Progress In Electromagnetics Research M, Vol. 9, 53–64, 2009

SPATIAL SOLITON PAIRING OF TWO CYLINDRICAL BEAMS IN SATURABLE NONLINEAR MEDIA

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Abstract—In this paper, we have extended the recently introduced theory of coupled propagation of two coaxially co-propagating and mutually incoherent bright 1-D beams to coupled propagation of two 2-D (cylindrical) bright beams and investigated the propagation behavior and spatial soliton pair formation of such beams by a recently introduced simple approach. We have considered saturable form of the nonlinear medium in this paper as 2-D spatial solitons are unstable in Kerr type media. We found that many of the propagation features of two 2-D beams in saturable media are same that of 1-D beams in Kerr type media. However, many features are different and to the best of our knowledge, reported in this paper for the first time. The present version of the theory is applicable in all possible physical situations and parameters.

1. INTRODUCTION

Formation of optical spatial soliton has attracted a lot of interest following the progress on photorefractive solitons [1], quadratic solitons [2], solitons in saturable nonlinear media [3] and topological solitons with time dependent coefficients [4]. Investigations of soliton formation, interaction and soliton induced wave guide are of high

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interest due to their potential applications in all-optical switching, all-optical interconnects [5] and wave guide applications [6, 7]. Coupled spatial soliton pairs are obtained using two co-propagating beams in nonlinear media and such pairing has always been an intriguing issue among spatial soliton interactions. Possibility of bright and/or dark soliton pairs has already been discussed in many papers, for example, References [8–12].

The present paper is the extended version of the theory developed in [12] for 1-D soliton pairing in Kerr type media. The present theory deals with 2-D soliton pairing in saturable media. Saturable form of nonlinearity is considered in the present paper as cylindrical beams are unstable and lead to either diffraction or collapse in Kerr type media. In [12] detailed investigations of the two coaxially co-propagating 1-D beams were carried out. In the present paper, we have carried out similar investigations on two cylindrical (2-D) beams. We found that many of the copropagation features of two 1-D beams are similar to two co-propagating 2-D beams, however, many are different and are reported here for the first time.

It is important to be mentioned here that using present approach, the evolution of the co-propagating 2-D beams with the distance of propagation could be obtained in a little time, for example, the figures like 2, 5–8 have been obtained in the time of the order of a minute with an ordinary processor, moreover, it requires ignorable memory space. Investigation of the same problem using Beam Propagation Method (BPM) would require time of the order of several hours and memory space that can overwhelm even a powerful computer. The results presented in this paper are not verified using (BPM), however, one can safely believe in these results as the 1-D version of the present theory has been tested using coupled Nonlinear Schrödinger equation in [12].

2. MUTUAL PROPAGATION OF BEAMS

We start by considering propagation of two cylindrically symmetric coaxial laser beams along the z-axis of a cylindrical coordinate system. The initial intensity distributions for the two beams (at $z = 0$) are assumed to be Gaussian and expressed as $A_1^2(z)\big|_{z=0} = E_{01}^2 \exp(-r^2/r_1^2)$ and $A_2^2(z)\big|_{z=0} = E_{02}^2 \exp(-r^2/r_2^2)$ respectively, where A_1 and A_2 are the real amplitudes of the electric vectors of two beams of angular frequencies ω_1 & ω_2 respectively, r the radial coordinate of the cylindrical coordinate system and r_1, r_2 represent dimensions of these beams. The effective dielectric constant of the medium corresponding to the two frequencies may respectively be written as

$\varepsilon(\omega_1) = \varepsilon_{10} + \varphi_1(A_1, A_2)$ and $\varepsilon(\omega_2) = \varepsilon_{20} + \varphi_2(A_1, A_2)$, where ε_{10} and ε_{20} are the dielectric constants at frequencies ω_1 and ω_2 respectively and φ_1 and φ_2 are the nonlinear dielectric constants may be expressed by the saturating profile $\varphi_1 = \varepsilon_{s1}X/(1+X); X = (\alpha_1 A_1^2 + \kappa\alpha_2 A_2^2)/2$ and $\varphi_2 = \varepsilon_{s2}Y/(1+Y); Y = (\kappa\alpha_1 A_1^2 + \alpha_2 A_2^2)/2$ respectively. Here α_1 and α_2 are constants with their ratio equal to the ratio of the nonlinear coefficients of the medium at frequencies ω_1 and ω_2 respectively ($\alpha_j A_j^2; j = 1, 2$, is the dimensionless electric field intensity), κ is the coupling coefficient of the two beams that depends on the experimental conditions and ε_{s1} and ε_{s2} are the saturated values of φ_1 and φ_2 respectively.

As done in [12], (Assuming the beams maintain their Gaussian shape while the widths vary along propagation), i.e., following WKB and paraxial ray approximation, we obtain two coupled equations that govern beam width parameters f_1, f_2 of the two beams with the propagation distance.

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{k_1^2 r_1^4 f_1^3} - \frac{\varepsilon_{s1} C}{2\varepsilon_{10} r_1^2 f_1^3 \Omega^2} - \frac{\kappa f_1 \varepsilon_{s1} D}{2\varepsilon_{10} r_2^2 f_2^4 \Omega^2} \tag{1}$$

$$\frac{\partial^2 f_2}{\partial z^2} = \frac{1}{k_2^2 r_2^4 f_2^3} - \frac{\varepsilon_{s2} D}{2\varepsilon_{20} r_2^2 f_2^3 \Lambda^2} - \frac{\kappa f_2 \varepsilon_{s2} C}{2\varepsilon_{20} r_1^2 f_1^4 \Lambda^2} \tag{2}$$

where, $C = \alpha_1 E_{01}^2$ and $D = \alpha_2 E_{02}^2$, $\Omega = 1 + \frac{C}{2f_1^2} + \frac{\kappa D}{2f_2^2}$ and $\Lambda = 1 + \frac{\kappa C}{2f_1^2} + \frac{D}{2f_2^2}$.

For self-trapped beams (spatial solitons), we must have $\frac{\partial f_j}{\partial z} = \frac{\partial^2 f_j}{\partial z^2} = 0$; (where: $j = 1, 2$). One can assume $\frac{\partial f_j}{\partial z} = 0$ as the initial condition of the beams. To have $\frac{\partial^2 f_j}{\partial z^2} = 0$, we must have from Equations (1) and (2).

$$D = \frac{2}{(\kappa)^2}\left[-(A) \pm \sqrt{(A)^2 - (\kappa)^2\left(1 + C + \frac{C^2}{4} - \frac{\varepsilon_{s1} C k_1^2 r_1^2}{2\varepsilon_{10}}\right)}\right] \tag{3}$$

and

$$D = 2\left[-(B) \pm \sqrt{(B)^2 - \left(1 + C\kappa + \frac{1}{4}(C\kappa)^2 - \frac{\varepsilon_{s2}\kappa C k_2^2 r_2^4}{2\varepsilon_{20} r_1^2}\right)}\right] \tag{4}$$

respectively.

Here $A = \kappa + \frac{C\kappa}{2} - \frac{\varepsilon_{s1}\kappa k_1^2 r_1^4}{2\varepsilon_{10} r_2^2}$ and $B = 1 + \frac{C\kappa}{2} - \frac{\varepsilon_{s2} k_2^2 r_2^2}{2\varepsilon_{20}}$.

3. NUMERICAL APPRECIATION AND DISCUSSION

We appreciate derived Equations (1)–(4) numerically by choosing following set of parameters $\omega_1 = \omega_2 = 2.7148 \times 10^{15}$ rad/s, $\varepsilon_{10} = \varepsilon_{20} = (1.6276)^2$, $\varepsilon_{s1} = \varepsilon_{s2} = 0.73 \times \varepsilon_{10}$, and $r_1 = r_2 = 10\,\mu$m. The parameters chosen in this paper are just for the numerical appreciation of the derived equations, the approach given here is valid for any other set of parameters.

We investigate coupled beam propagation for different coupling coefficients. As mentioned earlier [12], coupling coefficient κ depends on the experimental conditions and the present approach is applicable for arbitrary value of the coupling coefficient. However, in the present paper, we have investigated coupled beam propagation for $\kappa = 2$, $\kappa = 2/3$ and $\kappa = 1$ as these values of coupling coefficient have been discussed in earlier literature (see [9, 13, 14]).

3.1. Coupled Beam Propagation for Coupling Coefficient $\kappa = 2$

In Figure 1, we plot D with C for $\kappa = 2$ and above mentioned parameters and using Equations (3) and (4). Point P_1 corresponds to the power of the first beam required to self-trap itself in absence of the second beam (single soliton), similarly, point P_2 corresponds to the power of the second beam required to self-trap itself in absence of the first beam. The point of intersection S is the common solution of Equations (3) and (4) and provides the condition on beam powers of the two beams for mutual self-trapping (spatial soliton pairing). We verify mutual trapping by choosing beams' powers of the two beams from the point of intersection S which correspond to $C = D = 4.2103 \times 10^{-5}$.

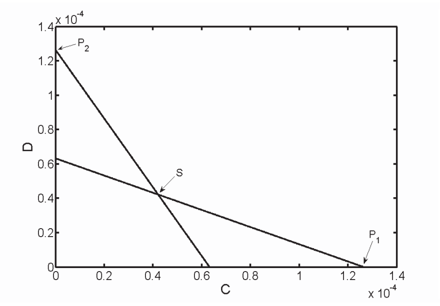

Figure 1. Using Equations (3) and (4) and parameters mentioned in the text, D with C has been plotted for $\kappa = 2$.

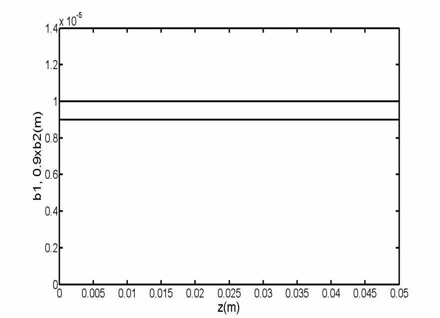

Figure 2. Evolution of the beams' widths with the propagation distance is obtained using Equations (1) and (2) as shown by solid lines in the figure. The chosen beams' widths are same, however, $0.9 \times b2(= r_2 f_2)$ has been plotted just to resolve $b1(= r_1 f_1)$ and $b2(= r_2 f_2)$. The figure shows solitonic pairing.

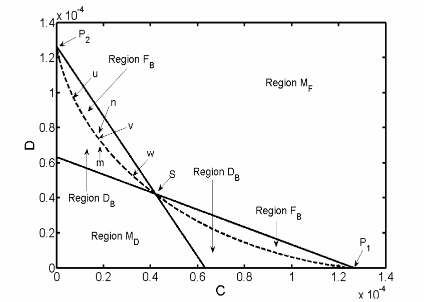

Figure 3. An arc of the circle that passes through P_1, S and P_2 has been drawn intuitively as shown in the figure. This arc has been identified as the existence curve of trapped breather pairs.

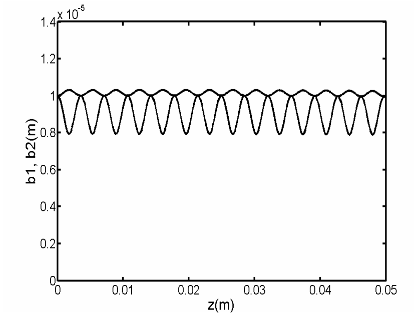

Figure 4. Trapped breather pair is obtained using beam powers correspond to point $u(C = 0.755 \times 10^{-5}, D = 1.0 \times 10^{-4})$ of Figure 3.

Evolution of the beams' widths with the propagation distance obtained using Equations (1) and (2) is shown in Figure 2. We have plotted $0.9 \times b2(= r_2 f_2)$ to resolve $b1(= r_1 f_1)$ and $b2(= r_2 f_2)$. Clearly both beams are mutually self-trapped or they form a spatial soliton pair. It can be observed in the Figure 1 that the beam power of each beam (corresponds to point S) in the soliton pair is one third of the power of the beam (correspond to points P_1, P_2) required to form a single soliton, moreover, there exists only one solution for soliton pairing.

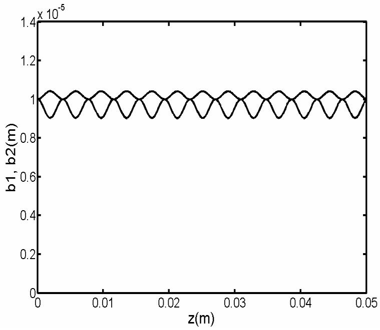

Figure 5. Trapped breather pair is obtained using beam powers correspond to point $v(C = 2.111 \times 10^{-5}, D = 6.9753 \times 10^{-5})$ of Figure 3.

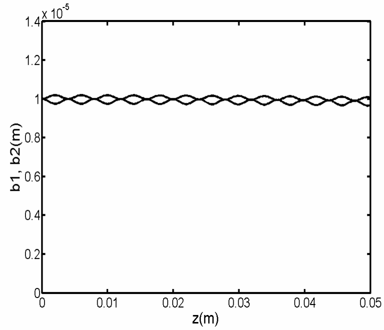

Figure 6. Trapped breather pair is obtained using beam powers correspond to point $w(C = 3.4859 \times 10^{-5}, D = 5.0 \times 10^{-5})$ of Figure 3.

We go further and as done in [12], draw an arc of the circle that passes through P_1, S and P_2 as shown in Figure 3. We confirmed that as in case of 1-D beams, this arc represents the existence curve of trapped breather pair. To show trapped breather pairs, we use beams' powers corresponding to points $u(C = 0.755 \times 10^{-5}, D = 1.0 \times 10^{-4})$, $v(C = 2.111 \times 10^{-5}, D = 6.9753 \times 10^{-5})$ and $w(C = 3.4859 \times 10^{-5}, D = 5.0 \times 10^{-5})$. The beam pairs form trapped breather pairs as shown in Figures 4–6 respectively. As we shift our point of interest on the arc towards point S, amplitude of breathing of the trapped breather pair decreases. In fact, at point S, amplitude of breathing becomes zero that leads to the soliton pairing. In other words, soliton pairing is just a special case of trapped breather pairing.

So far, propagation behavior of two co-propagating 2-D beams appears to be same that of the propagation behavior of two co-propagating 1-D beams of [12]. However, our detailed investigations with 2-D beams revealed some very interesting features of co-propagation which are different from the co-propagation of 1-D beams. We found that instead of just two regions (I and II) of distinct type of coupled propagation in case of 1-D beams, there exist four regions of distinct type of coupled propagation in case of 2-D beams. Those regions could be described as below.

(i) Region D_B is below the dashed arc. It is basically of two parts as shown in Figure 3. One part is bounded by the dashed arc and the solutions of Equation (3), while, the other part is bounded by the dashed arc and the solutions of Equation (4). (ii) Similarly region F_B

is the region above the dashed arc. One part of F_B is bounded by the dashed arc and the solutions of Equation (3), while, the other part is bounded by the dashed arc and the solutions of Equation (4). (iii) Region M_D is the region below the solutions of Equations (3) and (4). (iv) Region M_F is the region above the solutions of Equations (3) and (4).

Co-propagating beams with beams' powers corresponding to any point of region D_B (of Figure 3) form a breather pair that diffracts with the propagation distance as shown in Figure 7. In the figure, the diffracting breather is obtained using Equations (1) and (2) and by choosing beams' powers correspond to the point m of Figure 3. Co-propagating beams with beams' powers corresponding to all points of region F_B form a breather pair that focuses with the propagation distance as shown in Figure 7. The focusing breather is obtained by choosing beams' powers correspond to the point n of Figure 3. If beams' powers are chosen from Region M_D, both beams mutually diffract without breathing and if those are chosen from the region M_F, both mutually focus and form an oscillatory wave guide as seen in Figure 8. One can see the clear distinction between the focusing of co-propagating beams of region F_B and the focusing of the beams of region M_F. In the prior case, breathing is superimposed on the formation of oscillatory wave guide (focusing), while in later case, only oscillatory wave guide is formed and breathing is absent.

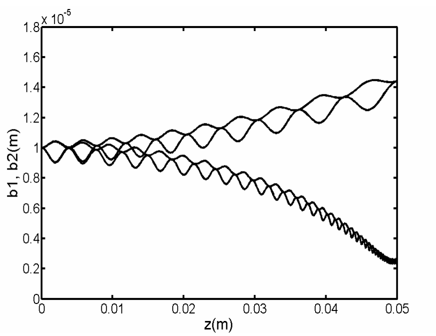

Figure 7. Co-propagating beams corresponding to all points of region D_B (of Figure 3) form a breather pair that diffracts with the propagation distance and beams corresponding to all points of region F_B form a breather pair that focuses with the propagation distance. In the figure, diffracting breather pair corresponds to the point m of Figure 3, while focusing breather pair corresponds to point n.

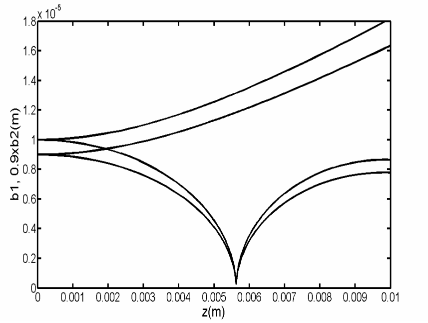

Figure 8. Both beams mutually diffract without breathing if beams' powers are chosen from Region M_D of Figure 3. If those are chosen from the region M_F, both mutually focus and form an oscillatory wave guide.

We must mention here that to avoid repetition, we have given only one example from each region to show the distinct type of propagation, however, we have confirmed through detailed numerical experimentation that two copropagating beams with powers corresponding to any one point of a given region show same propagation features to the co-propagating beams with powers corresponding to any other point of the same region.

3.2. Coupled Beam Propagation for Coupling Coefficient $\kappa = 2/3$

In case of $\kappa = 2/3$, Figure 3 modifies as Figure 9. Following two propagation features of two co-propagating 2-D beams could be seen;

(i) Beam power of each beam required for soliton pair is 60% of the power of the beam required to form a single soliton.

(ii) Only one solution exists for soliton pairing.

We confirmed that here also four regions D_B, M_D, M_F and F_B of distinct type of coupled propagation and propagation characteristics of co-propagating beams corresponding to these regions are exactly same as in the case of $\kappa = 2$.

3.3. Coupled Beam Propagation for Coupling Coefficient $\kappa = 1$

In case of $\kappa = 1$, solutions of Equations (3) and (4) merge and form a single existence line of solitonic pairing [11] as shown in the Figure 10.

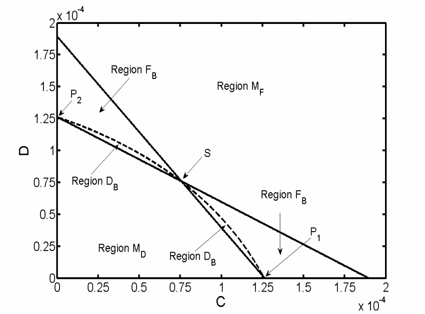

Figure 9. In case of $\kappa = 2/3$, Figure 3 modifies as shown. In this case the beam power of each beam required for soliton pair is 60% of the power of the beam required to form a single soliton. Only one solution exists for soliton pairing.

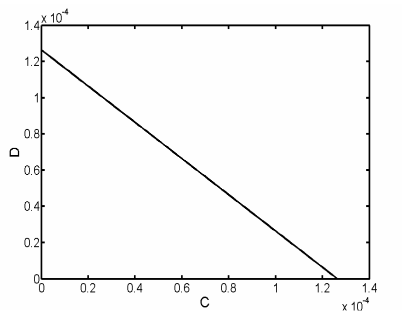

Figure 10. Interesting situation arises for $\kappa = 1$. The solutions of Equations (3) and (4) merge and form a single existence line of soliton pair as shown in the figure. Each point of existence line provides one soliton pair. Infinite solutions for solitonic pairing exist in this case.

Each point of this line is a common solution of Equations (3) and (4), and therefore, corresponds to one soliton pair as shown by solid lines in Figure 11 which has been obtained using Equations (1) and (2). If beams' powers are chosen from a point below the existence line, both beams mutually diffract and if those are chosen from a point above the existence line, they mutually focus as shown by dotted and dashed curves respectively in the Figure 11.

3.4. Bistability

In Figures 3, 9 and 10, we have shown only a small portion of solutions of Equations (3) and (4) that lye in the positive quadrant of x and y axes. The real solutions of Equations (3) and (4) in the entire range for $\kappa = 2$ is shown in Figure 12. Bistability of solitonic pairing of 2-D beams in saturable nonlinear media is revealed from the figure as two common solutions of Equations (3) and (4) exist in the positive quadrant, one corresponding to very high beam powers and other corresponding to lower beam powers. Entire real solutions of Equations (3) and (4) for $\kappa = 2/3$ and 1 are shown in Figures 13 and 14 respectively. It is evident from these figures and interesting to note that in case of $\kappa = 2/3$, two solutions exist similar to the case of $\kappa = 2$, while, there exist infinite solutions for solitonic pairing for $\kappa = 1$.

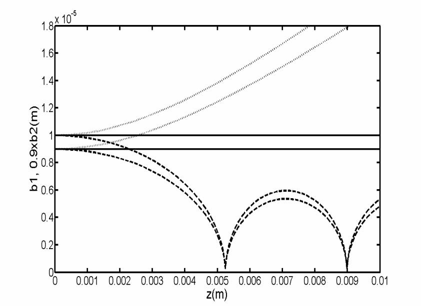

Figure 11. A point on the existence line of Figure 10 provides one soliton pair as shown by solid lines in the figure. If beams' powers are chosen from a point below the existence line, both beams mutually diffract and if those are chosen from a point above the existence line, they mutually focus shown by dotted and dashed curves respectively.

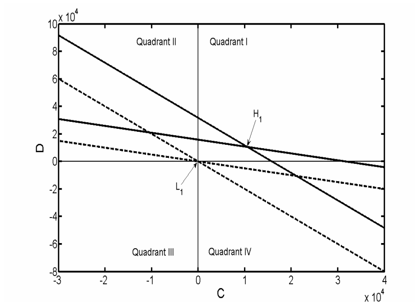

Figure 12. The real solutions of Equations (3) and (4) in the entire range are shown in the figure for $\kappa = 2$. Bistability of solitonic pairing of 2-D beams in saturable nonlinear media is reaveled from the figure as two common solutions of Equations (3) and (4) exist in the positive quadrant, one corresponding to very high beams' powers and other corresponding to lower beams' powers.

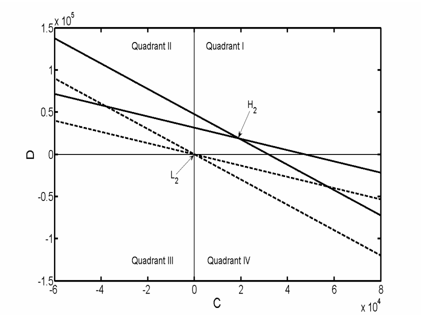

Figure 13. The real solutions of Equations (3) and (4) in the entire range is shown for $\kappa = 2/3$ in the figure. In this case, only two solutions exist similar to the case of $\kappa = 2$.

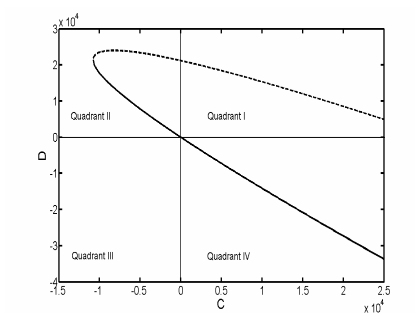

Figure 14. The real solutions of Equations (3) and (4) in the entire range for $\kappa = 1$ is shown in the figure. An infinite solution exists for solitonic pairing.

It is worth to be mentioned here that bistability never exists in the case of soliton pairing of two 1-D beams in Kerr type media, and also only one solution exists for solitonic pairing when $\kappa = 2$ and $2/3$.

4. CONCLUSION

In conclusion, coupled propagation of two coaxially co-propagating and mutually incoherent bright 2-D (cylindrical) beams in saturable nonlinear medium has been investigated in detail in this paper. Many of the propagation features of two 2-D beams in saturable media are same that of 1-D beams in Kerr type media, for example, beam power of each beam in a soliton pair is one third of the power of the beam required to form a single soliton when $\kappa = 2$. However, many features are different like instead of just two regions of distinct type of coupled propagation in case of 1-D beams, there exist four regions of distinct type of coupled propagation in case of 2-D beams, moreover, two solutions of pairing exist in contrast to one solution in case of 1-D beams.

ACKNOWLEDGMENT

SM thanks University Grants Commission (UGC), Government of India for financial assistance through a research project reference number: 34-24/2008 (SR). Moral support and encouragement from H. C. Pande and P. K. Barhai is gratefully acknowledged.

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