

Novel Finite Airy Array Beams Generated from Gaussian Array Beams Illuminating an Optical Airy Transform System

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Abstract—In this work, a novel family of Finite Airy array beams have been produced by an optical Airy transform system illuminated by Gaussian Array beams. Based on the generalized Huygens-Fresnel integral, an analytical expression is developed to describe the pattern properties of the beam generated at the output plan of the optical system. The well-known Finite Airy beam generated from the fundamental Gaussian beam using an optical Airy transform system is deduced, here, as a particular case of the main result of the actual study. Numerical calculations are performed to show the possibility to create a multitude of Finite Airy array beams with controllable parameters depending on the number of beamlets, the distance between the adjacent modules and the positions and orientations of the beamlets.

1. INTRODUCTION

In recent years, Airy beams have attracted great interest due to their unique features, including non-spreading, self-healing, self-acceleration (even in the absence of external force) and parabolic trajectories [1–5]. These characteristics lead them to be good candidate in many applications such as optical micro-manipulation [6], curved plasma channel generation [7, 8], optical microscopy and scanning microscopy using optical Airy bullets [9], vacuum electron acceleration [10], trapping and guiding micro-particles [11, 12]. Several methods have been devoted to generate the Airy beams with finite energy. Among these, we can cite the cubic phase, $3/2$ phase only pattern and three-wave mixing processes in asymmetric nonlinear photonic crystals [13–18].

The integral transformations including Fourier, Hilbert and Airy transforms have long been of interest due to their wide applications in solving many problems in applied mathematics, engineering sciences and optics. The Airy transform which was firstly introduced in mathematics by Widder in 1979 [19] has been used in physics to solve the Schrödinger equation for free particle and free fall [20]. In optics, the concept of Airy transform has been applied in solving the free-space wave equation of light beams [21]. Recently, Jiang et al. [22] have implanted the Airy transform technique as a novel tool to generate and control the Finite Airy beams. The authors have also derived the amplitude expression of the so-called Airy-related beam by using an optical Airy transform system illuminated by a flattened Gaussian beam [23].

On the other hand, increasing attention has been paid to laser Array beams over the last few decades due to their applications in high-power laser systems, inertial confinement fusion and high-energy weapons and free-space optical communications [24, 25]. Particularly, the Gaussian Array beams, which are constructed by combining multiple separate beamlets, have been the subject of several studies [26–32].

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In this work, by means of the two-dimensional optical Airy transform technique and with a setup including additional lens before a spatial light modulator [22], we introduce an analytical expression of a new Finite Airy Array beams family generated by an optical Airy transform system illuminated by Gaussian Array beams. Compared to a single conventional Airy beam, the creation of Airy Array beam can enable the parallel processing with intrinsic high processing efficiency for various applications; i.e., nanofabrication, generation of curve plasmon channel, light sheet microscopy, etc. Besides, compared to the Airy beam generated by Fourier transform of Gaussian beam modulated by a cubic phase [4], the Airy transform setup used in this work presents some practical advantages in controlling the output beam [22, 23]. It is provided that when the incident Gaussian beam is translated by a vector in the input plane, the output Airy beam will be translated by the same vector in the output plane. The optical Airy transform setup permits the control of the launch angle by varying the axial position of the beam waist. Furthermore, it is useful in directly producing and controlling new Airy beams from initial finite energy Airy beams [22]. In this paper, the properties of the new Airy Array beams generated are analyzed and discussed numerically. The obtained results are consistent with the previous works which can be regarded as particular cases of this study.

The paper is structured as follows. In the coming section, we present the distributions of the two-dimensional Gaussian Array beams as incident beams and their transverse intensity patterns at the source plan are illustrated. In Section 3, The mathematical model describing the propagation of a laser beam through an optical Airy transform system is presented, and an analytical expression of a Finite Airy Array beam generated from the Gaussian Array beam illuminating the optical Airy transform system is derived. The previous result concerning the generation of the Finite Airy beam from the fundamental Gaussian beam is deduced as a particular case of our main study. Some numerical calculations are carried out in Section 4 to illustrate the effects of the initial beam and the optical system parameters on the properties of generated beam. In the last section, we summarize the main results of our study.

2. GAUSSIAN ARRAY BEAM

At the source plane ($z = 0$), consider a Gaussian array beam composed of $M \times M$ beamlets. In the Cartesian coordinate system, the cross spectral density of the resulting beam is expressed as [26]

$$E(x_0, y_0, z = 0) = \sum_{m=-\frac{M-1}{2}}^{\frac{M-1}{2}} \exp \left[- \left(\frac{x_0 - mr_d}{w_0} \right)^2 \right] \cdot \sum_{n=-\frac{M-1}{2}}^{\frac{M-1}{2}} \exp \left[- \left(\frac{y_0 - nr_d}{w_0} \right)^2 \right], \quad (1)$$

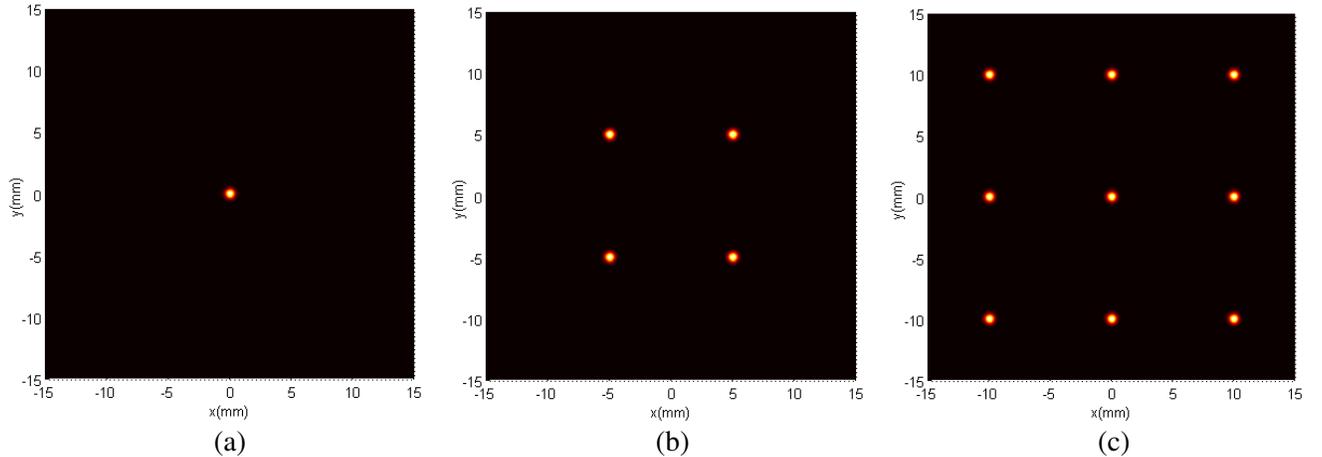


Figure 1. Intensity distribution contours of the Gaussian Array beam for different values of M : (a) $M = 1$, (b) $M = 2$, (c) $M = 3$, with $r_d = 10$ mm and $w_0 = 0.5$ mm.

where w_0 is the waist width of each Gaussian beamlet amplitude distribution, r_d the separation distance between adjacent modules and M the number of beamlets. Fig. 1 depicts the transverse intensity distribution of the Gaussian beam array at source plane at $z = 0$ for different $M \times M$. From the

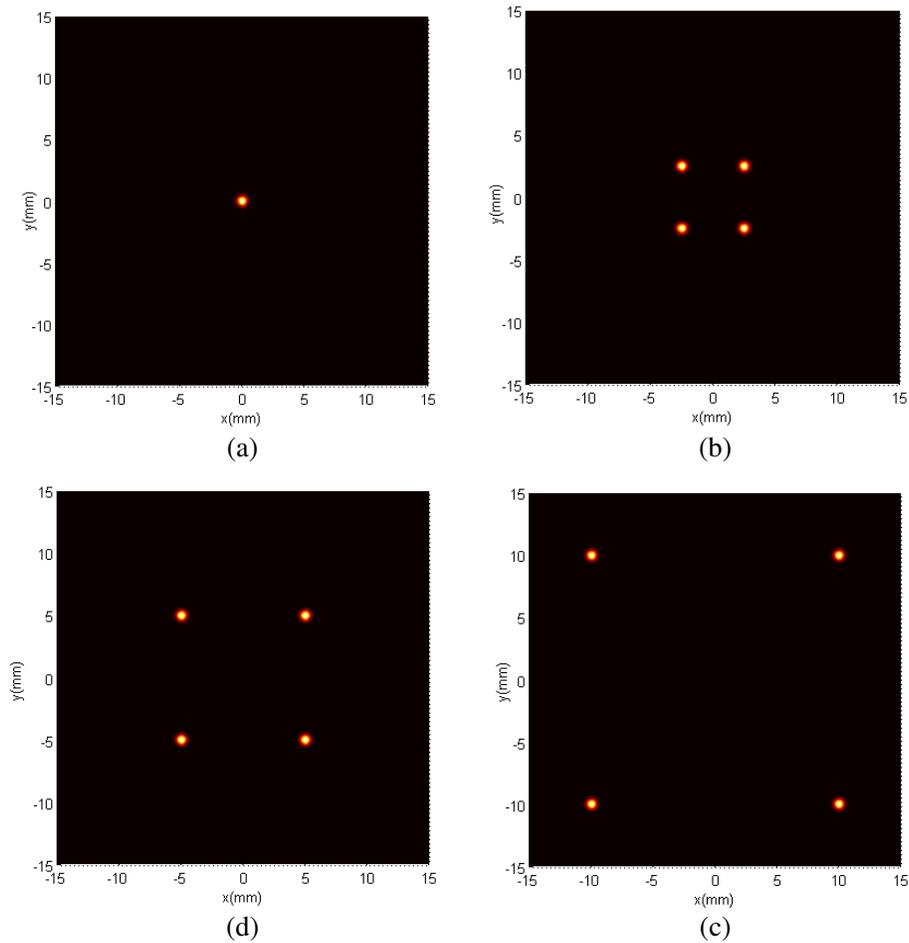


Figure 2. Intensity distribution contours of the Gaussian beam Arrays for different values of r_d : (a) $r_d = 0$, (b) $r_d = 5$ mm, (c) $r_d = 10$ mm and (d) $r_d = 20$ mm, with $M = 2$ and $w_0 = 0.5$ mm.

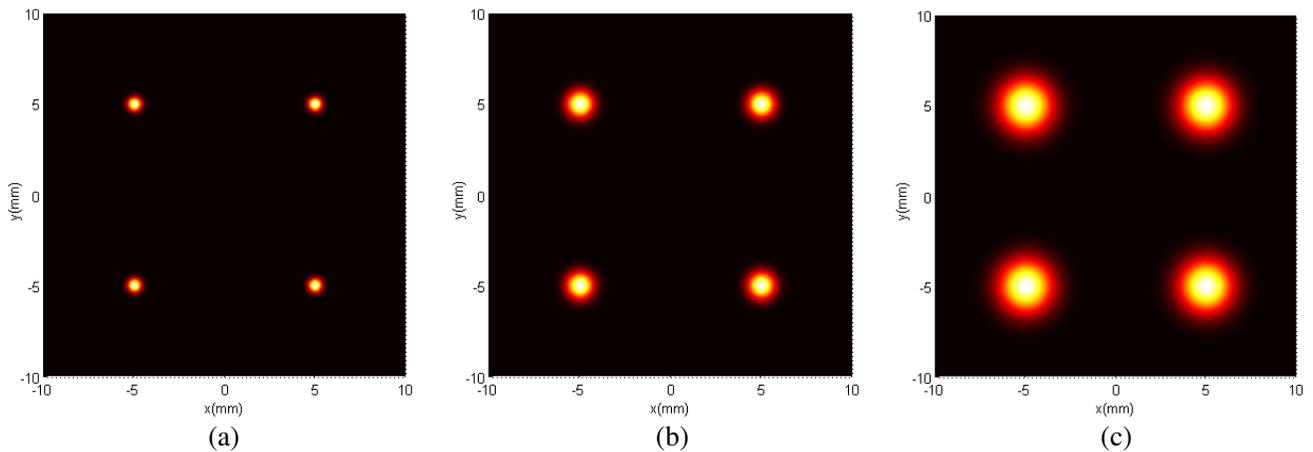


Figure 3. Intensity distribution contours of the incident Gaussian Arrays beam for different values of r_d : (a) $w_0 = 0.5$ mm, (b) $w_0 = 1$ mm and (c) $w_0 = 2$ mm, with $M = 2$ and $r_d = 10$ mm.

plots of this figure, it is shown that whatever the value of M , each set of M Gaussian Array beams presents $M \times M$ adjacent modules of Gaussian patterns. The parameters chosen in the simulations are $w_0 = 1$ mm, and the beamlets are separated by $r_d = 10$ mm.

Figure 2 illustrates the intensity contours for the Gaussian beam array $M \times M$ with $M = 3$ and for various r_d . It is clear that the distance between adjacent modules increases when r_d increases. Fig. 3 is as Figs. 1 and 2, but with a fixed $M \times M$, r_d respectively and for various w_0 . The graphics of this figure show that the beamlets keep their numbers and their separated distances and become very large when w_0 increases.

3. GENERATION OF FINITE AIRY ARRAY BEAM BY AN OPTICAL AIRY TRANSFORM SYSTEM

Consider an optical Airy transform system as schematized in Fig. 4. The optical system is composed of two thin lenses of focal length f and a spatial light modulator (SLM) used to convert the phase of the light heating its center by imposing on it a phase modulation ϕ .

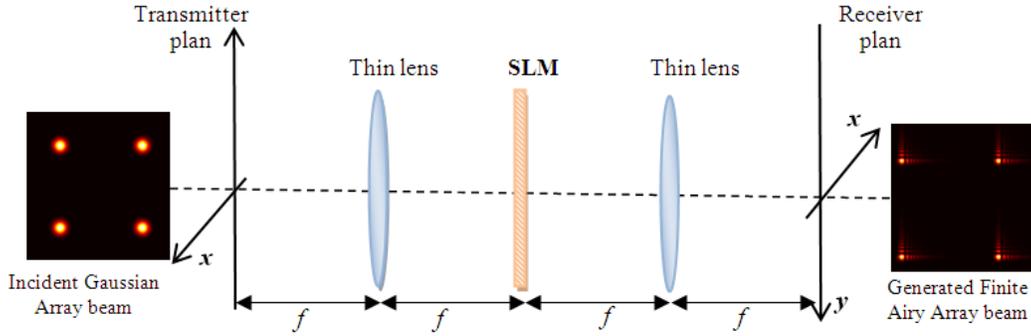


Figure 4. Schematic representation of the conversion of a Gaussian Array beam to a Finite Airy Array beam by means of the optical Airy transform system.

Assume that the transverse electrical field distribution of the incident beam is $E_0(x_0, y_0)$. So, the electrical field $E_1(x_1, y_1)$ of the beam received at an arbitrary point of the transverse SLM plane (just before the SLM) is given from $E_0(x_0, y_0)$ using the generalized Huygens-Fresnel integral [33] as

$$E_1(x_1, y_1) = \frac{2\pi}{ikf} \exp(ikf) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0(x_0, y_0) \exp\left(-\frac{ik}{f}(x_0x_1 + y_0y_1)\right) dx_0 dy_0, \quad (2)$$

where f is the focal length of the lens (see Fig. 4).

The phase pattern of the SLM is considered to be [20]

$$\phi(x_1, y_1) = \frac{\alpha^3 k_x^3 + \beta^3 k_y^3}{3} - (4kf + \pi), \quad (3)$$

where α, β are real constants, and k is the wave number, $k_x = kx_1/f$ and $k_y = ky_1/f$.

Thus, the electrical field of the beam just behind the SLM is

$$E_2(x_1, y_1) = E_1(x_1, y_1) \exp(i\phi(x_1, y_1)) \quad (4)$$

Finally, always based on the generalized Huygens-Fresnel diffraction principle [33], the outgoing electrical field of the beam exiting the output plane of the Airy transform system is written as

$$E(x, y) = \frac{2\pi}{ikf} \exp(ikf) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_2(x_1, y_1) \exp\left(-\frac{ik}{f}(xx_1 + yy_1)\right) dx_1 dy_1, \quad (5)$$

Inserting Eq. (4) into (5) and by taking account of Eqs. (2) and (3), the electric field at the output plane of the optical Airy transform system could be obtained directly from $E_0(x_0, y_0)$ by means of the mathematical Airy transform model which is expressed by two-dimensional double integral as [20]

$$E(x, y) = \frac{1}{|\alpha\beta|} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0(x_0, y_0) Ai\left(\frac{x-x_0}{\alpha}\right) Ai\left(\frac{y-y_0}{\beta}\right) dx_0 dy_0, \quad (6)$$

where $Ai(\cdot)$ is the Airy function.

Suppose that the optical Airy transform system is illuminated by a Gaussian Array beam whose electric field is given by Eq. (1). The electrical field of the created beam at the outgoing plane of the Airy optical system is obtained by the introduction of Eq. (1) into Eq. (3). This electric field reads as

$$E(x, y) = \frac{A_0}{|\alpha\beta|} \sum_{m=-\frac{M-1}{2}}^{\frac{M-1}{2}} \exp\left(-\frac{m^2 r_d^2}{w_0^2}\right) E_m(x) \cdot \sum_{n=-\frac{M-1}{2}}^{\frac{M-1}{2}} \exp\left(-\frac{n^2 r_d^2}{w_0^2}\right) \cdot E_n(y), \quad (7a)$$

where

$$E_m(x) = \int_{-\infty}^{+\infty} Ai\left(\frac{x-x_0}{\alpha}\right) \exp\left(-\frac{1}{w_0^2}x_0^2 + \frac{2mr_d}{w_0^2}x_0\right) dx_0, \quad (7b)$$

and

$$E_n(y) = \int_{-\infty}^{+\infty} Ai\left(\frac{y-y_0}{\beta}\right) \exp\left(-\frac{1}{w_0^2}y_0^2 + \frac{2nr_d}{w_0^2}y_0\right) dy_0. \quad (7c)$$

The calculation of the electrical field at receiver plane of the optical Airy transform system will halt on solving the integrals $E_m(x)$ and $E_n(y)$ which are similar and could be calculated by the same way. So, in this case, we develop $E_m(x)$ and derive from it $E_n(y)$.

Then, for this reason, we recall the representation of Airy function into an infinite integral which is written by [20]

$$Ai(s) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\left(i\frac{u^3}{3} + ius\right) du, \quad (8)$$

according to this expression, we obtain

$$Ai\left(\frac{x-x_0}{\alpha}\right) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\left(i\frac{u^3}{3} + \frac{iux}{\alpha} - \frac{iux_0}{\alpha}\right) du. \quad (9)$$

Substituting this last integral in the above expression of Eq. (7b), $E_m(x)$ becomes

$$E_m(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp\left(i\frac{u^3}{3} + \frac{iux}{\alpha}\right) \left\{ \int_{-\infty}^{+\infty} \exp\left[-\frac{1}{w_0^2}x_0^2 + \left(\frac{2mr_d}{w_0^2} - \frac{iux_0}{\alpha}\right) x_0\right] dx_0 \right\} du. \quad (10)$$

By the recall of the well-known integral of Ref. [34]

$$\int_{-\infty}^{+\infty} \exp(-p^2 y^2 \pm qy) dy = \frac{\sqrt{\pi}}{p} \exp\left(\frac{q^2}{4p^2}\right), \quad (11)$$

with $\text{Re}(p^2) > 0$, the x -electrical field component of the beam generated at the receiver plan of the optical Airy transform yields to

$$E_m(x) = \frac{w_0\sqrt{\pi}}{2\pi} \exp\left(\frac{m^2 r_d^2}{w_0^2}\right) \int_{-\infty}^{+\infty} \exp\left[i\frac{u^3}{3} + i\left(\frac{iw_0^2}{4\alpha^2}\right)u^2 + i\left(\frac{x}{\alpha} - \frac{mr_d}{\alpha}\right)u\right] du, \quad (12)$$

and by applying the following well-known integral [20],

$$\int_{-\infty}^{+\infty} \exp\left(i\frac{u^3}{3} + iau^2 + ibu\right) du = 2\pi \exp\left(ia\left(\frac{2a^2}{3} - b\right)\right) Ai(b - a^2). \quad (13)$$

Finally, the integral $E_m(x)$ will be changed to

$$E_m(x) = \sqrt{\pi}w_0 \exp\left(\frac{m^2r_d^2}{w_0^2}\right) \exp\left[\frac{w_0^2}{4\alpha^2}\left(\frac{x}{\alpha} - \frac{mr_d}{\alpha} + \frac{w_0^4}{24\alpha^4}\right)\right] Ai\left(\frac{x}{\alpha} - \frac{mr_d}{\alpha} + \frac{w_0^4}{16\alpha^4}\right), \quad (14a)$$

and similarly, the integral $E_n(y)$ could be deduced as

$$E_n(y) = \sqrt{\pi}w_0 \exp\left(\frac{n^2r_d^2}{w_0^2}\right) \exp\left[\frac{w_0^2}{4\beta^2}\left(\frac{y}{\beta} - \frac{nr_d}{\beta} + \frac{w_0^4}{24\beta^4}\right)\right] Ai\left(\frac{y}{\beta} - \frac{nr_d}{\beta} + \frac{w_0^4}{16\beta^4}\right). \quad (14b)$$

By replacing the expressions of $E_m(x)$ and $E_n(y)$ into Eq. (7a), the final expression of the electric field $E(x, y)$ of the Gaussian beam array propagating through an optical Airy transform in the output plane of this system is given by

$$\begin{aligned} E(x, y) &= \frac{A_0\pi w_0^2}{|\alpha\beta|} \sum_{m=-\frac{M-1}{2}}^{\frac{M-1}{2}} \exp\left[\frac{w_0^2}{4\alpha^2}\left(\frac{x}{\alpha} - \frac{mr_d}{\alpha} + \frac{w_0^4}{24\alpha^4}\right)\right] Ai\left(\frac{x}{\alpha} - \frac{mr_d}{\alpha} + \frac{w_0^4}{16\alpha^4}\right) \\ &\times \sum_{n=-\frac{M-1}{2}}^{\frac{M-1}{2}} \exp\left[\frac{w_0^2}{4\beta^2}\left(\frac{y}{\beta} - \frac{nr_d}{\beta} + \frac{w_0^4}{24\beta^4}\right)\right] Ai\left(\frac{y}{\beta} - \frac{nr_d}{\beta} + \frac{w_0^4}{16\beta^4}\right). \end{aligned} \quad (15)$$

Equation (15) expresses the electrical field distribution of a novel Finite Airy Array beam produced at an arbitrary point of the output plan of an optical Airy transform system illuminated by an Array Gaussian beam. It is worthy to note that this novel generated beam can be controlled numerically, by means of the different parameters corresponding to the incident beam and the optical Airy transform system.

When the number of beamlets M is equal to 1 ($M = 1$), the incident Gaussian beam array changes to a fundamental Gaussian beam expressed by

$$E(x_0, y_0, z = 0) = A_0 \exp\left(-\frac{x_0^2 + y_0^2}{w_0}\right)^2. \quad (16)$$

Under the same condition, the expression of the beam generated at the receiver plane of optical Airy transform system reduces to

$$E(x, y) = \frac{A_0\pi w_0^2}{|\alpha\beta|} \exp\left[\frac{w_0^2}{4\alpha^2}\left(\frac{x}{\alpha} + \frac{w_0^4}{24\alpha^4}\right)\right] Ai\left(\frac{x}{\alpha} + \frac{w_0^4}{16\alpha^4}\right) \times \exp\left[\frac{w_0^2}{4\beta^2}\left(\frac{y}{\beta} + \frac{w_0^4}{24\beta^4}\right)\right] Ai\left(\frac{y}{\beta} + \frac{w_0^4}{16\beta^4}\right). \quad (17)$$

This formula is similar to Eq. (11) of Ref. [22] corresponding to the generation of the Finite Airy beam from the fundamental Gaussian beam using the Airy transform system. So, our finding generalizes the result of Ref. [22].

It is worthy to note that the self-healing property of the Airy array beams, considered the common characteristics for the nondiffracting beams, can be demonstrated theoretically from considering the self-healing character of each conventional single Airy beam which is contained in the Array beam.

4. NUMERICAL SIMULATIONS

In this section, based on the main expression established in Eq. (15), some numerical calculations are performed in order to study the effects of some parameters on the properties patterns of the beam generated at the receiver plan of the optical system.

Figure 5 presents the transverse intensity distributions of the Finite Airy Array beams generated at the receiver plan of the optical Airy transform illuminated by a Gaussian array beam for various

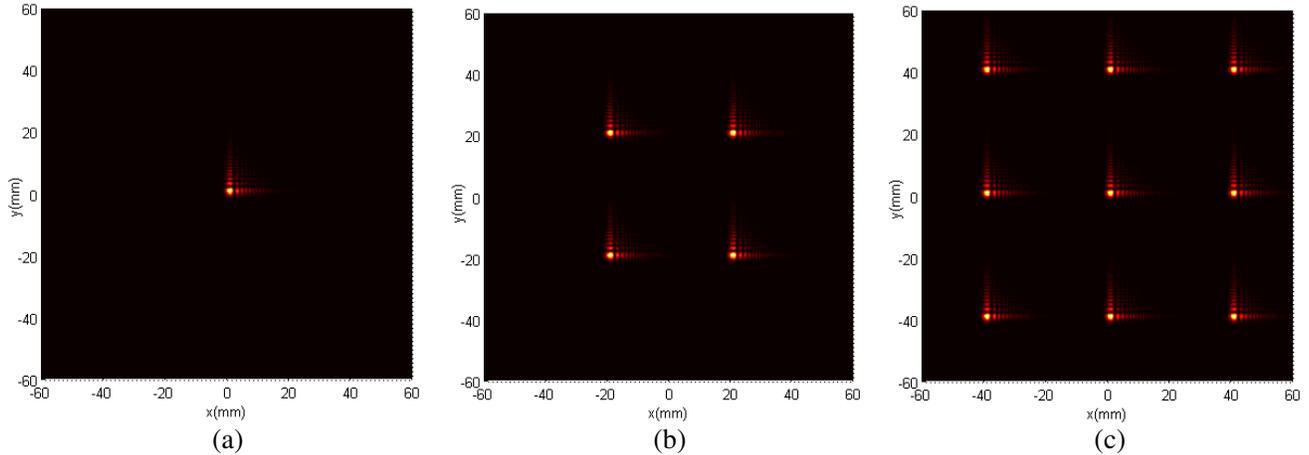


Figure 5. Transverse intensity distributions of the generated Finite Airy Array beams for different values of M : (a) $M = 1$, (b) $M = 2$, and (c) $M = 3$, with $r_d = 40$ mm, $w_0 = 0.5$ mm and $\alpha = \beta = -1$.

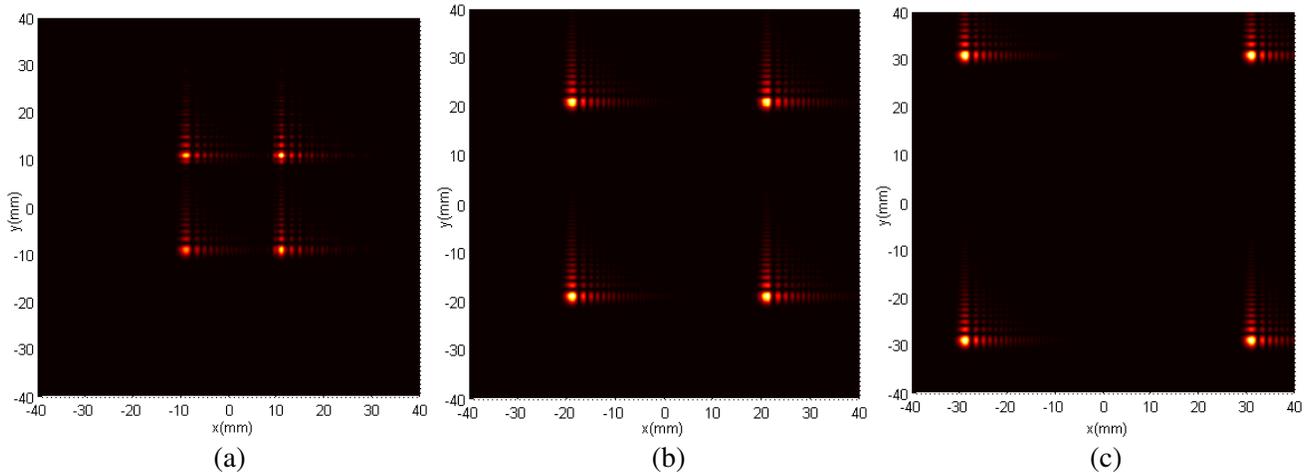


Figure 6. Transverse intensity distributions of the generated Finite Airy beams Arrays for different values of r_d : (a) $r_d = 20$ mm, (b) $r_d = 40$ mm and (c) $r_d = 60$ mm, with $M = 2$ and $w_0 = 0.5$ mm.

parameters $M \times M$, with $r_d = 40$ mm, $w_0 = 0.5$ mm and $\alpha = \beta = -1$. As can be seen from the drawings, we can control the number of beamlets in the produced Finite Airy array beam by convenient choice of number M of the beamlets contained in the input Gaussian array beam. Especially, the single Airy beam is obtained when a single Gaussian is involved.

Moreover, in Fig. 6, we display the transverse intensity of the generated Finite Airy Array beams for different separation distances between the adjacent modules, (a) $r_d = 20$ mm, (b) $r_d = 40$ mm and (c) $r_d = 60$ mm, with $M = 2$ and $w_0 = 0.5$ mm. From the plots of this figure, it is noticeable that as the separation distance between the adjacent modules of the incident beam increases, the separation distance between adjacent modules increases too.

The plots in Fig. 7 show the possibility of inverting the orientations of the obtained Airy array beams by changing the signs of the optical Airy parameters α and β . This effect is due to the change of localization of the single Airy modes contained in the Array beam from the third to the fourth quadrant for example when α and β change from positive to negative values as demonstrated in [22].

Such a direct controlling of the structured Array beam may be convenient for several applications in optics topic such as nanofabrication, generation of curve plasmon channel, light sheet microscopy, etc.

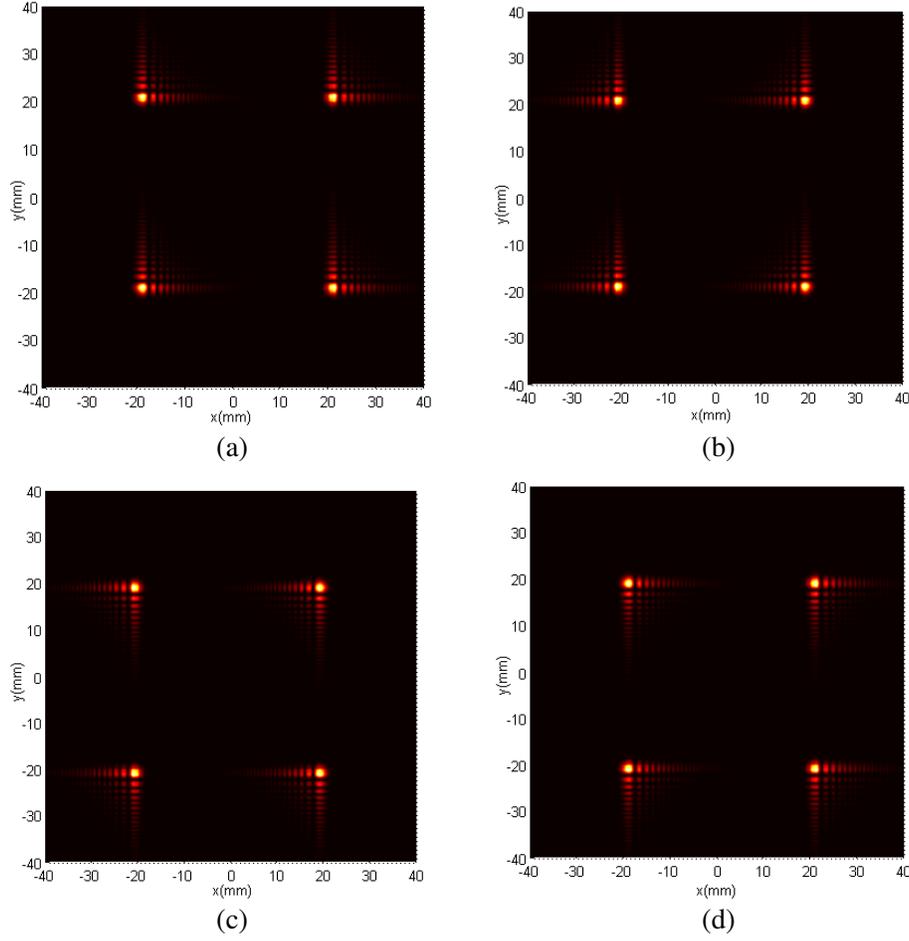


Figure 7. Transverse intensity distributions of the generated Finite Airy Array beams for different values of α and β : (a) $\alpha = \beta = -1$; (b) $\alpha = -\beta = -1$; (c) $\alpha = \beta = 1$ and (d) $\alpha = -\beta = 1$, with $M = 2$, $r_d = 40$ mm and $w_0 = 0.5$ mm.

5. CONCLUSION

Based on the generalized Huygens-Fresnel integral, a closed form expression of a novel Finite Airy array beam generated from the Gaussian array beam through an optical Airy transform system is developed. The investigation about the generation of the Finite Airy beam from the fundamental Gaussian beam, using an optical Airy transform system, is deduced as a particular case of our principal result. The numerical calculations show that the Airy optical system can be used as a tool to convert the Gaussian array beam into a Finite Airy Array beam with controllable parameters: number of beamlets, distance between the adjacent modules, positions and orientations of the modules of the Airy array beams produced by the optical Airy transform system. The findings of this research may be used in several practical applications such as manipulating, transport, guiding and trapping of particles.

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