

Mutual Inductance Calculation between Misalignment Coils for Wireless Power Transfer of Energy

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Abstract—In this paper we present a detailed theoretical analysis of lateral and angular misalignment effects in RF coils. Radio-frequency (RF) coils are used extensively in the design of implantable devices for transdermal power and data transmission. A design procedure is established to maximize coil coupling for a given configuration to reduce the effects of misalignment on transmission efficiency. Formulas are derived for the mutual inductance between all possible coil configurations including the coils of cross section, thin solenoids, pancakes and filamentary circular coils whose axes are laterally and angularly displaced. Coils are in air. In this approach we used the filament method and the mutual inductance between filamentary circular coils placed in any desired position. We completely describe all mathematical procedures to define coil positions that lead to relatively easy method for calculating the mutual inductance between previously mentioned coils. The practical coils in implantable devices fall into two categories: disk coils (pancakes) and solenoid coils. From the general approach for calculating the mutual inductance between coils of rectangular cross section with lateral and angular misalignments the mutual inductance between misalignment solenoids and disks will be calculated easily and accurately.

1. INTRODUCTION

Many contributions have been made in literature in relation to the problem of mutual inductance calculation for coaxial circular coils [1–4]. These contributions have been based on the application of Maxwell’s formula, Neumann’s formula, and the Biot Savart law. The calculation of the mutual inductance of inclined circular coils is of fundamental practical interests to electrical engineers and physicists. A survey of past literature shows that the greater part of this work was concentrated on the mutual induction calculation between an inclined filamentary circular coil and a thin wall solenoid or between two inclined thin wall solenoids. The mutual inductance of such configurations has been obtained over series expressed by Legendre polynomials [1–3]. Recently, a considerable work has been done in the calculation of the mutual inductance between circular coils with parallel and inclined axes using improved Grover’s formulas elliptic integrals, Bessel and Strouve functions [5–13]. The problem can be directly tackled using purely numerical methods such as finite and boundary element methods. However, sometimes analytical or semi-analytical methods might be possible, even though the problem is purely 3-D because coil axes can be parallel or inclined. In this paper, we will study the most general case for calculating the mutual inductance between coils of rectangular cross section placed in any desired positions. We use the filament method [5] and the formula for the mutual inductance between filamentary circular coils placed in any desired position [6]. The presented method covers all possible coil combinations either with inclined or parallel axes. This method is the most general then that one given in [5] in which one has to take into account center’s positions of secondary filamentary

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coils replacing the real secondary coil of rectangular cross section. In this case Grover's formula for inclined filamentary circular coils has to be modified. With the general formula for two filamentary inclined coils [6], we cover uniquely all inclined coils either with rectangular cross section or inclined coils with negligible cross section. The presented approach can be successfully used in applications such as transcutaneous energy transfer for medical implants, wireless power transfer through human skin based on the resonant techniques, inductive power pickup systems, RFID technology, PEEC modeling, magnetically controllable devices and sensors, wireless charging pads for portable electronic products such as mobile phones and iPods [14–24].

2. BASIC EXPRESSIONS

In [6] we calculated the mutual inductance between inclined filamentary circular coils placed in any desired position (see Figure 1),

$$M = \frac{\mu_0 R_s}{\pi} \int_0^{2\pi} \frac{[p_1 \cos \phi + p_2 \sin \phi + p_3] \Psi(k)}{k \sqrt{V_0^3}} d\phi \quad (1)$$

where R_s and R_p are the radius of secondary (small coil) and primary (large) coils, respectively. Here after we assume the primary coil is located in the plane “ $z = 0$ ” centered at the origin $(0, 0, 0)$, while R_s is placed in the inclined plane, whose general equation is $\lambda \equiv ax + by + cz$ centered at (x_c, y_c, z_c) , being a , b and c the coefficients of the plane's transformations. The constant $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space (vacuum) and the coefficients: $p_1 = \frac{\gamma c}{l}$ with $\gamma = \frac{y_c}{R_p}$ and $l = \sqrt{a^2 + c^2}$; $p_2 = \frac{\beta l^2 + \gamma ab}{lL}$ with $\beta = \frac{x_c}{R_p}$ and $L = \sqrt{b^2 + l^2}$; $p_3 = \frac{\alpha c}{L}$, with $\alpha = \frac{R_s}{R_p}$ and $\Psi(k) = (1 - \frac{k^2}{2})K(k) - E(k)$, with $k^2 = \frac{4V_0}{A_0 + 2V_0}$ the argument $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind [25, 26], respectively. Symbols:

$V_0^2 = \beta^2 + \gamma^2 + \alpha^2 [l_1 \cos^2 \phi + l_2 \sin^2 \phi + l_3 \sin 2\phi] + 2\alpha [q_1 \cos \phi + q_2 \sin \phi]$, whose parameters are: $l_1 = 1 - \frac{b^2 c^2}{l^2 L^2}$, $l_2 = \frac{c^2}{l^2}$, $l_3 = \frac{abc}{l^2 L}$ and $q_1 = \frac{\gamma l^2 - \beta ab}{lL}$, $q_2 = -\frac{\beta c}{l}$ and $A_0 = 1 + \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2\alpha [p_4 \cos \phi + p_5 \sin \phi]$ with: $\delta = \frac{z_c}{R_p}$, $p_4 = \frac{\gamma l^2 - \beta a - \delta bc}{lL}$, and $p_5 = \frac{\delta a - \beta c}{l}$.

Equation (1) describes the most general case, however there is one no contemplated and that is when the a secondary coil is aligned with the y -axis with normal $\mathbf{N} (0, 1, 0)$. In this case $a = c = 0$ and alternative definitions for above parameters can be found at the limit $l \rightarrow 0$, i.e., [6]:

$$\begin{aligned} p_1 &= 0, & p_2 &= -\gamma, & p_3 &= 0, & p_4 &= \beta, & p_5 &= \delta \\ l_1 &= 1, & l_2 &= 0, & l_3 &= 0, & q_1 &= \beta, & q_2 &= 0 \\ V_0^2 &= \beta^2 + \gamma^2 + \alpha^2 \cos^2 \phi + 2\alpha q_1 \cos \phi \end{aligned}$$

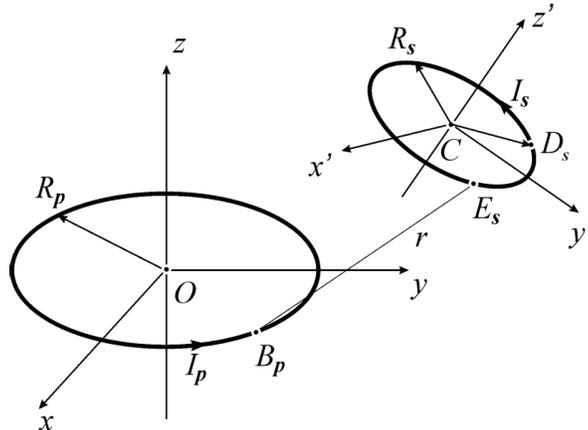


Figure 1. Filamentary circular coils with angular and lateral misalignment (most general case).

$$A_0 = 1 + \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2\alpha [p_4 \cos \phi + p_5 \sin \phi]$$

For this singular case it is also possible to find other expressions for previously mentioned coefficients which lead to the same results [6]. Note that “ λ ” defines the plane where the secondary coil is in three dimensional space with respect to the primary coil therefore its inclusion crucial for the mutual inductance calculations.

3. CALCULATION METHOD

For the purpose of computation the well-known filament method [5] is used. Our group recently use this approach for calculation of the self- and mutual-inductance to design actual coaxial coils with good experimental agreements [27]. Table 1 summarizes the notation used to distinguish the primary coil (larger coil) with respect to the secondary coil (smaller coil). The center of the primary coil is positioned at the point $O (0; 0; 0)$ and the plane $z = 0$ while the secondary coil is positioned at the point C in the plane λ as depicted in Figure 1.

If coils are such the packing factors along the axial and azimuth direction are small enough [27], then the electrical currents in these coils can be considered uniformly distributed over the whole cross sections on the winding. Using the well-known filamentary approach the current density in the coil cross section is assumed to be uniform, so that the filament currents are equal for each coil. This means that it is possible to take into consideration the pair of filamentary unit turn coils for which the mutual inductance is given by (1). Therefore, the fine dimension of the coil can be discretized into subdivisions along the x - and y -axis, respectively. Figure 2 illustrates the discretization of the primary coil into $(2N+1)(2S+1)$ cells while the secondary coil into $(2n+1)(2m+1)$ cells. The coil’s centers cuts the axial length a_a (primary coil) b_a (secondary coil) into two equal fractions as depicted in Figure 2.

Using the same procedures given in [5, 6] the mutual inductance can be expressed in the following form:

$$M = N_1 N_2 \sum_{g=-S}^S \sum_{h=-N}^N \sum_{p=-m}^m \sum_{q=-n}^n \frac{M(g, h, p, q)}{(2S+1)(2N+1)(2m+1)(2n+1)} \tag{2}$$

Table 1. Notation and location of coils.

Coil	Inner radius	Outer radius	Axial length	Radial length	Center	Plane
Primary	R_1	R_2	a_a	$R_2 - R_1$	$O (0, 0, 0)$	$z = 0$
Secondary	R_3	R_4	b_a	$R_4 - R_3$	$C (x_C; y_C; z_C)$	Λ

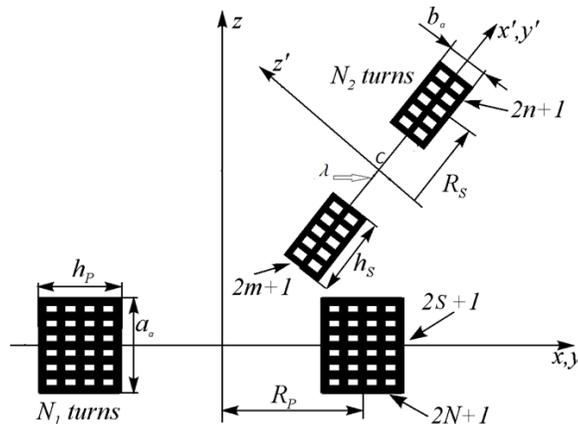


Figure 2. Discretization of the finite dimensions of two circular coils of rectangular cross section.

where N_1 and N_2 are the total number of turns for the primary and secondary coil, respectively. And inner coefficient of the series given by (1), i.e.,

$$M(g, h, p, q) = \frac{\mu_0 R_S(q)}{\pi} \int_0^{2\pi} \frac{[p_1 \cos \phi + p_2 \sin \phi + p_3] \Psi(k)}{k \sqrt{V_0^3}} d\phi, \quad (3)$$

with dependence on g, h, p and q as given below:

$$\begin{aligned} \alpha &= \frac{R_S(q)}{R_P(h)}, \quad \beta = \frac{x(p)}{R_P(h)}, \quad \gamma = \frac{y(p)}{R_P(h)}, \quad \delta = \frac{z(g, p)}{R_P(h)} \\ R_P(h) &= R_P + \frac{h_P}{(2N+1)}h, \quad h = -N, \dots, 0, \dots, N \\ R_S(q) &= R_S + \frac{h_S}{(2n+1)}q, \quad q = -n, \dots, 0, \dots, n \\ R_P &= \frac{R_1 + R_2}{2}, \quad R_S = \frac{R_3 + R_4}{2}, \quad h_P = R_2 - R_1, \quad h_S = R_4 - R_3 \\ x(p) &= x_C + \frac{b_a a}{(2m+1)}p, \quad p = -m, \dots, 0, \dots, m \\ y(p) &= y_C + \frac{b_a b}{(2m+1)}p, \quad p = -m, \dots, 0, \dots, m \\ z(g, p) &= z_C + \frac{a_a}{(2S+1)}g + \frac{b_a c}{(2m+1)}p \\ g &= -S, \dots, 0, S; \quad p = -m, \dots, 0, \dots, m \end{aligned}$$

Table 2. Formulas of mutual inductance for typical configurations of inclined coils.

Configurations	Simplifications	Equation (2)
1. A thick circular coil of rectangular cross section and a filamentary circular coil with inclined axis	$N_2 = 1; h_s = 0; b_a = 0$ and $R_S(q) = R_S$	$M = N_1 \sum_{g=-S}^S \sum_{h=-N}^N \frac{M(g, h)}{(2S+1)(2N+1)}$
2. A thick circular coil of rectangular cross section and an inclined thin disk	$b_a = 0$	$M = N_1 N_2 \sum_{g=-S}^S \sum_{h=-N}^N \sum_{q=-n}^n \frac{M(g, h, q)}{(2S+1)(2N+1)(2n+1)}$
3. A circular coil and an inclined thin wall solenoid	$h_s = 0; R_S(q) = R_S$	$M = N_1 N_2 \sum_{g=-S}^S \sum_{h=-N}^N \sum_{p=-m}^m \frac{M(g, h, p)}{(2S+1)(2N+1)(2m+1)}$
4. Two thin inclined wall solenoids	$h_P = h_S = 0;$ $R_P(h) = R_P; R_S(q) = R_S$	$M = N_1 N_2 \sum_{g=-S}^S \sum_{p=-m}^m \frac{M(g, p)}{(2S+1)(2m+1)}$
5. A thin solenoid and an inclined thin disk coil	$h_P = 0; b_a = 0;$ $R_P(h) = R_P$	$M = N_1 N_2 \sum_{g=-S}^S \sum_{q=-n}^n \frac{M(g, q)}{(2S+1)(2n+1)}$
6. A thin solenoid and an inclined filamentary circular coil	$N_2=1; h_P = h_S = 0;$ $b_a = 0; R_P(h) = R_P;$ $R_S(q) = R_S$	$M = \sum_{g=-S}^S \frac{M(g)}{(2S+1)}$
7. Two thin inclined disks	$a_a = b_a = 0$	$M = N_1 N_2 \sum_{h=-N}^N \sum_{q=-n}^n \frac{M(h, q)}{(2N+1)(2n+1)}$
8. A thin disk and an inclined filamentary coil	$N_2 = 1; a_a = b_a = 0;$ $h_s = 0; R_S(q) = R_S$	$M = \sum_{h=-N}^N \frac{M(h)}{(2N+1)}$

Note that (2) has the same singularity as in (1) in the case $a = c = l = 0$, therefore coefficients are likewise determined using the limit $l \rightarrow 0$ as pointed out earlier. There are number of configurations that can be extracted from (2). Table 2 summarizes most probable combinations of circular coils with rectangular cross sections (e.g., disk coils, thin wall solenoids, filamentary, etc.) engineers may have in designing [15–24].

4. EXAMPLES

In the following examples we give some comparative results obtained by the presented method and those obtained in literature. As we previously said the practical coils in implantable devices fall into two categories: disk coils (pancakes) and solenoid coils. In the following examples we will treat them as well as their combinations with filamentary circular coils.

4.1. Example 1

Two reactance coils of rectangular cross section with parallel axes with coil characteristics (see for instance [9, 11]): $R_P = 7.8232$ cm; $R_S = 11.7729$ cm; $l_a = 14.2748$ cm; $l_a = 2.413$ cm; $h_P = 1.397$ cm; $h_S = 4.1529$ cm; $c = 7.366$ cm; $d = 30.988$ cm. The numbers of turns are $N_1 = 1142$ and $N_2 = 516$. c is the axial displacement of the centers of the two coils and d is the relative perpendicular displacement of the axes of two coils. Calculate the mutual inductance between reactance coils. Table 3 summarizes the dimensions according to the present method. The mutual inductance calculation the formulae to be applied is the general case according to (2).

Table 3. Physical dimension and localization of coils of Example 1.

Coil	$N_{1(2)}$	$R_{1(3)}$ (cm)	$R_{2(4)}$ (cm)	$a_a(b_a)$ (cm)	Center (cm)	Plane position (cm)
Primary coil	1142	7.1247	8.5217	14.2748	$O(0, 0, 0)$	$z = 0$
Secondary coil	516	9.69645	13.84935	2.413	$C(0, 30.988, 7.366)$	$z = 7.366$

The mutual inductance for this configuration was numerically computed, using Mathematica software, by Conway [11]:

$$M = -1.42256038 \mu\text{H}.$$

Using the method reported by Akyel et al. [7] the mutual inductance for this system was:

$$M = -1.42262284 \mu\text{H}.$$

If we assume that the total number of subdivisions of both coils is 12 (i.e., $N = S = n = m = 12$), the computation using (2) after 291 s yielded:

$$M = -1.42262284 \mu\text{H}.$$

The agreement between these two computations is good. The small relative error ($\sim 10^{-5}$) between the above values is not significant for experimental determination of the mutual inductance. The accuracy of cutting edge technologies for impedance measurements does not exceed 0.08% (e.g., Agilent 4294A Precision Impedance Analyzer Technologies Inc., USA).

In order to assess the impact of number of subdivisions instead of 12 we use 20 (e.g., $N = S = n = m = 20$). And the mutual inductance was recalculated yielding to:

$$M = -1.422583607 \mu\text{H}.$$

However, the computational time increases from 291 s to 32.3 min. The absolute discrepancy between these values was 0.0028%. As pointed earlier the latter discrepancy exceeds the experimental accuracy; therefore increasing the number of subdivisions does not impact accuracy but increase undesirably the computational time. One conclusion we can draw from this example is that there is an optimum number of subdivisions to reduce the computational time of the mutual inductance of actual coils.

Table 4. Physical dimensions and localization of Example 2.

Coil	$N_{1(2)}$	R_P (cm)	R_S (cm)	$a_a(b_a)$ (cm)	Center (cm)	Plane position (cm)
Primary coil	120	7		12	$O(0, 0, 0)$	$z = 0$
Secondary coil	60		5	4	$C(0, 0, 0)$	$\sin \alpha \cdot y + \cos \alpha \cdot z = 0$

4.2. Example 2

Next example the calculation of the mutual inductance of two solenoids reported by Snow was considered [2]. One solenoid has radius 6 cm, length 12 cm, and 10 turns, while the other 5 cm, 4 cm, 15 turns, respectively. Their axes were inclined at an angle whose cosine is 0.8. Center of the second solenoid is in the middle plane of the primary solenoid. This is the combination of two thin inclined wall solenoids that was described in Table 2 (see configuration 4). The physical dimensions of solenoids are given in Table 4.

The computation of the mutual inductance (see configuration 4 in Table 2) after 257 s considering $S = m = 300$ gave:

$$M = 336.8474 \mu\text{H}.$$

In [2] Snow obtained the mutual inductance for this system equal to:

$$M = 336.8473 \mu\text{H}.$$

In an early study the mutual inductance was obtained [5] yielding to:

$$M = 336.8483 \mu\text{H},$$

and using Fast-Henry software gave [14]:

$$M = 337.4905 \mu\text{H}.$$

The mutual inductance of this configuration (see configuration 4 in Table 2) was recalculated and yielded to:

$$M = 336.8489 \mu\text{H}$$

Once again the time was dropped to 19s when the number of subdivisions was $S = m = 80$ instead of $K = m = 300$. The absolute discrepancy between these values was small ($\sim 0.0001\%$).

In an attempt to reduce even further the computational time the number of subdivision was set to $S = m = 30$. The calculation after 5 s for this combination yielded to:

$$M = 336.8539 \mu\text{H},$$

Yet, about 0.001% discrepancy between the values was observed. Decreasing further the number of subdivision was not warranted without deteriorating accuracy. Therefore for this type of configuration it is recommended set the number of subdivision to 30.

4.3. Example 3

In this example let's consider a solenoid (radius 6 cm, length 12 cm, number of turns 120) and a circular filament coil of radius 5 cm. The latter is centered at different points on the axis of the solenoid with different angles of inclination. Axes are inclined at an angle whose cosine is 0.4. Center of the circle on the axis of the solenoid is 6 cm outside the end plane as described in [2]. For this case the computation of the mutual inductance is given by the configuration 6 on Table 2. Physical dimension and localization of the filament coil and solenoid are the same of Table 4 with the exception N_2 and b_a are not applicable and the center of the origin of coordinates for the secondary coil $C(0, 0, 0.12)$ and localized in same plane as before but at $z = 0.12$ cm; with $\cos \alpha = 0.4$, ($a = 0$; $b = \sin \alpha$; $c = \cos \alpha$).

In our first computation the number of subdivision was set to $S = 1700$, then after 2.1 s yields to:

$$M = 0.5735 \mu\text{H}.$$

The mutual inductance obtained by Snow [2] was,

$$M = 0.5735 \mu\text{H}.$$

Using one of our earliest approaches for the calculation of the mutual inductance [5] results,

$$M = 0.5735 \mu\text{H}.$$

The recalculation of the mutual inductance but this time setting $S = 30$, then after 0.1 s gave:

$$M = 0.5734 \mu\text{H}$$

A discrepancy between these previous computation of 0.0173% let us to conclude $S = 30$ is a good approximation for the discretization of the solenoid involved in this example.

4.4. Example 4

In this example we treat two disk coils (pancakes) where the centers of filamentary coils that replace the second inclined disc are on the axis of the primary coil [5]. Calculate the mutual inductance between these pancakes when their axes are inclined at an angle whose cosine is 0.9. This is the combination of two thin inclined disks with N_1 and N_2 turns as described in Table 2 (see configuration 7). Physical dimension and localization of pancakes are depicted in Table 5.

Table 5. Physical dimension and localization of coils of Example 4.

Coil	$N_{1(2)}$	$R_{1(3)}$ (cm)	$R_{2(4)}$ (cm)	$a_a(b_a)$ (cm)	Center (cm)	Plane position (cm)
Primary coil	200	0.4	0.6	0	O(0,0,0)	$z = 0$
Secondary coil	100	0.15	0.25	0	C (0, 0, 0)	$\sin \alpha \cdot x + \cos \alpha \cdot z$

Setting the total number of subdivision to $N = n = 60$ the mutual inductance gave:

$$M = 98.6965 \mu\text{H}.$$

In [5] the mutual inductance has been obtained by the presented approach,

$$M = 98.6965 \mu\text{H},$$

and also by the software Fast-Henry [14],

$$M = 97.6160 \mu\text{H}$$

Once again the above results are in good agreement.

4.5. Example 5

Two annular disk coils have each an inner radius of 20 cm and an outer radius 50 cm. They each have 100 turns and lie in parallel planes of constant $z = 25$ cm. The distance between axes is 10 cm. Physical dimensions and localizations of these disks are depicted in Table 6. To calculate the mutual inductance between these disks we use the mutual inductance of configuration 7 (see Table 2).

Table 6. Physical dimension and localization of coils of Example 5.

Coil	$N_{1(2)}$	$R_{1(3)}$ (cm)	$R_{2(4)}$ (cm)	$a_a(b_a)$ (cm)	Center (cm)	Plane position (cm)
Primary coil	100	20	50	0	O (0, 0, 0)	$z = 0$
Secondary coil	100	20	50	0	C (10, 0, 25)	$z = 25$ cm

We compare the accuracy of the presented method for different number of subdivisions with an independent exact approach for thin coils with parallel axes based on Bessel functions [13].

In [13], Conway obtained by his approach the mutual inductance between previously mentioned disks,

$$M = 2.278193661584031 \mu\text{H}$$

In his approach using the software Mathematica, Conway gave 16 significant figures. We will use this result to test the accuracy and the computational time for different number of subdivisions of presented method in this paper. The mutual inductance calculation presented in this paper was made in Matlab and Mathematica implementation.

In Table 7 we give the mutual inductance calculation for different number of subdivisions, computational time and discrepancy regarding to Conway's result. From Table 7 we can see that all results are in a very good agreement with the result given in [13]. The significant figures were bolded to compare results of the presented approach with the result given in [13]. Also we can see that increasing in the number of subdivisions of disks doesn't give considerable accuracy but the computational time increases considerably. Thus, for practical engineering applications we don't need a lot of subdivisions to better approximate coils. It is enough to take 4, 8 or 16 disks' subdivisions to have a very good accuracy with minimum computational time. The same conclusion can be used in the treatment of the mutual inductance between all types of coils treated in this paper. All presented examples for these coils show very accurate calculation of the mutual inductance with reduced computational time. Thus the presented approach can be considered as the simplest method in the calculation of the mutual inductance between real circular coils under lateral and angular misalignments.

The effect of the total number of subdivisions upon accuracy is better understood to the light of Figure 3. The data plotted corresponds to that of Table 7. On the left y -axis the mutual inductance was plotted while on the right y -axis the discrepancy. As mentioned earlier the experimental uncertainty on measurements does not exceed 0.08%. In Figure 3 at ($N = n = 10$) the discrepancy between calculated values is 0.075% and therefore it is the optimum number of subdivisions for this arrangement. Here, the optimum value means to attain a desired accuracy, within a reasonable computational time, for a minimum number of cell subdivisions. This fact becomes clear from Figure 4 where the total number of subdivisions was plotted vs the computational time as well as the mutual inductance. Likewise, the data is the one given on Table 7.

Figure 4 illustrates the monotonically increasing dependence of the computational time with respect to number of subdivisions. This increase is exponential and therefore the optimum number of subdivisions is crucial to optimize the time and microprocessor resources. The former is important in the case the corrections of misalignments are done in real time. In this case the control is *via* closed-loop and feedback type of technique where decisions whether or not corrections need to be made are based on previous computation stages. In this case the computational time is critical and may decide the sampling rate of this application.

Table 7. Comparison of computational efficiency.

N/n All	M — This work (μH)	Computational Time (Seconds)	Discrepancy (%)
4	2.280116287235428	0.077170	-0.08439255
8	2.278731101248461	0.228908	-0.02359060
16	2.278336179839532	0.856784	-0.00625576
32	2.278230388412927	3.238267	-0.00161210
64	2.278202985704301	13.139142	-0.00093241
100	2.278197502109049	31.422768	-0.00016858
150	2.278195374152472	76.183236	-0.00009130
200	2.278194626504623	131.765031	-0.00004235
250	2.278194279749312	197.192871	-0.00002713
300	2.278194091151046	288.381211	-0.00001886
350	2.278193977334336	379.562336	-0.00001386
400	2.278193903416549	493.767600	-0.00001062
450	2.278193852714596	649.236990	-0.00000839
500	2.278193816434139	887.394346	-0.00000680

4.6. Example 6

Let's take into consideration wireless domino-resonator systems with non-coaxial resonators [23, 24]. The configuration of a circular domino-resonator is the system with n identical circular resonators in which all the centers of the resonators are placed on a circular path with radius $R = 0.6$ m and the center of each resonator is placed in a same plane with the center of the circular path (See Figure 5). If we consider resonators as simple circular coils of the radius $R_R = 0.5$ m calculate the mutual inductance between two adjacent resonators for $n = 2; 3; 4; 5; 6; 7$ and 8 .

In Table 8 we give values of the mutual inductance for different number of resonators. From this table we can see that the mutual inductance between two adjacent resonator increases that was expected [23, 24].

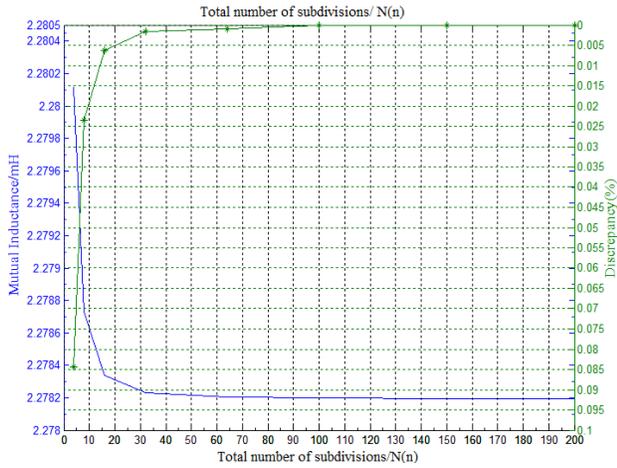


Figure 3. Dependence of number of subdivision on the computational time.

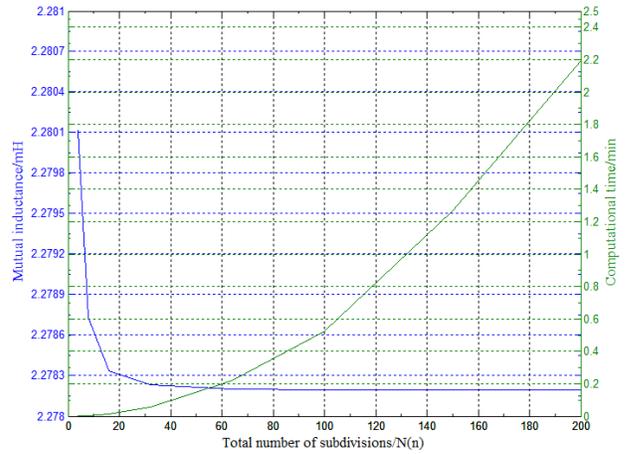


Figure 4. Computational time with respect to total number of subdivisions.

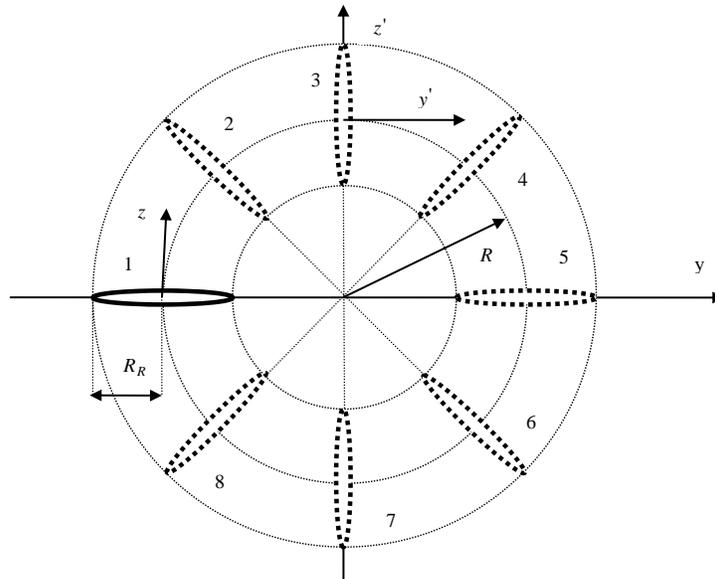


Figure 5. Configuration of a circular domino-resonator system with 8 identical circular resonators.

Table 8. Mutual inductance between two adjacent resonator (Equation (1)).

n	Θ	$C(x_C, y_C, z_C)$ (m)	$\{a; b; c\}$	M (nH)
2	π	$C(0; 1.2; 0)$	$\{0; 0; -1\}$	62.90315288
3	$2\pi/3$	$C(0; 0.9; 0.6 \cdot 0.3^{0.5})$	$\{0; 3^{0.5}; -1\}$	94.39741981
4	$\pi/2$	$C(0; 0.6; 0.6)$	$\{0; 1; 0\}$	149.3340559
5	$2\pi/5$	$C(0; 0.6 \cdot (1 - \cos 2\pi/5); 0.6 \cdot \sin 2\pi/5)$	$\{0; \tan 2\pi/5; 1\}$	209.9549206
6	$\pi/3$	$C(0; 0.3; 0.6 \cdot 0.3^{0.5})$	$\{0; 3^{0.5}; 1\}$	270.4709781
7	$2\pi/7$	$C(0; 0.6 \cdot (1 - \cos 2\pi/7); 0.6 \cdot \sin 2\pi/7)$	$\{0; \tan 2\pi/7; 1\}$	329.4709781
8	$\pi/4$	$C(0; 0.3(2 - 2^{0.5}); 0.3 \cdot 2^{0.5})$	$\{0; 1; 1\}$	385.2920506

5. CONCLUSION

This paper presents a consistent analytical derivation of the mutual inductance for real coils under lateral and angular misalignments. The mutual inductance between circular coils of rectangular cross section placed in any desired position is obtained by simple integral whose kernel function is the combination of the complete elliptic integrals of the first and second kind. This method is suitable for any combinations of inclined coils such as coils of rectangular cross section, disk coils, solenoids and filamentary circular coils. Also, the presented approach is suitable either for large coils or for micro coils that was confirmed by several examples. It is notable that the presented method gives high accuracy for reasonable computational time so that it can be useful tool for engineers and physicists which do not have to use complicated numerical methods to calculate the mutual inductance between inclined circular coils of different shapes that are demonstrated in this paper. From presented examples one can conclude that an easy and suitable method for calculating the mutual inductance between real coils which are positioned in any desired position (angular and lateral misalignments) has been given. By our knowledge the presented method is the simplest known method in the calculation of the mutual inductance between circular coils under lateral and angular misalignments. This general method includes also either lateral or angular misalignment as the special cases.

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