximation of the far total field in the specular reflection direction. The integral (6) is similar to (2) except that the square bracket is calculated on the rough surface $z' = \zeta(x')$ and no more on the smooth plane $z' = 0$.

Then, taking into account (1) and (2a), we may write (6)

$$\phi(x,\, z) = \int\limits_{-\infty}^{\infty} d\beta \exp(i\beta x) F_1(\beta),$$
$$F_1(\beta) = (1/4\pi) \int\limits_{-\infty}^{\infty} dx' \exp(-i\beta x')[A(x',\, z')]_{z'=z(x')} \tag{7}$$

with $A(x',\, z')$ given by (4a) and on the rough plane $z' = \zeta(x')$ we have the $0(\bar{\zeta}^2)$ approximation

$$[A(x',\, z')]_{z'=z(x')} = [A(x',\, z')]_{z'=0} + \zeta(x')[\partial_{z'} A(x',\, z')]_{z'=0} + 0(\bar{\zeta}^2) \tag{8}$$

$[A(x',\, z')]_{z'=0}$ is the expression (5) and using the relations $\partial_{z'}|z - z'|_{z'=0} = \partial_{z'}|z^\dagger - z'|_{z'=0} = 1$, we get

$$[\partial_{z'} A(x',\, z')]_{z'=0} = -2 \exp(ik_x x') \sin(\chi_z z) \tag{8a}$$

so that taking into account (5) and (8a) the approximation (8) is now with the 0-symbol deleted

$$[A_S(x',\, z')]_{z'=z(x')} = 2k_z/\chi_z \exp(ik_x x') \sin(\chi_z z)[1 - \chi_z \zeta(x')] \tag{9}$$

Substituting (9) into the expression (7), $F_1(\beta)$ becomes with $F(\beta)$ supplied by (5a)

$$F_s(\beta) = F(\beta) - (k_z/2\pi) \sin(\chi_z z) \gamma(k_x - \beta) \tag{10}$$

$$\gamma(k_x - \beta) = \int\limits_{-\infty}^{\infty} dx' \exp[i(k_x - \beta)x']\zeta(x') \tag{10a}$$

Finally with (10) the integral representation (7) of $\phi(x,\, z)$ becomes to the $0(\bar{\zeta}^2)$ order

$$\phi(x,\, z) = \psi(x,\, z) - (k_z/2\pi) \int\limits_{-\infty}^{\infty} d\beta \sin(\kappa_z z) \exp(i\beta x) \gamma(k_x - \beta) \tag{11}$$

in which $\psi(x,\, z)$ is the field (1) and $\kappa_z^2 = k_x^2 + k_z^2 - \beta^2$.

As previously said, this approximation of the total field in the specular direction does not satisfy the Dirichlet boundary condition $[\phi(x,\, z)]_{z'=z(x')} = 0$ since to the $0(\bar{\zeta}^2)$ order

$$[\phi(x,\, z)]_{z'=z(x')} = [\phi(x,\, z)]_{z'=0} + \zeta(x)[\partial_z \phi(x,\, z)]_{z'=0} + 0(\bar{\zeta}^2) \tag{12}$$

with according to (10)

$$[\phi(x,\, z)]_{z=0} = 0, \quad [\partial \phi_z(x,\, z)]_{z=0} = k_z \exp(ik_x x) + 0(\bar{\zeta}) \tag{12a}$$

It is easy to transpose this integral representation to a total field $\psi^\circ(x, z) = \exp(ik_x x)\cos(k_z z)$ satisfying the Neumann boundary condition $[\partial_z \psi^\circ(x, z)]_{z=0} = 0$ on the perfectly reflecting smooth plane $z = 0$

$$\psi^\circ(x, z) = -\int dx'[\psi^\circ(x', z')\partial_{z'} g^\circ(x, z; x', z')]_{z'=0} \tag{13}$$

with the Green's function [10]

$$\begin{aligned} g^\circ(x, z; x', z') \;=\; (1/4\pi) \int_{-\infty}^{\infty} d\beta k_z^{-1} \exp(i\beta x - i\beta x') \\ [\exp(i\chi_z|z - z'|) + \exp(i\chi_z|z^\dagger - z'|)] \end{aligned} \tag{13a}$$

so that

$$\psi^\circ(x, z) = \int_{-\infty}^{\infty} d\beta \exp(i\beta x) F^\circ(\beta), \quad F^\circ(\beta) = \cos(\chi_z z)\delta(k_x - \beta) \tag{14}$$

And, provided that the previous three conditions are fulfilled, the coherent field approximation of the total far field in the specular direction is just obtained by changing $\sin(\kappa_z z)$ into $\cos(k_z z)$ in (11) and ψ into ψ°:

$$\psi^\circ(x, z) = \psi^\circ(x, z) - (k_z/2\pi) \int_{-\infty}^{\infty} d\beta \cos(\chi_z z) \exp(i\beta x)\gamma(k_x - \beta) \tag{15}$$

In both cases, we need the characteristic function $\gamma(k_x - \beta)$ of the roughness amplitude distribution.

3. GENERALIZED ZIG-ZAG FUNCTIONS

The zig-zag functions recently introduced by Murakami [9] to describe fracture lines in stressed materials can be generalized to get

$$\zeta(x) = \sum_{n=-\infty}^{\infty} M_n(x) V_n(x) \tag{16}$$

$$\begin{aligned} M_n(x) &= h_n a_n^{-1}(-1)^n[2x - (2n - 1)a_n], \\ V_n(x) &= U[x - (n - 1)a_n] - U(x - na_n) \end{aligned} \tag{16a}$$

in which U is the unit step function $U(x) = 1$ for $x \geq 0$ and $U(x) = 0$ for $x < 0$. The function (16) is linear inside any interval $-a_n \leq x \leq a_n$

and takes its extremum values $\pm h_n$ for $|x| = |a_n|$ so that $\bar{\zeta} = 0$ in agreement with the first condition to be satisfied by roughness.

The zig-zag function (16) with a judicious choice of the a_n, h_n parameters supplies many different approximations for the description of 1D-rough planes with in the more regular case $h_n = h$, $a_n = a$. In addition, to avoid mathematical difficulties attached to infinite series, we may either assume that h_n is strongly decreasing with increasing $|n|$ from some rank N or that h_n is weighted with a convenient function of n. These two possibilities are used here and we first consider the zig-zag function

$$\zeta(x) = \sum_{n=-N}^{N} M_n(x) V_n(x) \tag{17}$$

$$\begin{aligned} M_n(x) &= ha^{-1}(-1)^n[2x - (2n-1)a], \\ V_n(x) &= U[x - (n-1)a] - U(x - na) \end{aligned} \tag{17a}$$

To apply the integral representations (11), (15) to harmonic plane wave scattering from the 1D-rough surface (17), we need as stated in Section 2 the characteristic function $\gamma(t)$ of the roughness amplitude distribution

$$\gamma(t) = \int_{-\infty}^{\infty} dx \exp(itx)\zeta(x) \tag{18}$$

$$= ha^{-1} \sum_{n=-N}^{N} (-1)^n \int_{(n-1)a}^{na} dx \exp(itx)[2x - (2n-1)a] \tag{18a}$$

A simple calculation gives

$$\begin{aligned} &\int_{(n-1)a}^{na} dx \exp(itx)[2x - (2n-1)a] \\ &= \ 2t^{-2} \exp(inat)[1 - iat/2 - (1 + iat/2)\exp(-iat)] \end{aligned} \tag{19}$$

and substituting (19) into (18) we get

$$\begin{aligned} \gamma(t) &= (2h/at^2)b(t) \sum_{n=-N}^{N} (-1)^n \exp(inat), \\ b(t) &= 1 - iat/2 - (1 + iat/2)\exp(-iat) \end{aligned} \tag{20}$$

But the summation in (20) is easy to perform

$$\begin{aligned}\sum_{n=-N}^{N}(-1)^n\exp(inat) &= \sum_{n=0}^{N}\varepsilon_n\cos(nat),\ \varepsilon_0=1,\ \varepsilon_n=2(-1)^n n>0\\ &= (-1)^N\cos[(N+1/2)at]/\cos(at/2) \qquad (20a)\end{aligned}$$

and finally

$$\gamma(t)=(-1)^N(4h/at^2)b(t)\cos[(N+1/2)at)]/\cos(at/2) \qquad (21)$$

We now consider the second possibility mentioned earlier to deal with infinite series

$$\begin{aligned}\zeta^*(x) &= \sum_{n=-\infty}^{\infty}M_n^*(x)V_n(x),\\ M_n^*(x) &= h\exp(-\alpha^2n^2)a^{-1}(-1)^n[2x-(2n-1)a]\end{aligned} \qquad (22)$$

in which the roughness amplitude h decreases with increasing $|n|$ at a rate fixed by the parameter α. The characteristic function becomes

$$\begin{aligned}\gamma^*(t) &= (2h/at^2)b(t)\sum_{n=-\infty}^{\infty}(-1)^n\exp(-\alpha^2n^2)\exp(inat)\\ &= (2h/at^2)b(t)\exp[-(\pi+at)^2/4\alpha^2]\sum_{n=-\infty}^{\infty}\exp[-\alpha^2(n-\mu)^2],\\ &\mu=-ia^{-2}(\pi+at) \qquad (23)\end{aligned}$$

and, approximating the sum $\sum\limits_{n=-\infty}^{\infty}$ with the integral $\int\limits_{-\infty}^{\infty} dn$, we get

$$\gamma^*(t)=(2h/at^2)b(t)\pi^{1/2}\alpha^{-1}\exp[-(\pi+at)^2/4\alpha^2)] \qquad (24)$$

We may now discard the constraint imposed on the infinite series to get since $b(-\pi/a)=2$

$$\lim_{N fi\infty}\gamma(t)=\lim_{a fi 0}\gamma^*(t)=(8h/\pi)\delta(t+\pi/a) \qquad (25)$$

The zig-zag functions (17) and (22) may be termed regular since all the intervals inside which the rough function is linear have the same length $2a$ and a modulated regular zig-zag function is obtained for instance with $a_n=a\cos\theta_n$ in (16). The most general situation is met when a is a random variable with a distribution $P(a)$ then, $\gamma(t,\,a)$ becomes a random function whose successive moments have to be obtained to get the properties of the scattering kernel $A(x',\,z')$.

We could also consider zig-zag functions nonlinear inside the intervals $(-a, a)$ for instance with the functions $M_n(x)$ in (17a) changed into

$$\begin{aligned} M_n(x) \ &= \ ha^{-2}(-1)^n[x^2-(2n+1)ax+(n+n-1)a^2], \\ &\quad (n-1)a \leq x \leq na \end{aligned} \tag{26}$$

This discussion shows the huge possibilities of zig-zag functions to describe rough planes but, except for the regular ones, we cannot expect to get analytical expressions of scattered acoustic and electromagnetic fields from rough surfaces and an important numerical work re-mains to be performed to implement efficient zig-zag functions.

Some similarity exists with the spline functions [12] used in interpolation theory but with an important difference since these last functions are continuously differentiable.

4. HARMONIC PLANE WAVE REFLECTION FROM 1D ZIG-ZAG ROUGH PLANES

The approximation (11) of the integral representation approach to harmonic plane wave reflection in the specular direction from 1D-rough planes has to be used for soft acoustic fields and for the E_y component of the TE electromagnetic field satisfying on the perfectly reflecting plane the Dirichlet boundary condition. The approximation (15) corresponds to hard acoustic fields, to the H_y component of the TM electromagnetic wave and to the Neumann boundary condition.

We only consider the integral representation (11) that we write with the variable $p = k_x - \beta$

$$\begin{aligned} &\phi(x, z) = \psi(x, z) + \psi_1(x, z), \\ &\psi_1(x, z) = -(k_z/2\pi)\exp(ik_x x)\int_{-\infty}^{\infty} d\beta \sin(\chi_z z)\exp(-ipx)\gamma(p) \end{aligned} \tag{27}$$

in which $\chi_z = [k_x^2 + k_z^2 - (k_x - p)^2]^{1/2}$ and

$$\gamma(p) = \int_{-\infty}^{\infty} dx' \exp(ipx')\zeta(x') \tag{27a}$$

For an 1D-rough plane described by the zig-zag function (17) we have according to (21) and assuming N even

$$\gamma(p) = (4h/ap^2)b(p)\cos[(N+1/2)ap]/\cos(ap/2) \tag{28}$$

in which from (20)

$$b(p) = 2i\exp(-iap/2)[\sin(ap/2) - ap/2\cos(ap/2)] \tag{28a}$$

To perform the integration (27) is a difficult task because of the branch points introduced by κ_z, and to avoid lenghty developments of the Brillouin-Sommerfeld type [13], we impose further constraints on roughness.

We first suppose that k_x is not too far from its value β for a smooth plane so that p/k_z is small and χ_z has the $0(p^2/k_z^2)$ approximation:

$$\chi_z = k_z + k_x p/k_z + 0(p^2/k_z^2) \tag{29}$$

We further assume a roughness function made of dense spikes with short interval length $2a$ so that ap is small and a simple calculation gives

$$\begin{aligned} b(p)/a^2p^2 &= iap/12 + 0(a^2p^2) \\ \cos[(N+1/2)ap]/\cos(ap/2) &= \cos(Nap) - \tan(ap/2)\sin(Nap) \\ &= 1 + 0(a^2p^2) \end{aligned} \tag{30}$$

Substituting (30) into (28) we get

$$\gamma(p) = ia^2hp/3 + 0(a^2p^2) \tag{31}$$

and with (29) and (31) the expression (27) of y_1 becomes

$$\psi_1(x,z) = -\,(ia^2k_zh/6\pi)\exp(ik_xx)I(x,\, z) \tag{32}$$

$$I(x,z) = \int_{-\infty}^{\infty} pdp\sin[(k_z+pk_x/k_z)z]\exp(-ipx)+0(a^2p^2, p^2/k_z^2) \tag{32a}$$

and we get deleting the Landau symbol

$$\begin{aligned} I(x,\, z) \;&=\; (1/2)\partial_x \int_{-\infty}^{\infty} dp\{\exp(ik_zz)\exp[-ip(x-k_xz/k_z)] \\ &\qquad - \exp(-ik_zz)\exp[-ip(x+k_xz/k_z)]\} \\ &=\; \pi\partial x[\exp(ik_zz)\delta(x-k_xz/k_z)-\exp(-ikz)\delta(x+k_xz/k_z)] \\ &=\; \pi[\exp(ik_zz)\delta'(x-k_xz/k_z)-\exp(-ikz)\delta'(x+k_xz/k_z)] \end{aligned} \tag{33}$$

Substituting (33) into (32) gives since $f(x)\delta'(x) = -f'(x)\delta(x)$

$$\begin{aligned} \psi_1(x,\, z) \;=\; & ia^2k_zk_xh/6)[\exp(ik_xx+ik_zz)\delta(x-k_xz/k_z) \\ & -\exp(ik_xx-ik_zz)\delta(x+k_xz/k_z)] \end{aligned} \tag{34}$$

This simple result is due to the drastic approximations (29), (30) and specially (29). A better approximation of $\psi_1(x,\, z)$ could be obtained with the saddle point method of integration [13, 14], at the expense of more intricate calculations.

5. CONCLUSIONS

The coherent field approximation of scattered harmonic plane waves in the direction of specular reflection is, when the diffuse field is negligible, a suitable tool for analyzing the back-ward radiation from an illuminated surface in the perspective of a minimum signal-to-noise ratio. This approach leads to tractable analytical expressions with integrals of the Brillouin-Sommerfeld type [13] and some more mathematical works are needed to get better approximations. The recently developped nanocomposite materials [15] supplying rough surfaces with small roughness amplitudes will make the coherent field approximation particularly attractive.

The integral relation (2) may be generalized to the coherent reflection of arbitrary harmonic plane waves incident on 2D-mirrors: this generalization with no conceptual difficulties [10] requires double instead of simple integrals to get the form factors F. The zig-zag ridge line used to describe 1D-roughness can be interpreted as a grooved surface for 2D-roughness.

An important generalization of the present approach concerns the harmonic plane wave scattering from non-perfectly reflecting mirror on which the total field $\psi(x,\, z)$ satisfies a Robin boundary condition [16]

$$[\partial_z \psi(x,\, z) + is\psi(x,\, z)]_{z=0} = 0 \tag{35}$$

where s is a simple constant or a function of position on the boundary.

Assuming s constant, the total field incident and specularly reflected has the form

$$\psi(x,\, z) = \exp(ik_x x)[\exp(ik_z z) + R(s,\, k_z)\exp(-ik_z z)] \tag{36}$$

in which R is a reflection coefficient and substituting (36) into (35) gives

$$R(s,\, k_z) = (k_z + s)(k_z - s)^{-1} \tag{37}$$

so that

$$\psi(x,\, z) = 2(k_z - s)^{-1}\exp(ik_x x)[k_z\cos(k_z z) - is\sin(k_z z)] \tag{36a}$$

and the integral relation (2) becomes

$$\psi(x,\, z) = \int_{-\infty}^{\infty} dx'[g_s(x,\, z;\, x',\, z')\partial_{z'} y(x',\, z')]_{z'=0} \tag{38}$$

$$g_s(x,\, z;\, x',\, z') = [\Gamma(s)/4\pi]\int_{-\infty}^{\infty} d\beta\chi_z^{-1}\exp(i\beta x - i\beta x')$$
$$[\exp(i\chi_z|z - z'|) + R(s,\, \chi_z)\exp(i\chi_z|z^{\dagger} - z'|)] \tag{38a}$$

with a constant $\Gamma(s)$ to be determined and $R(s,\,\chi_z)$ obtained from (37) by changing k_z into χ_z. We check easily that g_s satisfies the Robin boundary condition (35). Then, taking into account (38a) we may write (38)

$$\begin{aligned}\psi(x,\,z) &= \int_{-\infty}^{\infty} d\beta \exp(i\beta x) F_s(\beta),\\ F_s(\beta) &= [\Gamma(s)/4\pi] \int_{-\infty}^{\infty} dx' \exp(-i\beta x')[A_s(x',\,z')]_{z'=0}\end{aligned} \tag{39}$$

with

$$\begin{aligned}A_s(x',\,z') &= \chi_z^{-1}\partial_{z'}\psi(x',\,z')[\exp(i\chi_z|z-z'|)\\ &\quad +R(s,\,\chi_z)\exp(i\chi_z|z^\dagger - z'|)]\end{aligned} \tag{40}$$

We get from (36a)

$$[\partial_{z'}\psi(x',\,z')]_{z'=0} = -2ik_z s(k_z - s)^{-1}\exp(ik_x x') \tag{41a}$$

while

$$\begin{aligned}&[\exp(i\chi_z|z-z'|) + R(s,\,\chi_z)\exp(i\chi_z|z^\dagger - z'|)]_{z'=0}\\ &= 2(\chi_z - s)^{-1}[\chi_z\cos(\chi_z z) - is\sin(\chi_z z)]\end{aligned} \tag{41b}$$

and substituting (41a), (41b) into (40) we get

$$\begin{aligned}[A_s(x',\,z')]_{z'=0} &= -4ik_z s[\kappa_z(k_z - s)(\kappa_z - s)]^{-1}\\ &\quad \exp(ik_x x')[\kappa_z\cos(\kappa_z z) - is\sin(\kappa_z z)]\end{aligned} \tag{42}$$

Taking into account (42) the form factor $F_s(\beta)$ becomes according to (39)

$$\begin{aligned}F_s(\beta) &= -4i\Gamma(s)k_z s[\chi_z(k_z - s)(\chi_z - s)]^{-1}\\ &\quad [\chi_z\cos(\kappa_z z) - is\sin(\chi_z z)]\delta(k_x - \beta)\\ &= -2i\Gamma(s)s(k_z - s)^{-2}[k_z\cos(k_z z) - is\sin(k_z z)]\delta(k_x - \beta)\end{aligned} \tag{43}$$

where we used the relation $\chi_z^2 = k_x^2 + k_z^2 - \beta^2$ and $\beta = k_x$ according to the Dirac distribution then, substituting (43) into the expression (39) of $\psi(x,\,z)$ gives (36a) with $\Gamma(s) = i(k_z - s)s^{-1}$.

We observe at once that for $s \Rightarrow \infty$, (35) becomes the Dirichlet boundary condition and the relations (39)–(43) reduce to the expressions (2)–(5). For $s = 0$, (35) is the Neumann boundary condition and making $s = 0$ in (39)–(43) gives the relations (13), (14).

The situation is not so simple when s is function of x and it is better work with the transverse.

Fourier transform of fields $\Psi(\zeta, z) = \int_{-\infty}^{\infty} dx \exp(i\zeta x)\psi(x, z)$ so that the boundary condition (35) becomes

$$[\partial_z \Psi(\zeta, z) + is(\zeta)\Psi(\zeta, z)]_{z=0} = 0 \tag{44}$$

We shall investigate later the coherent field approximation for this type of boundary condition and its extension to harmonic plane wave scattering from non-perfectly reflecting rough mirrors.

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