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ON THE SYSTEM MODELING OF ANTENNAS

Y. Duroc

Grenoble-INP
Laboratoire de Conception et d'Intégration des Systèmes (LCIS)
50, rue Barthélémy de Laffemas, BP54, 26902 Valence Cedex 9, France

Abstract—With the emergence of UWB systems and, in particular, the pulsed modulations, the modeling of antennas as Linear Time Invariant (LTI) systems has been studied in the last years extensively. This approach offers the advantage of modeling the antennas in frequency domain as well as in time domain. Further, the performance in terms of dispersion is taken into account implicitly in the modeling. This paper presents and compares methods in order to model antennas as LTI systems. The models are analyzed and discussed: physical interpretations are specified; differences between the models are highlighted; alternatives are proposed; advantages and drawbacks of approaches are emphasized.

1. INTRODUCTION

To describe and specify transient radiation and reception characteristics of antennas, the effective lengths (or effective heights) have been firstly considered in the literature [1, 2]. More recently, with the emergence of the UWB technology, the transfer function (i.e., frequency response) and associated impulse response (i.e., time response), which are derived from effective length of antenna, have been preferred to describe these characteristics. Therefore, UWB antennas are considered as linear time invariant (LTI) systems for which the performance will affect the overall performance of the wireless communication system. In the literature, several approaches [3–10] for characterizing the antennas as LTI systems have been proposed with diverse objectives: determine the spectral energy density of emitted pulses in order to comply with regulation masks, carry out a specific link budget, evaluate the effects of antenna dispersion on the waveform of pulses, achieve compact models, integrate models in co-simulation designs, etc. The

Corresponding author: Y. Duroc (Yvan.Duroc@grenoble-inp.fr).

theoretical approaches are generally the same: the radio link including antennas is modeled as a two-port network and characterized using the network theory based on S-parameters, Z-parameters, Y-parameters, h-parameters or $ABCD$-parameters. However, a careful reading of these different studies shows several issues even when the free space propagation (i.e., ideal propagation condition) is assumed. The antenna models can be different, and several variants exist: the models of the transmitting (TX) and receiving (RX) antennas are not always the same; the effect of the channel is occasionally included in the antenna model; the assumed conditions of the impedance matching can be also different. A few errors can even appear in some papers where, for example, the causality principle is not verified.

In this paper, new and critical views on the possible options to achieve the antenna models are reported. A unified approach to system modeling of antennas is presented: several inputs and outputs are taken into account in the modeling; the results are normalized and given when the impedances are matched; different theoretical approaches are compared, showing potential choices for modeling TX and RX antennas; the radio link model always remains coherent with the well-known Friis formula. Section 2 outlines the problematic of the antenna modeling. Section 3 develops the theoretical concepts of modeling and the associated analytical equations. Section 4 evaluates and discusses the different approaches and their results. Conclusion and perspectives are given in the last section.

2. PRESENTATION OF THE MODELING APPROACH

A wireless communications system is commonly modeled as illustrated in Fig. 1. The transmitter is composed of the emission circuit and the TX antenna, and the receiver comprises the reception circuit and the RX antenna. Further, the radio link can be defined by the association of the propagation channel and the TX/RX antennas.

In order to characterize the complete radio link, the functional

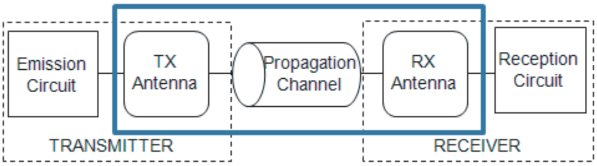

Figure 1. Block diagram of wireless communication system.

modeling, illustrated in Fig. 2, is usually used. It is constituted by two blocks, a transmission block and a reception block. This approach directly leads to the analytical expressions of the transmitting and receiving models. The input signal e excites the TX antenna which emits a radiated electric field $\vec{E}_{rad}$, while the RX antenna collects an incident electric field $\vec{E}_{inc}$ and provides the output signal s. Each block can then be characterized by a transfer function. The "transmission" transfer {input e — Radiated field $\vec{E}_{rad}$} is defined by a transfer function $\vec{H}_{rad}(f,\ \theta_{TX},\ \varphi_{TX})$ (where f is the frequency, and θ and ϕ are the polar and azimuth angles). Its time response, i.e., the associated impulse response, $\vec{h}_{rad}(t,\ \theta_{TX},\ \varphi_{TX})$ can be easily deduced from the inverse Fourier transform. In the same way, the "reception" transfer {incident field $\vec{E}_{inc}$ — output s} is characterized by a transfer function $\vec{H}_{rec}(f,\ \theta_{RX},\ \varphi_{RX})$ and the associated impulse response $\vec{h}_{rec}(t,\ \theta_{RX},\ \varphi_{RX})$.

Under some assumptions, the model of the complete transfer {input-output} can be deduced from both in frequency and time domains. The transfer function $H(f,\theta,\varphi,)$ and the associated impulse response $h(t,\theta,\varphi)$ are given by

$$H(f,\theta,\varphi) = \vec{H}_{rad}(f,\theta_{TX},\varphi_{TX}) \cdot \vec{H}_{rec}(f,\theta_{RX},\varphi_{RX}) \tag{1}$$

$$h(t,\theta,\varphi) = \vec{h}_{rad}(t,\theta_{TX},\varphi_{TX}) * \vec{h}_{rec}(t,\theta_{RX},\varphi_{RX}) \tag{2}$$

where the operator '$*$' denotes the convolution product.

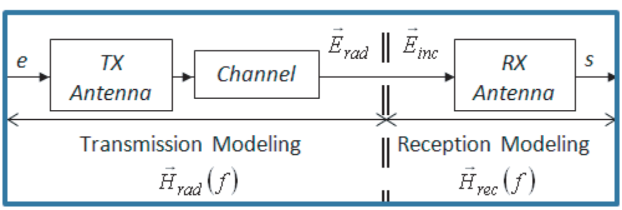

Figure 2. Block description of the modeling of the radio link.

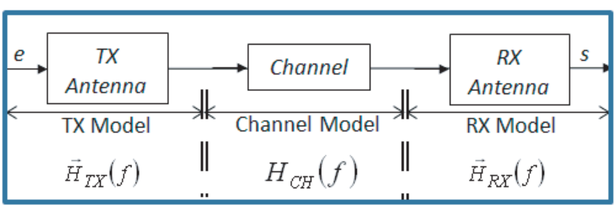

Figure 3. Radio link modeled from three functional blocks.

From these results, a second step can consist to achieve an intrinsic model of each element constituting the radio link which is then decomposed in three blocks as illustrated in Fig. 3: the TX antenna, the channel and the RX antenna. The transfer function $H(f,\theta,\varphi)$ and the associated impulse response $h(t,\theta,\varphi)$ are written as

$$\begin{aligned} H(f,\theta,\varphi) &= S(f)/E(f) \\ &= \vec{H}_{TX}(f,\theta_{TX},\varphi_{TX}) \cdot H_{CH}(f) \cdot \vec{H}_{RX}(f,\theta_{RX},\varphi_{RX}) \quad (3) \\ h(t,\theta,\varphi) &= \vec{h}_{TX}(t,\theta_{TX},\varphi_{TX}) * h_{CH}(t) * \vec{h}_{RX}(t,\theta_{RX},\varphi_{RX}) \quad (4) \end{aligned}$$

The following section presents the analytical expressions for achieving the antenna models from these modeling approaches.

3. ANALYTICAL EXPRESSIONS OF THE ANTENNA MODELS

3.1. Approach from Z-parameters

As presented in the introduction, the general principle is to consider the radio link (including the channel and the antennas) as a two-port network, and to analyze it from network theory. Among the potential approaches, the use of Z-parameters is based on equivalent electric circuits, and has the advantage to use electrical quantities. Thus, the equivalent circuit in the emission mode is modeled by a Thevenin equivalent circuit (V_G, Z_G), where V_G is the open-circuit voltage output of the generator, and Z_G is its output impedance loaded by the input impedance Z_A^{TX} of the TX antenna. Similarly, the equivalent circuit in reception mode is also modeled by a Thevenin equivalent circuit $\left(V_{OC},\ Z_A^{RX}\right)$ corresponding to the RX antenna, and a load impedance Z_L. This modeling is illustrated in Fig. 4.

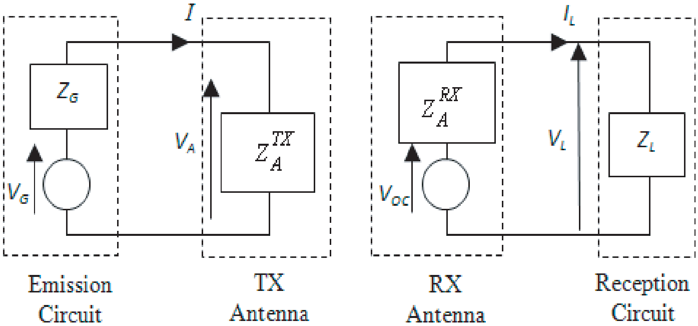

Figure 4. Modeling from equivalent electric circuits.

3.2. Transfer Function and Impulse Response of the Radio Link

Assuming free space, i.e., line-of-sight (LOS) propagation and far-field conditions, the TX and RX antennas constitutes an ideal channel which can be easily described from well-known electromagnetic principles. The radiated field in emission $\vec{E}_{rad}$ and the open circuit voltage in reception V_{OC} can be defined in the frequency domain from the effective lengths $\vec{h}_{eTX}(\omega, \theta_{RX}, \varphi_{RX})$ and $\vec{h}_{eRX}(\omega, \theta_{RX}, \varphi_{RX})$ of the TX antenna and RX antenna respectively as [11]:

$$\vec{E}_{rad}(\omega, \theta_{TX}, \varphi_{TX}) = j\frac{\omega}{c} Z_0 \frac{e^{-j\omega\frac{d}{c}}}{4\pi d} I(\omega) \cdot \vec{h}_{e_{TX}}(\omega, \theta_{TX}, \varphi_{TX}) \quad (5)$$

$$V_{OC}(\omega) = \vec{E}_{inc}(\omega, \theta_{RX}, \varphi_{RX}) \cdot \vec{h}_{e_{RX}}(\omega, \theta_{RX}, \varphi_{RX}) \quad (6)$$

where ω is the pulsation ($\omega = 2\pi f$), c is the speed of light, d is the distance between two antennas, Z_0 is the free space impedance. The inputs are the excitation current $I(\omega)$ for the emission, and the incident field $\vec{E}_{inc}$ for the reception. It should be highlighted that the introduced effective lengths are vectors. They depend on the orientation of antennas, and are related to the polarization, as if to say that the description is complete.

From these equations, the transfer functions $\vec{H}_{rad}(f, \theta_{TX}, \varphi_{TX})$ and $\vec{H}_{rec}(f, \theta_{RX}, \varphi_{RX})$ can be directly deduced as

$$\vec{H}_{rad}(\omega, \theta_{TX}, \varphi_{TX}) = \frac{\vec{E}_{rad}(\omega, \theta_{TX}, \varphi_{TX})}{I(\omega)}$$

$$= j\frac{\omega}{c} Z_0 \frac{e^{-j\omega\frac{d}{c}}}{4\pi d}\ \vec{h}_{e_{TX}}(\omega, \theta_{TX}, \varphi_{TX})\, \vec{E}_{rad}(\omega, \theta_{TX}, \varphi_{TX}) \quad (7)$$

$$\vec{H}_{rec}(\omega, \theta_{RX}, \varphi_{RX}) = \frac{V_{OC}(\omega)}{\vec{E}_{inc}(\omega, \theta_{RX}, \varphi_{RX})} = \vec{h}_{e_{RX}}(\omega, \theta_{RX}, \varphi_{RX}) \quad (8)$$

They can be also generalized using the other inputs, such as the voltages V_G or V_A for the emission, and the voltage V_L or the current I_L for the reception. Thus, sufficient coefficients Λ_{TX} and Λ_{RX}, which are function of the impedances, have to be introduced and adapted according to the considered cases, as follows:

$$\vec{H}_{rad}(\omega, \theta_{TX}, \varphi_{TX}) = \frac{\vec{E}_{rad}(\omega, \theta_{TX}, \varphi_{TX})}{e(\omega)}$$

$$= j\frac{\omega}{c} \Lambda_{TX} Z_0 \frac{e^{-j\omega\frac{d}{c}}}{4\pi d}\ \vec{h}_{e_{TX}}(\omega, \theta_{TX}, \varphi_{TX}) \quad (9)$$

$$\vec{H}_{rec}(\omega,\theta_{RX},\varphi_{RX}) = \frac{s(\omega)}{\vec{E}_{inc}(\omega,\theta_{RX},\varphi_{RX})} = \Lambda_{RX}\ \vec{h}_{e_{RX}}(\omega,\theta_{RX},\varphi_{RX}) \quad (10)$$

with $\Lambda_{TX} = 1,\ 1/Z_A^{TX}$ or $1/(Z_G + Z_A^{TX})$ for $e = I,\ V_A$ or V_G, respectively and $\Lambda_{RX} = 1,\ Z_L/(Z_L + Z_A^{RX})$ or $1/(Z_L + Z_A^{RX})$ for $s = V_{OC},\ V_L$ or I_L, respectively.

Assuming a LOS link, the radiated incident field and the generated electric field can be considered as equal. In consequence, the transfer function of the radio link is written as

$$H(\omega,\theta,\varphi) = \underbrace{j\frac{\omega}{c}Z_0\Lambda_{TX}\frac{e^{-j\omega\frac{d}{c}}}{4\pi d}\vec{h}_{e_{TX}}(\omega,\theta_{TX},\varphi_{TX})}_{\vec{H}_{rad}(\omega,\theta_{TX},\varphi_{TX})} \cdot \underbrace{\Lambda_{RX}\ \vec{h}_{e_{RX}}(\omega,\theta_{RX},\varphi_{RX})}_{\vec{H}_{rec}(\omega,\theta_{RX},\varphi_{RX})} \quad (11)$$

Equally, its impulse response (i.e., the response in time domain) is given by

$$h(t,\theta,\varphi) = \underbrace{\frac{Z_0\Lambda_{TX}}{4\pi dc}\delta\left(t-\frac{d}{c}\right) * \vec{h}_{e_{TX}}(t,\theta_{TX},\varphi_{TX})}_{\vec{h}_{rad}(t,\theta_{TX},\varphi_{TX})} * \underbrace{\Lambda_{RX}\vec{h}_{e_{RX}}(t,\theta_{RX},\varphi_{RX})}_{\vec{h}_{rec}(t,\theta_{RX},\varphi_{RX})} \quad (12)$$

where δ(t) is the Dirac distribution.

Finally, this study allows the calculation of the analytical expressions corresponding to the modeling of Fig. 2 and its related Equations (1) and (2).

3.3. Transfer Functions and Impulse Responses of Antennas

The objective is now to mathematically characterize each block of the radio link as shown in Fig. 3. Firstly, always assuming a far-field propagation, the transfer function of the free space is introduced and given by

$$H_{CH}(\omega) = \frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}} \quad (13)$$

where λ is the wavelength (with $\lambda = c/f = 2\pi c/\omega$).

It is important to notice that the transfer function of the channel, defined as above, clearly shows that:

- the loss factor of the channel (which is a function of the frequencies) is defined by the gain $|H_{CH}(\omega)|^2 = (\lambda/4\pi d)^2$, as it usually appears in Friis transmission equation;
- the phase $\phi = -\omega d/c$ is linear, due to the delay created by the propagation in free space and depending on the distance d between antennas;
- the group delay is defined from the derivative of the phase such as $\tau_g = -d\phi/d\omega = d/c$; the group delay is obviously a positive value corresponding to the propagation time.

The transfer functions of the TX and RX antennas can then be identified from the transfer function of the complete radio link (11) and the presented modeling in (3). Different formulations are possible.

Considering the voltages V_G and V_L (arbitrarily chosen) as input and output of the system respectively, the transfer function $H(\omega, \theta, \varphi)$ in (11) becomes

$$\frac{V_L(\omega)}{V_G(\omega)} = j\frac{\omega}{c} Z_0 \frac{1}{Z_A^{TX} + Z_G} \frac{Z_L}{Z_A^{RX} + Z_L} \frac{e^{-j\omega\frac{d}{c}}}{4\pi d} \vec{h}_{e_{TX}}(\omega) \cdot \vec{h}_{e_{RX}}(\omega) \quad (14)$$

The variables θ and φ are not explicitly mentioned in (14) and also in the following expressions for compactness. Introducing the transfer function of the free space channel, (14) can be rewritten such as

$$\frac{V_L(\omega)}{V_G(\omega)} = \underbrace{\left\{ j\frac{\omega}{c} Z_0 \frac{1}{Z_A^{TX} + Z_G} \frac{Z_L}{Z_A^{RX} + Z_L} \vec{h}_{e_{TX}}(\omega) \cdot \vec{h}_{e_{RX}}(\omega) \right\} \cdot \frac{\omega}{2\pi c}}_{H_{TX}(\omega) H_{RX}(\omega)} \cdot \underbrace{\left\{ \frac{\lambda}{4\pi d} e^{-j\omega\frac{d}{c}} \right\}}_{H_{CH}(\omega)} \quad (15)$$

From this equation, several options are possible to define the transfer function of TX and RX antennas. The term $(\omega/2\pi c)$ can be considered in the modeling of antennas according to several ways. Hence, this term can be distributed between emission and reception either as

$$\frac{V_L(\omega)}{V_G(\omega)}^{(1)} = \underbrace{\left\{ j\frac{\omega}{c} \sqrt{\frac{\omega}{2\pi c}} \sqrt{\frac{R_0}{Z_G}} \frac{\sqrt{Z_G Z_0}}{Z_A^{TX} + Z_G} \vec{h}_{e_{TX}}(\omega) \right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{ \frac{\lambda}{4\pi d} e^{-j\omega\frac{d}{c}} \right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{ \sqrt{\frac{\omega}{2\pi c}} \sqrt{\frac{Z_L}{R_0}} \frac{\sqrt{Z_L Z_0}}{Z_A^{TX} + Z_L} \vec{h}_{e_{RX}}(\omega) \right\}}_{H_{RX}(\omega)} \quad (16)$$

or taken into account in the transmission model

$$\frac{V_L(\omega)}{V_G(\omega)}^{(2)} = \underbrace{\left\{ j\frac{\omega}{c}\frac{\omega}{2\pi c}\sqrt{\frac{R_0}{Z_G}}\frac{\sqrt{Z_G Z_0}}{Z_A^{TX}+Z_G}\vec{h}_{e_{TX}}(\omega)\right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{\frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{\sqrt{\frac{Z_L}{R_0}}\frac{\sqrt{Z_L Z_0}}{Z_A^{TX}+Z_L}\vec{h}_{e_{RX}}(\omega)\right\}}_{H_{RX}(\omega)} \quad (17)$$

or taken into account in the reception model

$$\frac{V_L(\omega)}{V_G(\omega)}^{(3)} = \underbrace{\left\{ j\frac{\omega}{c}\sqrt{\frac{R_0}{Z_G}}\frac{\sqrt{Z_G Z_0}}{Z_A^{TX}+Z_G}\vec{h}_{e_{TX}}(\omega)\right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{\frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{\frac{\omega}{2\pi c}\sqrt{\frac{Z_L}{R_0}}\frac{\sqrt{Z_L Z_0}}{Z_A^{TX}+Z_L}\vec{h}_{e_{RX}}(\omega)\right\}}_{H_{RX}(\omega)} \quad (18)$$

The reference impedance R_0 is introduced in order to facilitate the comparative analysis with the S-parameters approach presented below. It should be also noticed that R_0 and Z_0 represent different physical quantities: R_0 is a reference impedance (generally equal to $50\,\Omega$), and Z_0 is the vacuum impedance ($\approx 120\pi\Omega$).

As suggested in [1], in order to obtain expressions including the impedances, the three transfers V_L/V_G can be simplified by the introduction of the normalized effective lengths $\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)$ and $\vec{h}_{eN_{RX}}(\omega,\theta,\varphi)$, defined as follows

$$\vec{h}_{eN_{TX}}(\omega) = \frac{\sqrt{Z_G Z_0}}{Z_A^{TX}+Z_G}\vec{h}_{e_{TX}}(\omega) = \frac{1-\Gamma_{TX}}{2}\frac{\sqrt{Z_G Z_0}}{R_G}\vec{h}_{e_{TX}}(\omega) \quad (19)$$

$$\vec{h}_{eN_{RX}}(\omega) = \frac{\sqrt{Z_L Z_0}}{Z_A^{TX}+Z_L}\vec{h}_{e_{TX}}(\omega) = \frac{1-\Gamma_{RX}}{2}\frac{\sqrt{Z_L Z_0}}{R_L}\vec{h}_{e_{TX}}(\omega) \quad (20)$$

where the reflection coefficients Γ_{TX} and Γ_{RX} are given by

$$\Gamma_{TX}(\omega) = \frac{Z_A^{TX}-Z_G^*}{Z_A^{TX}+Z_G} \quad (21)$$

$$\Gamma_{RX}(\omega) = \frac{Z_A^{RX}-Z_L^*}{Z_A^{RX}+Z_L} \quad (22)$$

with Z_G^* and Z_L^* the conjugate complex impedances of Z_G and Z_L.

Supposing that the generator impedance Z_G and the load impedance Z_L are equal to the reference impedance R_0, i.e., $Z_G = Z_L = R_0$ (impedance matching condition), (16), (17) and (18) can be rewritten as

$$\frac{V_L(\omega)}{V_{G(\omega)}}^{(1)} = \underbrace{\left\{j\frac{\omega}{c}\sqrt{\frac{\omega}{2\pi c}}\vec{h}_{eN_{TX}}(\omega)\right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{\frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{\sqrt{\frac{\omega}{2\pi c}}\vec{h}_{eN_{RX}}(\omega)\right\}}_{H_{RX}(\omega)} \quad (23)$$

$$\frac{V_L(\omega)}{V_{G(\omega)}}^{(2)} = \underbrace{\left\{j\frac{\omega}{c}\frac{\omega}{2\pi c}\vec{h}_{eN_{TX}}(\omega)\right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{\frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{\vec{h}_{eN_{RX}}(\omega)\right\}}_{H_{RX}(\omega)} \quad (24)$$

$$\frac{V_L(\omega)}{V_{G(\omega)}}^{(3)} = \underbrace{\left\{j\frac{\omega}{c}\vec{h}_{eN_{TX}}(\omega)\right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{\frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{\frac{\omega}{2\pi c}\vec{h}_{eN_{RX}}(\omega)\right\}}_{H_{RX}(\omega)} \quad (25)$$

where the normalized effective lengths $\vec{h}_{eN_{TX}}(\omega)$ and $\vec{h}_{eN_{RX}}(\omega)$ are given by

$$\vec{h}_{eN_{TX}}(\omega) = \frac{1-\Gamma_{TX}}{2}\sqrt{\frac{Z_0}{R_0}}\vec{h}_{e_{TX}}(\omega) \quad (26)$$

$$\vec{h}_{eN_{RX}}(\omega) = \frac{1-\Gamma_{RX}}{2}\sqrt{\frac{Z_0}{R_0}}\vec{h}_{e_{RX}}(\omega) \quad (27)$$

In order to complete this study, another solution could be to model the antenna in emission and in reception with a unique model. The fourth proposed model is then given by

$$\frac{V_L(\omega)}{V_{G(\omega)}}^{(4)} = \underbrace{\left\{\frac{\omega}{c}\sqrt{\frac{j}{2}}\vec{h}_{eN_{TX}}(\omega)\right\}}_{H_{TX}(\omega)} \cdot \underbrace{\left\{\frac{\lambda}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}}_{H_{CH}(\omega)} \cdot \underbrace{\left\{\frac{\omega}{c}\sqrt{\frac{j}{2}}\vec{h}_{eN_{RX}}(\omega)\right\}}_{H_{RX}(\omega)} \quad (28)$$

The introduced transfer functions and impulse responses of antennas are directional and complex vectors which depend on the effective lengths. The suggested description of the radio link takes into account the polarization and the impedance matching. It is also coherent with the Friis transmission equation and even more complete for the reason

Table 1. Summary of the models of the wireless radio link.

Model	$\vec{H}_{TX}(\omega,\theta,\varphi)$	$H_{CH}(\omega)$ [-]	$\vec{H}_{RX}(\omega,\theta,\varphi)$	$G_r(\omega,\theta,\varphi)$ [-]
(0)	$j\frac{\omega}{c}\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)\frac{e^{-j\omega\frac{d}{c}}}{4\pi d}$ $[m^{-1}]$		$\vec{h}_{eN_{RX}}(\omega,\theta,\varphi)$ $[m]$	$\frac{\omega^2}{\pi c^2}\left\|\vec{H}^{(0)}_{RX}(\omega,\theta,\varphi)\right\|^2$
(1)	$j\frac{\omega}{c}\sqrt{\frac{\omega}{2\pi c}}\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)$ $[m^{-1/2}]$	$\frac{\lambda e^{-j\omega\frac{d}{c}}}{4\pi d}$	$\sqrt{\frac{\omega}{2\pi c}}\vec{h}_{eN_{RX}}(\omega,\theta,\varphi)$ $[m^{1/2}]$	$\frac{2\omega}{c}\left\|\vec{H}^{(1)}_{RX}(\omega,\theta,\varphi)\right\|^2$ $\frac{2c}{\omega}\left\|\vec{H}^{(1)}_{TX}(\omega,\theta,\varphi)\right\|^2$
(2)	$j\frac{\omega}{c}\frac{\omega}{2\pi c}\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)$ $[m^{-1}]$	$\frac{\lambda e^{-j\omega\frac{d}{c}}}{4\pi d}$	$\vec{h}_{eN_{RX}}(\omega,\theta,\varphi)$ $[m]$	$\frac{\omega^2}{\pi c^2}\left\|\vec{H}^{(2)}_{RX}(\omega,\theta,\varphi)\right\|^2$ $\frac{4\pi c^2}{\omega^2}\left\|\vec{H}^{(2)}_{TX}(\omega,\theta,\varphi)\right\|^2$
(3)	$j\frac{\omega}{c}\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)$ [-]	$\frac{\lambda e^{-j\omega\frac{d}{c}}}{4\pi d}$	$\frac{\omega}{2\pi c}\vec{h}_{eN_{RX}}(\omega,\theta,\varphi)$ [-]	$4\pi\left\|\vec{H}^{(3)}_{RX}(\omega,\theta,\varphi)\right\|^2$ $\frac{1}{\pi}\left\|\vec{H}^{(3)}_{TX}(\omega,\theta,\varphi)\right\|^2$
(4)	$\frac{\omega}{c}\sqrt{\frac{j}{2\pi}}\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)$ $[rad^{-1/2}]$	$\frac{\lambda e^{-j\omega\frac{d}{c}}}{4\pi d}$	$\frac{\omega}{c}\sqrt{\frac{j}{2\pi}}\vec{h}_{eN_{TX}}(\omega,\theta,\varphi)$ $[rad^{1/2}]$	$2\left\|\vec{H}^{(4)}_{RX}(\omega,\theta,\varphi)\right\|^2$ $2\left\|\vec{H}^{(4)}_{TX}(\omega,\theta,\varphi)\right\|^2$

that the phase is considered. This property is relevant because the effects of antenna dispersion are included in the phase. In order to discuss and compare these results with the relation directly defining the transfer, (10) can be written from the normalized effective lengths such as:

$$\frac{V_L(\omega)}{V_{G(\omega)}}^{(0)} = \left\{j\frac{\omega}{c}\vec{h}_{eN_{TX}}(\omega)\frac{1}{4\pi d}e^{-j\omega\frac{d}{c}}\right\}\cdot\left\{\vec{h}_{eN_{RX}}(\omega)\right\} \tag{29}$$

Finally, Tab. 1 summarizes the presented different models.

3.4. Relation between Transfer Functions and Gains

The definition of the IEEE standard antenna gain $G(\omega,\theta,\varphi)$ which excludes the losses due to mismatching, is given by the following relation [12]

$$G(\omega,\theta,\varphi) = \underbrace{\frac{\omega^2}{\pi c^2}\left|\vec{h}_e(\omega,\theta,\varphi)\right|^2}_{=G_r(\omega,\theta,\varphi)} / \underbrace{\left(1-\Gamma|(\omega)|^2\right)}_{=\rho} = G_r(\omega,\theta,\varphi)/\rho \tag{30}$$

where $\Gamma(\omega)$ is the reflection coefficient of the antenna, $G_r(\omega,\theta,\varphi)$ is the effective continuous wave gain pattern (i.e., the realized gain), and is the reflection efficiency.

From the transfer function of TX and RX antennas, the realized gain is then given by

$$G_r(\omega,\theta,\varphi) = 2\left|\vec{H}_{TX}(\omega,\theta_{TX},\varphi_{TX})\right| \cdot \left|\vec{H}_{RX}(\omega,\theta_{TX},\varphi_{TX})\right| \quad (31)$$

$$G_r(\omega,\theta,\varphi)_{\mathrm{dB}} = \left|\vec{H}_{TX}(\omega,\theta_{TX},\varphi_{TX})\right|_{\mathrm{dB}} + \left|\vec{H}_{RX}(\omega,\theta_{TX},\varphi_{TX})\right|_{\mathrm{dB}} + 6\,\mathrm{dB} \quad (32)$$

It is also possible to define the realized gain using either the TX antenna or the RX antenna as indicated in Tab. 1.

4. DISCUSSION ON THE MODELING OF ANTENNAS

4.1. Analysis of Models

The model (2) presents the advantage of describing the physical behavior. In effect, the radiated transfer function {TX antenna + channel} is considered and divided in two blocks. Otherwise, the RX model is directly linked to the effective length of antenna, and so easy to define. Further, the Lorentz reciprocity relation is verified and leads to the following equations in the frequency domain and time domain respectively

$$H_{TX}(\omega) = \frac{j\omega}{c} H_{RX}(\omega) \quad (33)$$

$$h_{TX}(t) = \frac{1}{c}\frac{dh_{RX}(t)}{dt} \quad (34)$$

This property implies that unlike the antenna pattern, the gain or the impedance, the transfer functions for emission or reception are not equal, and their dimensions are different (as indicated on Tab. 1). In consequence, the realized gain expresses with two equations according to the considered model (TX or RX). Moreover, the definitions related to gains show a multiplication or division operation by the square of the variable pulsation.

It should be noticed that this model is generally adopted in the literature but not only. These results were expected. However, the presented approach is interesting because other models have been determined and can also present some relevant particularities.

Otherwise, some papers introduce the effective lengths of antenna directly as transfer functions. This approach of modeling is enough different and may present some disadvantages. Hence, an operation of differentiation has to be introduced in the model in emission. Further, the transfer function of the channel does not verify the presented properties in 3.3, notably for the loss factor.

In some situations, it can be appropriate to have models not directly connected to the physical behavior. Although the model (4) does not allow the calculation of the radiated electric field, it offers the advantage of having the two same transfer functions for the emission and the reception, i.e., a unique model of antenna. The gain of antenna can be then expressed in the same way whatever the model taken into account. This remark is relevant when a link budget is considered because of an identification with the Friis transmission equation can be easily done. The general and well-known Friis formula is given by

$$\frac{P_R}{P_T} = e_{pol} G_{TX} \left(\frac{\lambda}{4\pi d} \right)^2 G_{RX} \tag{35}$$

where P_T and P_R are the transmitted and received powers respectively, G_{TX} and G_{RX} are the realized gains of TX and RX antennas respectively, and e_{pol} represents the polarization loss factor.

It is possible to deduce an equivalent formulation of Friis formula such as

$$\begin{aligned} |H(f,\theta,\varphi)|^2 =& \underbrace{\left| \hat{h}_{e_{TX}}(f,\theta_{TX},\varphi_{TX}) \cdot \hat{h}_{e_{RX}}(f,\theta_{RX},\varphi_{RX}) \right|^2}_{=e_{pol}} \\ & \cdot \underbrace{|H_{TX}(f,\theta_{TX},\varphi_{TX})|^2}_{=G_{TX}/2} \cdot \underbrace{|H_{CH}(f)|^2}_{=\left(\frac{\lambda}{4\pi d}\right)^2} \cdot \underbrace{|H_{RX}(f,\theta_{RX},\varphi_{RX})|^2}_{=G_{RX}/2} \end{aligned} \tag{36}$$

where $\hat{h}_{e_{TX}}(f,\theta_{TX},\varphi_{TX})$ and $\hat{h}_{e_{RX}}(f,\theta_{RX},\varphi_{RX})$ are the unitary vectors of the effective lengths of TX and RX antennas respectively.

The advantage of this modeling is to present a single model for TX and RX antennas, and also a direct relation between the transfer functions and the gain.

It should be noticed that several approaches in the literature are based on this idea [13–16]. The radio link is then written by a scalar form such as

$$H(f,\theta,\varphi) = H_{TX}(f,\theta_{TX},\varphi_{TX}) \cdot H_{CH}(f) \cdot H_{RX}(f,\theta_{RX},\varphi_{RX}) \tag{37}$$

Assuming that the transmitting and receiving antennas have the same transfer function characteristics $H_a(f)$, θ_{TX} and θ_{RX} set to 90°, and φ_{TX} and φ_{RX} set to 0, the transfer function of each antenna is computed from the following equation

$$H_a(f) = \sqrt{H(f,\theta,\varphi)/H_{CH}(f)} \tag{38}$$

In consequence, the transfer function $H_a(f)$ characterizing the antenna is equivalent to the transfer function obtained in the presented model (4).

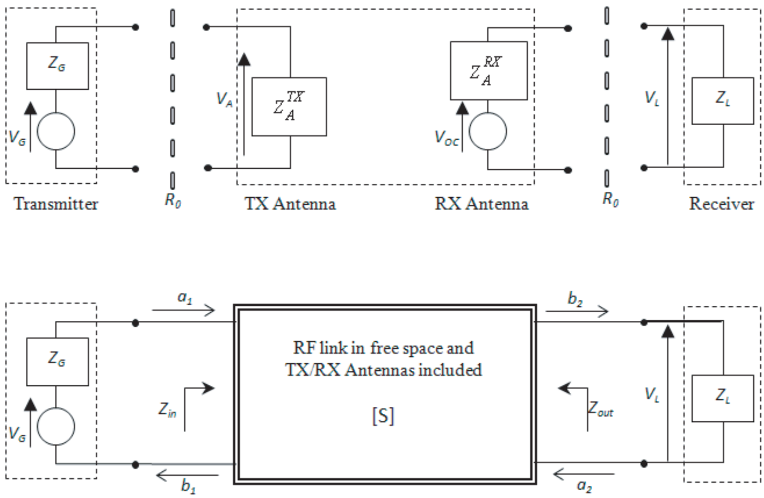

Figure 5. Modeling of the transmission: equivalent circuit model and S-parameters model.

4.2. Analysis and Comparison in Terms of S-parameters

In the study of Section 2, Z-parameters were considered. It is also relevant to analyze the modeling in terms of S-parameters. For a theoretical point of view, the modeling with equivalent electric circuits can be modified using a two-port network described by S-parameters, as illustrated in Fig. 5. R_0 is the reference impedance, a_i and b_j $(i, j = 1, 2)$ are the incident and reflected waves, Z_{in} and Z_{out} are the input and output impedances of the two-port network respectively.

From this modeling and, (11) can be written in terms of scattering parameters. If the impedances Z_G and Z_L are normalized and equal to $50\,\Omega$, i.e., $Z_G = Z_L = R_0$, (11) is then written as

$$\frac{V_{L\to R_0}(\omega)}{V_{G\to R_0}(\omega)} = j\frac{\omega}{c}Z_0\frac{R_0}{\left(Z_A^{TX}+R_0\right)\left(Z_A^{RX}+R_0\right)}\frac{e^{-j\omega\frac{d}{c}}}{4\pi d}\vec{h}_{e_{TX}}(\omega)\cdot\vec{h}_{e_{RX}}(\omega) \tag{39}$$

Otherwise, this transfer is defined by the transmission parameter S_{21} such as

$$\frac{V_{L\to R_0}(\omega)}{V_{G\to R_0}(\omega)} = \frac{S_{21}(\omega,\theta,\varphi)}{2} \tag{40}$$

In consequence, for any generator and any load (whatever the impedances), the transfer function between the voltage of the generator

V_G and the voltage at the load V_L is given by

$$\frac{V_L}{V_G} = \frac{\left(Z_A^{TX} + R_0\right)\left(Z_A^{RX} + R_0\right)}{\left(Z_A^{TX} + Z_G\right)\left(Z_A^{RX} + Z_L\right)} \frac{Z_L}{R_0} \frac{S_{21}\left(\omega, \theta, \varphi\right)}{2} \tag{41}$$

Introducing the input and output reflection coefficients $\Gamma_G\left(\omega\right)$ and $\Gamma_L\left(\omega\right)$ defined as below, (39) becomes

$$\frac{V_L}{V_G} = \frac{1 - \Gamma_G\left(\omega\right)}{1 - S_{11}\left(\omega\right)\Gamma_G\left(\omega\right)} \cdot \frac{S_{21}\left(\omega, \theta, \varphi\right)}{2} \cdot \frac{1 + \Gamma_L\left(\omega\right)}{1 - S_{22}\left(\omega\right)\Gamma_L\left(\omega\right)} \tag{42}$$

with

$$\Gamma_G\left(\omega\right) = \frac{Z_G\left(\omega\right) - R_0}{Z_G\left(\omega\right) + R_0} \tag{43}$$

$$\Gamma_L\left(\omega\right) = \frac{Z_L\left(\omega\right) - R_0}{Z_L\left(\omega\right) + R_0} \tag{44}$$

The matching terms are given respectively as follows:

$$\frac{1 - \Gamma_G\left(\omega\right)}{1 - S_{11}\left(\omega\right)\Gamma_G\left(\omega\right)} = \frac{Z_A^{TX}\left(\omega\right) + R_0}{Z_A^{TX}\left(\omega\right) + Z_G\left(\omega\right)} \tag{45}$$

$$\frac{1 + \Gamma_L\left(\omega\right)}{1 - S_{22}\left(\omega\right)\Gamma_L\left(\omega\right)} = \frac{Z_L\left(\omega\right)}{R_0} \frac{Z_A^{RX}\left(\omega\right) + R_0}{Z_A^{RX}\left(\omega\right) + Z_L\left(\omega\right)} \tag{46}$$

From the network theory, this transfer function can also be directly calculated [17] as

$$\left.\frac{V_L}{V_G}\right|_{ref} = \frac{1 - \Gamma_G\left(\omega\right)}{1 - \Gamma_{in}\left(\omega\right)\Gamma_G\left(\omega\right)} \cdot \frac{S_{21}\left(\omega, \theta, \varphi\right)}{2} \cdot \frac{1 + \Gamma_L\left(\omega\right)}{1 - S_{22}\left(\omega\right)\Gamma_L\left(\omega\right)} \tag{47}$$

where $\Gamma_{in}\left(\omega\right)$ is the input reflection coefficient of the network loaded by the impedance Z_L given by

$$\Gamma_{in}\left(\omega\right) = S_{11}\left(\omega\right) + \frac{S_{21}\left(\omega, \theta, \varphi\right) S_{12}\left(\omega, \theta, \varphi\right) \Gamma_L\left(\omega\right)}{1 - S_{22}\left(\omega\right)\Gamma_L\left(\omega\right)} \tag{48}$$

Equations (42) and (45) are dissimilar because of the different matching: $S_{11}\left(\omega\right)$ or $\Gamma_{in}\left(\omega\right)$. This difference can be explained. In the approach using the equivalent electric circuit model, the re-emitted power by the RX antenna has been neglected. Considering omni-directional antennas, the re-emitted power in a given direction is weak and so, it can be eliminated. It should be noticed that if the load is matched, i.e., $\Gamma_L\left(\omega\right) = 0$, or if the input is matched, i.e., $\Gamma_G\left(\omega\right) = 0$, then the two equations become equal. Further, if these two conditions are verified simultaneously, then (42) and (47) can be simplified as

$$\frac{V_L}{V_G} = \frac{S_{21}\left(\omega, \theta, \varphi\right)}{2} \tag{49}$$

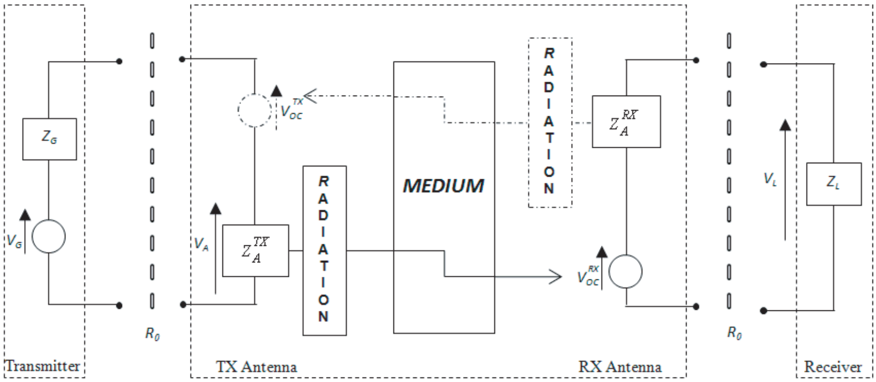

Figure 6. Electric model of the transmission taking into account the mutual coupling.

Figure 6 presents a more complete electric model which takes into account the previous remark. If the mutual coupling can be neglected, there is no radiation by the RX antenna and the voltage V_{OC}^{TX} is equal to zero.

Finally, for a practical point of view, it should be noticed that the effective lengths can be determined from the simple procedure presented in [18], which is based on the knowledge of the measured or simulated S-parameters. Afterwards, a specific model of TX antenna or RX antenna can be obtained from the equations summarized in Tab. 1.

5. CONCLUSION

The characterization of antennas as LTI systems presents the advantage to achieve time-frequency models. Therefore, the modeling can equally be done in the time domain and the frequency domain. The computations are easier in the frequency domain than in the time domain, because the convolution is replaced by multiplication. However, with the knowledge of the frequency responses, the time responses can be easily deduced. In addition to frequency domain parameters (gain, delay group, etc.), several time domain parameters have been introduced for describing the effects of antenna dispersion: such as peak output voltage from an incident waveform, envelope width, ringing, and transient gain [9]. These parameters are calculated directly from impulse responses.

This paper presents a study on the modeling of antennas as LTI systems. Several approaches of the modeling have been developed and discussed. The proposed antenna modeling techniques can be useful for system-level simulations of communication systems using pulsed modulations, as UWB technology. Further, it should be also noticed that the recent development of terahertz techniques is moving towards time approaches, and in consequence, such models could be very convenient.

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