Progress In Electromagnetics Research Letters, Vol. 12, 87–97, 2009

SYNTHESIS OF THINNED PLANAR CIRCULAR ARRAY ANTENNAS USING MODIFIED PARTICLE SWARM OPTIMIZATION

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Abstract—In this paper, the authors present an optimization method based on modified Particle Swarm Optimization (PSO) algorithm for thinning large multiple concentric circular ring arrays of uniformly excited isotropic antennas that will generate a pencil beam in the vertical plane with minimum relative side lobe level (SLL). Two different cases have been studied, one with fixed uniform inter-element spacing and another with optimum uniform inter-element spacing. In both the cases, the number of switched off elements is made equal to 220 or more. The half- power beam width of the synthesized pattern is attempted to make equal to that of a fully populated array with uniform spacing of 0.5λ. Simulation results of the proposed thinned arrays are compared with a fully populated array to illustrate the effectiveness of our proposed method.

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1. INTRODUCTION

Circular antenna arrays find various applications in sonar, radar, mobile and commercial satellite communications systems [1–5]. It consists of a number of elements arranged on a circle [1] with uniform spacing and can be used for beam forming in the azimuth plane for example at the base stations of the mobile radio communications system [2–5].

A very popular type of antenna arrays is the circular array that has several advantages over other type of array antenna configurations; such as all-azimuth scan capability, invariant beam pattern in every ϕ-cut. Concentric Circular Antenna Array (CCAA) that contains many concentric circular rings of different radii and number of elements have several advantages such as ϕ symmetric pattern, flexibility in array pattern synthesis etc. [2–5]. For Direction of Arrival (DOA) applications, CCAA provides almost invariant azimuth angle coverage.

Uniform CCA (UCCA) is one of the most important configurations of the CCA [2] where the inter-element spacing in individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited. The side lobe in the UCCA drops to about 17.5 dB, especially at larger number of rings [2] with uniform excitation.

Uniformly excited and equally spaced antenna arrays [1, 2] have high directivity but they usually suffer from high side lobe level. To reduce the side lobe level further, the array is made aperiodic by altering the positions of the antenna elements with all excitation amplitudes being uniform. Another possibility is to use an equally spaced array with radially tapered amplitude distribution [3, 4]. However, uniform excitation is desired to minimize the complexity in designing a feed network and to maximize the power input.

Thinning a large array will not only reduce side lobe level further but also reduce the number of antennas in the array and thereby cut down cost substantially. Due to the complexity in synthesis problem, analytical methods are not generally used in designing a thinned array. Therefore, global optimization tools such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO) etc. are used to solve these problems. PSO is an evolutionary algorithm and have been successfully used in the design of antenna arrays [5–10]. The PSO algorithm [11] has been shown to be an effective alternative to other evolutionary algorithms [12, 13] such as Genetic Algorithms (GA), Ant Colony Optimization (ACO) etc. in handling certain kinds of optimization problems. There are many published articles [13–17] dealing with the synthesis of thinned array. Element behavior in a thinned array is described in [18].

In this paper, we have proposed a method based on particle swarm optimization for thinning large multiple concentric circular ring arrays of isotropic antennas that will reduce the side lobe level of the generated pattern further with little compromise on half-power beam width. Results are compared with a fully populated array.

2. THINNED ARRAY DESIGN

Thinning an array means turning off some elements in a uniformly spaced or periodic array to generate a pattern with low side lobe level. In our proposed method, the positions of the elements are fixed and all the elements have two states either "on" or "off", depending on whether the element is connected to the feed network or not. In the "off" state, either the element is passively terminated to a matched load or open circuited. If there is no coupling between the elements, it is equivalent to removing them from the array.

Thinning an array [16] to produce low side lobes is much simpler than the more general problem of nonuniform spacing the elements. Nonuniform spacing has an infinite number of possibilities for placement of the elements.

The arrangement of elements in planar circular arrays [2, 3] may contain multiple concentric circular rings, which differ in radius and number of elements. Figure 1 shows the configuration of multiple concentric circular arrays [2, 3] in XY plane in which there are M

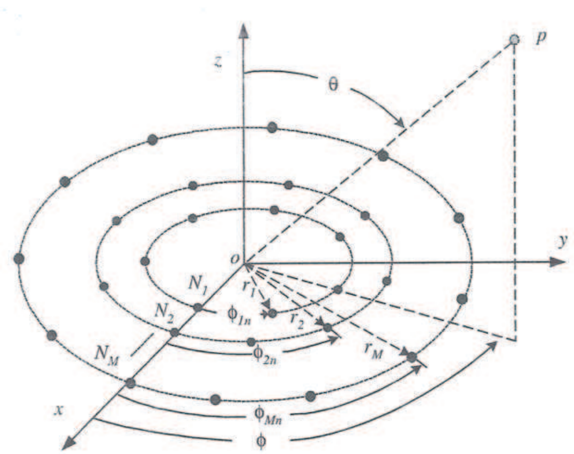

Figure 1. Multiple concentric circular ring arrays of isotropic antennas in XY plane.

concentric circular rings. The m-th ring has a radius r_m and number of isotropic elements N_m, where $m = 1, 2, \ldots, M$. Elements are equally placed along a common circle.

The far-field pattern [1] in free space is given by:

$$E(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N_m} I_{mn} e^{j 2\pi r_m \sin\theta \cos(\phi - \phi_{mn})} \tag{1}$$

Normalized absolute power pattern, $P(\theta, \phi)$ in dB can be expressed as follows:

$$P(\theta, \phi) = 10 \log 10 \left[\frac{|E(\theta, \phi)|}{|E(\theta, \phi)|_{\max}} \right]^2 = 20 \log 10 \left[\frac{|E(\theta, \phi)|}{|E(\theta, \phi)|_{\max}} \right] \tag{2}$$

where r_m = radius of m-th ring = $N_m d_m / 2\pi$, d_m = inter-element arc spacing of m-th circle, $\phi_{mn} = 2n\pi / N_m$ = angular position of mn-th element with $1 \leq n \leq N_m$, θ, ϕ = polar, azimuth angle, k = wave number = $2\pi/\lambda$, λ = wave length, I_{mn} = excitation amplitude of mn-th element. In our case, I_{mn} is 1 if the mn-th element is turned "on" and 0 if it is "off". All the elements have same excitation phase of zero degree.

3. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization [5–11] emulates the swarm behavior of insects, animals herding,birds flocking, and fish schooling where these swarms search for food in a collaborative manner. Each member in the swarm adapts its search patterns by learning from its own experience and other member's experiences. These phenomena are studied and mathematical models are constructed. In PSO, a member in the swarm, called a *particle*, represents a potential solution,which is a point in the search space. The global optimum is regarded as the location of food.Each particle has a fitness value and a velocity to adjust its flying direction according to the best experiences of the swarm to search for the global optimum in the D-dimensional solution space.The PSO algorithm is easy to implement and has been empirically shown to perform well on many optimization problems.

The PSO algorithm is an evolutionary algorithm capable of solving difficult multidimensional optimization problems in various fields. Since its introduction in 1995 by Kennedy and Eberhart [11], the PSO has gained an increasing popularity as an efficient alternative to GA, SA, ACO etc. in solving optimization design problems in antenna arrays. As an evolutionary algorithm,the PSO algorithm depends on the social interaction between independent agents, here called particles, during their search for the optimum solution using the concept of fitness.

PSO emulates the swarm behavior and the individuals represent points in the D-dimensional search space. A particle represents a potential solution. The particle swarm optimization used in this paper is a real-coded one.

The steps involved in modified PSO are given below:

***Step1*:** Initialize positions and associated velocity of all particles (potential solutions) in the population randomly in the D-dimension space.

***Step2*:** Evaluate the fitness value of all particles.

***Step3*:** Compare the personal best (*pbest*) of every particle with its current fitness value. If the current fitness value is better, then assign the current fitness value to *pbest* and assign the current coordinates to *pbest* coordinates.

Step 4: Determine the current best fitness value in the whole population and its coordinates. If the current best fitness value is better than global best (*gbest*), then assign the current best fitnessvalue to *gbest* and assign the current coordinates to *gbest* coordinates.

***Step5*:** Update velocity (V_{id}) and position (X_{id}) of the d-th dimension of the i-th particle using the following equations:

$$\begin{aligned} V_{id}^{t} &= w * V_{id}^{t-1} + c_1 * rand1_{id}^{t} * \left(pbest_{id}^{t-1} - X_{id}^{t-1}\right) \\ &\quad + c_2 * \left(1 - rand1_{id}^{t}\right) * \left(gbest_{d}^{t-1} - X_{id}^{t-1}\right) \end{aligned} \tag{3}$$

$$V_{id} = \mathrm{Min}\left(V_{\max}^{d}, \max\left(V_{\min}^{d}, V_{id}^{t}\right)\right) \tag{4}$$

$$X_{id}^{t} = X_{id}^{t-1} + V_{id}^{t} \tag{5}$$

If $X_{id}^{t} > X_{\max}^{d}$

$$\text{then } X_{id}^{t} = X_{\max}^{d} - 0.25 * rand\ 2_{id}^{t} * \left(X_{\max}^{d} - X_{\min}^{d}\right) \tag{6}$$

If $X_{id}^{t} < X_{\min}^{d}$

$$\text{then } X_{id}^{t} = X_{\min}^{d} + 0.25 * rand3_{id}^{t} * \left(X_{\max}^{d} - X_{\min}^{d}\right) \tag{7}$$

where c_1, c_2 = acceleration constants = 1.4945, w = inertia weight linearly damped with iterations starting at 0.9 and decreasing linearly to 0.4 at the last iteration, $rand1$, $rand2$ and $rand3$ are uniform random numbers [10] between 0 and 1, different value in different dimension, t is the current generation number.

Equations (4), (6) and (7) have been introduced to clamp the velocity and position along each dimension to ($V_{\max}^{d}$, X_{id}^{t}) and ($V_{\min}^{d}$, X_{id}^{t}) value if they try to cross the desired domain of interest. These clipping techniques are sometimes necessary to prevent particles from explosion. The maximum velocity is set to the upper limit of the dynamic range of the search ($V^d_{\max} = X^d_{\max}$) and the minimum velocity ($V^d_{\min}$) is set to ($-V^d_{\max}$).

Step 6: Repeat steps 2–5 until a stop criterion is satisfied or a prespecified number of iteration is completed, usually when there is no further update of best fitness value.

In the proposed modified PSO, velocity clipping technique is applied with time-varying maximum velocity [9], which decreases linearly from $V^d_{\max}$ to $0.1V^d_{\max}$ over the full range of search, because as the particles approach the optimal result it is preferred to have them move with lower velocities [9].

The fitness function to be minimized with the proposed modified PSO for optimal synthesis of thinned array is given in Eq. (8).

$$\begin{aligned} Fitness \ &= \ k_1 \max SLL \ + k_2(HPBW_o - HPBW_d)^2 \\ &\quad + k_3(T^{off}_o - T^{off}_d)^2 H(T) \end{aligned} \tag{8}$$

where max SLL is the value of maximum side lobe level, $HPBW_o$, $HPBW_d$ are obtained and desired value of half-power beam width respectively, T^{off}_o, T^{off}_d are obtained and desired value of number of switched off element respectively, k_1, k_2, k_3 are weighting coefficients to control the relative importance given to each term of Eq. (8).

$H(T)$ is Heaviside step functions defined as follows:

$$T = \left(T^{off}_o - T^{off}_d\right) \tag{9}$$

$$H(T) = \begin{cases} 1, & \text{if } T \leq 0, \\ 0 & \text{if } T > 0 \end{cases} \tag{10}$$

4. RESULTS

We consider a planar array of ten concentric circular rings. In the example, each ring of the antenna contained $8m$ equi-spaced isotropic elements (a total of 440), where m is the ring number counted from the innermost ring 1. Two cases have been studied and presented with results.

Case-I: In this case, inter-element arc spacing (d_m) in all the rings is fixed at 0.5λ.

For such a fully populated and uniformly excited array, the maximum side lobe level is calculated to be -17.37 dB and half-power beam width is approximately 4.5 degree.

Problem is now to find the optimal set of on and off elements that will generate a pencil beam in the XZ plane keeping the half-power beam width unchanged, fixing the number of switched off elements to be equal to 220 or more and reducing the maximum side lobe level further. Number of particles in modified particle swarm optimization is taken to be 50 and the algorithm is run for 60 generations.

The maximum number of generation is kept at a value when there is no further update of best fitness value.

Case-II: In the second case, inter-element arc spacing (d_m) in all the rings is made uniform and same but not fixed. Optimum value of inter-element arc spacing along with optimal set of on and off elements are found out using this modified PSO that will generate a pencil beam in the XZ plane with reduced side lobe level. The desired half-power beam width is kept at 4.5 degree and the desired number of switched off elements is made equal to 220 or more. Number of particles in this case is also taken to be 50 and the algorithm is run for 60 generations.

Obtained results for the above two cases and its comparison to a fully populated array are shown in Table 1. Results clearly show that the synthesized pattern of thinned array using modified PSO with fixed and optimum inter-element arc spacing is better than a fully populated array in terms of side lobe level and number of elements switched off with little compromise on half-power beam width in the fixed case. Moreover, the synthesized pattern of thinned array with optimum inter-element arc spacing is better than the synthesized pattern of thinned array with fixed inter-element arc spacing in terms of side lobe level, half-power beam width. Optimized inter-element arc spacing is found to be 0.6266λ.

Table 1. Obtained results.

Design parameters	**Synthesized thinned array with optimum** $d = 0.6266\lambda$	**Synthesized thinned array with fixed** $d = 0.5\lambda$	**Fully populated array with** $d = 0.5\lambda$
Side lobe level (SLL, in dB)	−23.85	−23.22	−17.37
Half-power beam width (HPBW, in degree)	4.0	4.6	4.5
Number of switched off elements	227	231	0

Table 2. Excitation amplitude distributions (I_{mn}) using modified PSO with fixed $d = 0.5\lambda$.

m \ n

11100011
1101011101100100
010101111100001101100101
11010001001010011011101111010100
0101000110011111000000111010100001011000
111100111110110100010111011001000110100100010001
11110111011111010011010100111000001101000111011010000100
00000100110111000000001010010001100010010000111100000101100101110
110101100001111100111110100010111001101010000111010101000110001110000101
10100111111011100010101001101000100110011010000101000000100110100000000000001011 0

Table 3. Excitation amplitude distributions (I_{mn}) using modified PSO with optimized $d = 0.6266\lambda$.

m \ n

01110011
0010101110000110
111011110110101001010010
01100011010111011010110110010001
1000101011111111101100010100110011011000
100000100000001000010010100101010010010100000000101
00001010111111100101100100001101011101111101111011001011
1011111011100010111100011111111000010110001010111110110100101010
100000111011111110001011011000100010000100100011010110111000110100000000101
00000000000010110010000101000100111111000111000010000100111101110000001110100001

Optimally obtained excitation amplitude distribution using modified PSO for fixed and optimized inter-element arc spacing is shown in Table 2 and Table 3 respectively. Figure 2 shows convergence curves for the thinned arrays using modified PSO with fixed and optimized inter-element arc spacing. Figure 3 shows normalized absolute power patterns in dB in XZ plane for fully populated array, thinned array with fixed inter-element arc spacing and thinned array with optimized inter-element arc spacing.

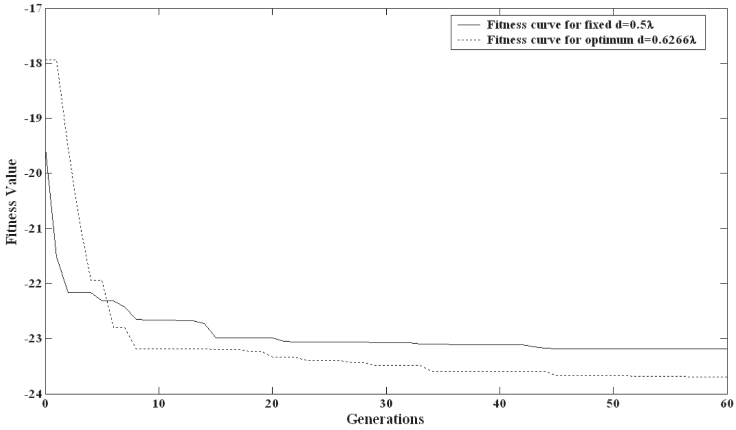

Figure 2. Convergence curves for thinned array design using modified PSO.

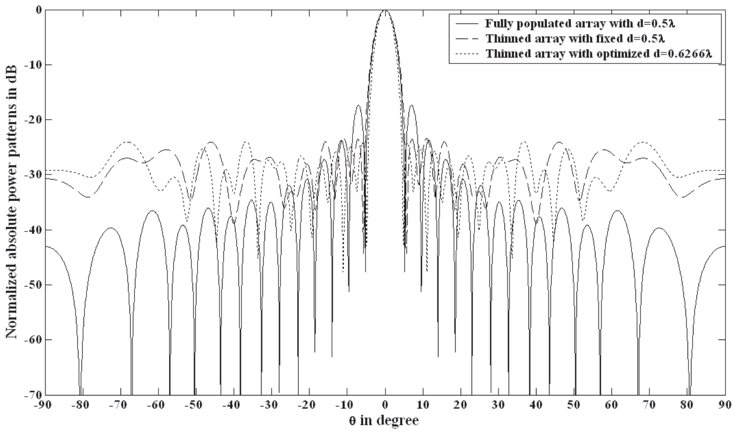

Figure 3. Normalized absolute power patterns in dB in XZ plane for fully populated array, thinned array with fixed inter-element spacing and thinned array with optimized inter-element spacing.

5. CONCLUSION

This paper presents a technique based on modified particle swarm optimization for thinning large multiple concentric circular ring antenna arrays of isotropic elements to generate a pencil beam in the vertical plane with reduced side lobe level. The synthesized thinned pattern with fixed inter-element arc spacing has half-power beam width very close to the value of a fully populated array of same size and shape and yet has better side lobe level. The synthesized thinned pattern with optimized inter-element arc spacing has even better half-power beam width than fully populated array and thinned array with fixed inter-element arc spacing. Both the synthesized thinned arrays using modified PSO has more than 220 elements switched off, i.e., a reduction of 50% or more of the total elements used in case of a fully populated array. This will reduce the cost of designing the arrays substantially.

Results clearly show a very good agreement between the desired and synthesized specifications. Results for thinned large multiple concentric circular ring isotropic antenna arrays have illustrated the performance of this proposed technique. This method is very simple and can be applied in practice to thin an array of other shapes.

ACKNOWLEDGMENT

The authors are grateful to Kalpana Chawla Space Technology Cell, Indian Institute of Technology, Kharagpur, India for supporting this work.

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