# Online-Calibrated CS-Based Indoor Localization over IEEE 802.11 Wireless Infrastructure

# Wei Ke<sup>1, 2, 3, \*</sup>, Jie Jin<sup>1</sup>, Hengkuan Xu<sup>1</sup>, Kunliang Yu<sup>1</sup>, and Jianhua Shao<sup>1</sup>

Abstract—Recent technological achievements have made it low cost to realize indoor localization using the received signal strength (RSS) information from Wi-Fi signals. However, the current RSS-based indoor localization techniques have two major challenges: one is that the RSS signal is quite sensitive to channel conditions, and the other is that sufficient number of access points (APs) is needed to provide enough RSS measurements for guaranteeing good performance. To solve these problems, this paper proposes an adaptive compressive sensing (CS) based indoor localization method based on the IEEE 802.11 Wi-Fi standard. The novel feature of this method is to dynamically adjust both the dictionary and the sparse solution using an online dictionary learning (DL) technology so that the location solution can better match the real-time RSS scenario. Meanwhile, an improved approximate  $l_0$ norm minimization algorithm is presented to enhance sparse recovery speed and reduce the number of APs required by indoor localization systems. The effectiveness of the proposed scheme is demonstrated by experimental results where the proposed algorithm yields substantial improvement for localization performance and reduces computation complexity.

# 1. INTRODUCTION

Although Global Positioning System (GPS) has been in service for many years, it is only available in GPS-enabled devices and may encounter problems in indoor environments because of its poor signal penetration capabilities [1]. Thus, the location estimation based on existing wireless infrastructures has been advanced rapidly in recent years. Nowadays, IEEE 802.11 based Wi-Fi has become a critical component of networking architecture and is available in most corporate environments and commercial buildings [2,3]. With the widespread deployment of Wi-Fi, indoor positioning based on Wi-Fi is especially favored because of little requirement for extra infrastructure investments. Since RSS can be easily obtained by a Wi-Fi-integrated mobile device without any additional hardware modification, many Wi-Fi positioning systems rely on the location information of RSS.

Although we have witnessed swift advances in Wi-Fi over the last decade, wireless channels in indoor environments are generally noisy, and the RSS measurement is quite sensitive to channel conditions. Generally, the main challenges for RSS-based positioning systems come from two aspects:

1) In indoor environments, a number of factors affect the RF signal propagation including multipath, channel fading, shadowing, temperature and humidity variations and the presence and mobility of human beings, etc. Many experiments have demonstrated the effects of these factors which result in the RSS variations as high as 7–20 dBm [4, 5]. Therefore, a practical RSS-based positioning system has to be adaptable to the variations of environmental dynamic factors.

Received 16 October 2016, Accepted 2 December 2016, Scheduled 15 December 2016

<sup>\*</sup> Corresponding author: Wei Ke (wkykw@sina.com).

<sup>&</sup>lt;sup>1</sup> Jiangsu Key Laboratory on Optoelectronic Technology in School of Physics and Technology, Nanjing Normal University, Nanjing 210023, China. <sup>2</sup> Jiangsu Key Laboratory of Meteorological Observation and Information Processing, Nanjing University of Information Science & Technology, Nanjing 210044, China. <sup>3</sup> Jiangsu Center for Collaborative Innovation in Geographical Information Resource Development and Application, Nanjing 210023, China.

2) Due to the deployment of access points (APs) in indoor environments, the total number of detectable APs is generally finite for positioning, which will lead to only a small number of RSS measurements in real scenarios. However, most presented location schemes require as many RSS measurements as possible for accurate position estimates [6]. The shortage of detectable RSS measurements reduces the accuracy of the location estimation considerably.

Therefore, building an indoor localization system to estimate the locations of targets is still a challenging problem. Since targets generally lie at a few points in the discrete spatial domain, this inherent sparsity can be exploited to convert the localization problem into a sparse recovery problem. In recent years, the compressive sensing (CS) theory that receives a great deal of attentions has been successfully applied to outdoor and indoor localization, which results in higher localization accuracy and reduces the dimensions of measurement vectors [7–12] Different from that the line-of-sight (LOS) path is dominant in an open outdoor environment, multipath is common in an indoor environment, and thus the change in RSS becomes unpredictable. Although these CS-based efforts are easy to implement, these algorithms ignore the effects of environmental variations, and thus they cannot achieve stable localization performance under complex indoor circumstances.

In this paper, we continue to investigate the CS-based indoor positioning problem with respect to the adverse impact of RSS changes under the condition of small number of detectable APs. Different from the previous works, we take advantage of CS theory to handle the spatial sparsity to reduce the number of APs required by indoor localization systems and exploit the online DL technique to deal with the problem of the RSS sensitivity. We also propose an improved sparse reconstruction algorithm based on approximate  $l_0$  norm minimization for the fast recovery of sparse signals. The notation used in this paper is according to the convention. Symbols for matrices (upper case) and vectors (lower case) are in boldface.  $(\cdot)^T$ ,  $\|\theta\|_0$ ,  $\|\theta\|_1$  and  $\|\theta\|_2$  denote transpose,  $l_0$  norm,  $l_1$  norm and  $l_2$  norm, respectively. The remainder of the paper is organized as follows. Section 2 presents the related works. Section 3 describes the system model and problem formulation. We introduce a new positioning scheme to calibrate the dictionary dynamically and estimate the sparse solution adaptively in Section 4. Experimental results are given in Section 5. Finally, Section 6 concludes the paper.

## 2. RELATED WORKS

Indoor localization is a promising technique that receives extensive attention, and lots of works have been carried out to realize localization based on Wi-Fi. In this section, we briefly summarize the most relevant research on the RSS-based indoor localization and the CS-based localization.

Generally, there are two kinds of main methods for the RSS-based indoor positioning, which are the model-based method [13, 14] and the fingerprinting based method [15, 16]. Existing model-based positioning approaches mainly depend on the specific path loss model that converts RSS measurements into corresponding distances, and then use the geometric method to find the position of each target. The main difficulty of this method is to establish a reliable signal propagation model and estimate its path loss parameters in an indoor environment due to the unpredictable nature of indoor radio channel. On the other hand, the fingerprinting method need record RSSs at every possible location through off-line training process and compare the RSS measurement of each target with the recorded RSSs to find the best matching RSS pattern. However, this method is also environment dependent, and any significant change on the topology implies a costly new recalibration.

Spatial sparsity, a main theme in CS, arises naturally in wireless localization. In [7], it was realized that the localization problem can be formulated as a distributed sparse approximation problem. However, in this method a localization dictionary has to be locally estimated at each sensor node. On the contrary, Feng et al. proposed a server-based sparse multiple target localization algorithm in [8], where the localization dictionary was constructed at the location center (i.e., the server) and each sensor only transmitted a small number of compressive measurements to the location center. In [9] Zhang and Tan adopted the  $l_1$  norm minimization to estimate the target location. However, the computational complexity of the  $l_1$  minimization algorithm is too high and not suitable for being adopted in Wi-Fi devices. To reduce the computational complexity reference [1] proposed a greedy matching pursuit (GMP) algorithm to efficiently realize sparse signal reconstruction. Although the above methods can achieve better performance for solving the indoor localization problem than

the traditional positioning methods, these works neglect a number of factors that affect the signal propagation in indoor environments. An affinity-based CS localization scheme (ACS) was proposed by exploiting affinity propagation and cluster matching to reduce the effects of RSS variations in [11]. However, this method may result in large positioning bias because the false cluster matching can take place due to environmental variations. In our previous work [12], we attempted to use the off-line DL technique for improving the accuracy of localization. However, that work can only achieve high location accuracy in stationary environments since the dictionary is only trained once at the beginning of localization due to high computational complexity.

# 3. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first present a linear model for relating the RSS measurements to the spatial locations of targets under the CS framework and then discuss the problem of dictionary mismatch.

# 3.1. System Model under the CS Framework

The proposed system is composed of Wi-Fi APs and off-the-shelf 802.11-compliant devices such as smart phones and panel computers, where APs with known locations serve as anchors The RSS measurements are made periodically by APs and transmitted to the location server.

Consider that K unknown-location targets are located in an area of interest which is divided into N grids, and assume M APs in the targeted area, which take RSS measurements from these targets. In general,  $N \gg M > K$ . Since targets located at one or a few grid points in the localization area, the positions of targets in the discrete spatial domain can be accurately represented as a sparse vector  $\theta$ . The elements of vector  $\theta$  are equal to ones if targets are located at the corresponding positions in the localization problem is converted to determine non-zero elements and their specific locations in the sparse vector  $\theta$  according to the received signals. Based on the CS theory, the sparse localization model can be represented as:

$$\mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\theta} + \mathbf{v} \tag{1}$$

where  $\mathbf{y}$  is the RSS measurement vector,  $\Psi$  the dictionary, and  $\mathbf{v}$  the measurement noises. Since the dictionary  $\Psi$  is a key factor for the sparse reconstruction, this paper firstly sets up an initial dictionary based on the radio propagation model in [17] and then exploits the DL technique to modify the initial dictionary for overcoming the model errors. According to [17], the *mn*th received RSS coming from the target located at grid *n* by the *m*th AP can be expressed as:

$$\psi_{mn} = P(d_0) + 10\alpha \log_{10}(d_{mn}/d_0), \quad m = 1, \dots, M, \quad n = 1, \dots, N$$
(2)

where  $P(d_0)$  is the path loss at a close-in reference distance  $d_0$ ,  $d_{mn}$  is distance between the *n*th grid and the *m*th AP, and  $\alpha$  the path loss exponent. The path-loss exponent is usually set between 2 and 5 according to [17], and  $\alpha = 2.6$  is suggested to be used in an indoor environment with hard partitions in [17].

#### 3.2. Problem of Dictionary Mismatch

It should be noted that  $\Psi$  can be known in advance since grid locations and AP locations are known, which means that we can estimate the coordinates of targets as long as we find the positions of nonzero values in  $\theta$ . Moreover, the number of these dominant nonzero values gives K.

However, the model described in Eq. (1) is only an approximate model, and the non-ideal factors are inevitable in a practical localization system. In fact, since the RSS measure is susceptible to several wireless propagation factors discussed in the above section, it is difficult to obtain a nice and close-form theoretical model. Therefore, the predefined dictionary cannot effectively express the actual signal, and may easily cause performance degradation in the sparse recovery process. We assume the error dictionary matrix  $\Gamma$  which describes the difference between the predefined dictionary and the practical received signals. Note that the error matrix  $\Gamma$  is time-varying and cannot be known in advance. Thus, the sparse positioning model is correspondingly modified as:

$$\mathbf{y} = (\mathbf{\Psi} + \mathbf{\Gamma})\boldsymbol{\theta} + \mathbf{v} = \mathbf{D}\boldsymbol{\theta} + \mathbf{v}$$
(3)

where  $\mathbf{D} = \mathbf{\Psi} + \mathbf{\Gamma}$  denotes the actual dictionary with time-varying interferences. Since the mismatch exists between the columns of  $\mathbf{D}$  and the corresponding columns of the ideal basis  $\Psi$ , the performance degradation is inevitable in the sparse recovery process. To obtain accurate localization results, we will exploit online DL technique to correct the dictionary variations dynamically.

# 4. ONLINE-CALIBRATED INDOOR LOCALIZATION ALGORITHM

Focused on the above problem, an adaptive sparse recovery algorithm combining online DL is proposed in this section. So far, most DL methods are generally based on alternating minimization [18]. In one step, a sparse recovery algorithm finds sparse representations of the training samples with a fixed dictionary. In the other step, the dictionary is updated to decrease the average approximation error while the sparse coefficients remain fixed. The proposed method in this paper also uses this formulation of alternating minimization.

#### 4.1. Sparse Recovery Phase

In the sparse reconstruction stagethe dictionary  $\mathbf{D}$  is fixed, and thus the sparse signal recovery problem can be converted into a following optimization problem,

$$\min \|\theta\|_0 \quad s.t. \mathbf{y} = \mathbf{D}\theta \tag{4}$$

Note that (4) is NP-hard to solve. An alternative is to use  $l_1$  norm instead of  $l_0$  norm to enforce sparsity, which leads to

$$\min \|\theta\|_1 \quad s.t. \mathbf{y} = \mathbf{D}\theta \tag{5}$$

However, it should be emphasized that larger coefficients in  $\theta$  are penalized more heavily in the  $l_1$  norm than smaller coefficients, unlike the more democratic penalization of the  $l_0$  norm [19]. In practice, large coefficients are usually the entries corresponding to the actual positions of targets, while small coefficients commonly represent the noise entries. The imbalance of the  $l_1$  norm penalty will seriously influence the recovery accuracy, which may result in many false targets. To overcome the mismatch between  $l_0$  norm minimization and  $l_1$  norm minimization, in this section we propose an improved approximate  $l_0$  norm minimization algorithm (called IAL0 in the following sections) as our sparse recovery method. The IAL0 algorithm can not only avoid the drawbacks of using  $l_1$  norm to approximate  $l_0$  norm, but also overcome the NP-hard problem of  $l_0$  norm constraints.

The problem of using  $l_0$  norm is the need for a combinatorial search for its minimization, since the  $l_0$  norm of a vector is a discontinuous function of that vector. The main idea of IAL0 is to approximate  $l_0$  discontinuous function by a suitable continuous one, which is the similar idea as the approach in [20]. However, different from the Gaussian function used in [20], we design the following combination function, which is expressed as:

$$f_{\sigma}(x) = \lambda \left( e^{x^2/2\sigma^2} - e^{-x^2/2\sigma^2} \right) / \left( e^{x^2/2\sigma^2} + e^{-x^2/2\sigma^2} \right) + (1 - \lambda)(1 - e^{-x^2/\lambda\sigma^2})$$
(6)

where  $0 < \lambda < 1$ , which arranges the different proportions between the Gaussian function and hyperbolic tangent function. Under an appropriate parameter  $\sigma$ , this combination function is steeper than the Gaussian function around x = 0, and thus it can be more accurately approximated to  $l_0$  norm. Moreover,  $f_{\sigma}(x)$  is a continuous function and has the following properties:

$$\lim \sigma \to 0 \tag{7}$$

Therefore, this function can meet the conditions required for a smoothing function as ] pointed out in [20. Let  $F_{\sigma}(\theta) = \sum_{i=1}^{N} f_{\sigma}(\theta_i)$  when  $\sigma$  is very small, we have  $F_{\sigma}(\theta) \approx \|\theta\|_0$  Then, Eq. (4) can be converted into

$$\arg\min_{\theta} F_{\sigma}(\theta) , \ s.t. \ \mathbf{y} = \mathbf{D}\theta \tag{8}$$

Using the Lagrange multiplier method, Eq. (8) is converted into an unconstrained optimization problem:

$$\min_{\theta} L(\theta) = F_{\sigma}(\theta) + \beta \|\mathbf{y} - \mathbf{D}\theta\|_2^2$$
(9)

where  $\beta$  is the Lagrange multiplier. Now, we exploit the Fletcher-Reeves (FR) algorithm [21] to solve the optimization problem in Eq. (9), instead of the steepest descent method used in [2]. The FR algorithm belongs to the class of conjugate gradient methods which can be faster convergent than the steepest descent method and has good numerical stability. Moreover, the FR algorithm does not need to calculate the second derivative of the cost function (9) as the Newton method. According to the FR algorithm, in the  $j^{th}$  iteration  $\theta_{j+1}$  is updated as

$$\theta_{j+1} = \theta_j + \mu_j \mathbf{h}_j \tag{10}$$

where  $\mu_j$  is the step-size parameter and  $\mathbf{h}_j$  the conjugate direction which can be computed as

$$\mathbf{h}_{j} = \begin{cases} -\mathbf{g}_{0}, & j = 0\\ -\mathbf{g}_{j} + \eta_{j-1}\mathbf{h}_{j-1}, & j \neq 0 \end{cases}$$
(11)

where  $\mathbf{g}_j$  is the gradient vector when  $\theta = \theta_j$ , and  $\eta_{j-1} = \|\mathbf{g}_j\|_2^2 / \|\mathbf{g}_{j-1}\|_2^2$ . For completeness, a full description of the IALO algorithm is given in the sparse recovery part of Fig. 1.



Figure 1. The proposed algorithm.

# 4.2. Dictionary Learning Phase

At this stage, the sparse vector is fixed, and we update the dictionary by using the DL algorithm. Currently, there are some common DL algorithms, such as the K-SVD algorithm, agent function optimization method, and recursive least squares method [18, 22]. However, these methods can only effectively handle training data off-line, so they are unable to meet the requirement of rapid positioning. To overcome this drawback, we tend to optimize the dictionary in the online manner. Fortunately, Mairal et al. [23] proposed a new online optimization algorithm based on stochastic approximations for DL, which can adapt to the dynamic training data changing over time.

Therefore, in order to be fit for the time-varying RSS measurements, this paper chooses the online DL algorithm in [23], whose key idea is to simply add an increment element to each column of the original dictionary. This method can avoid performing operations on large matrices and realize fast positioning. According to [23], each column of the dictionary is updated as

$$\mathbf{u}_j \leftarrow \mathbf{d}_j + (\mathbf{b}_j - \mathbf{D}\mathbf{a}_j) / \mathbf{A}(j, j), \quad j = 1, 2, \dots, N$$
(12)

$$\mathbf{d}_j \leftarrow \frac{1}{\max(\|\mathbf{u}_j\|_2, 1)} \mathbf{u}_j \tag{13}$$

where  $\mathbf{d}_j$ ,  $\mathbf{b}_j$ ,  $\mathbf{a}_j$  are the *j*th column vector of  $\mathbf{D}$ ,  $\mathbf{B}_t$  and  $\mathbf{A}_t$ , respectively.  $\mathbf{B}_t$  and  $\mathbf{A}_t$  are intermediate variables of the online DL algorithm.  $\mathbf{A}(j, j)$  represents the element at the *j*th column and *j*th row of  $\mathbf{A}_t$ .

The corresponding procedure is summarized in Fig. 1, and the details can be found in [23].

#### 5. EXPERIMENTAL RESULTS

To evaluate the actual localization results of the proposed method under the time-varying environment, RSS data were collected from an office building during different times (6:00 a.m.-10:00 p.m.) by the monitor device at testing locations. The localization area of the scene is the part area of XingJian Building in Nanjing Normal University, which is covered by 802.11 Wi-Fi signals. The selected area  $(25 \text{ m} \times 20 \text{ m})$  is a typical office environment as shown in Fig. 2. A total of 12 detectable APs are used to measure RSS values in the localization area, and each experiment selects 6 RSS measurements from 12 APs to realize its corresponding algorithms. For undetected APs, we set a default value,  $-95 \,\mathrm{dBm}$ , as the threshold value of detectable RSS. In experiments, the localization area is divided into  $N = 25 \times 20$ grids which means yielding a 1 m resolution along both x and y axes. The testing locations are selected at random, within the measurement area. A laptop with 3.1 GHz processor and 4 GB memory is used to gather the RSS signatures from nearby APs, and saves them for real-time processing. Positioning performance is evaluated by average positioning error and root mean square error (RMSE). The results of each time are obtained based on 30 experiments. In order to evaluate the performance of the proposed algorithm, we compared it with the previously-reported CS-based indoor localization algorithms, namely the convex-optimization based  $l_1$  minimization approach in [7], greedy-based GMP method in [10] and t non-CS based least squares (LS) algorithm in [24] under the same conditions.



Figure 2. The configuration of localization area. The black dots show the locations of the APs.



Figure 3. Location errors under different testing times.

For a single target, the influences on localization accuracies at different times are shown in Fig. 3. From Fig. 3, the proposed algorithm could achieve the more stable localization performance than other algorithms in one day. Especially, the variations of RMSE in the GMP and  $l_1$  algorithms are near 1.5 m, while the variations in the proposed algorithm are small under the same conditions. It should be pointed out that although the  $l_1$  algorithm and GMP algorithm are CS-based algorithms, they achieve nearly the same results as that of the LS algorithm. This is because of the fact that without the accurate measurement matrix, the  $l_1$  algorithm and GMP algorithm could not reconstruct the sparse signal effectively. The Fig. 3 demonstrates that the proposed algorithm is a robust and effective sparse signal reconstruction algorithm without the need of the accurate knowledge about the measurement matrix. Moreover, these results also confirm that the proposed algorithm is suitable for being adopted in the positioning applications where electromagnetic environments are time-varying

The detailed statistical results of the localization performance of different algorithms are summarized in Table 1. Compared with that of the  $l_1$  and GMP algorithms, the mean localization error of the IAL0 algorithm decreases by 25.8% and 31.1%, respectively. Meanwhile, the proposed approach has significantly better performance than the other two CS-based methods in terms of RMSE.

Algorithm	Average (m)	RMSE (m)	Running Time (ms)
$l_1$	1.82	2.39	390.55
$\operatorname{GMP}$	1.96	2.67	68.83
IAL0	1.35	1.44	115.67
LS	1.93	2.71	61.36

Table 1. Comparisons of localization error and average running time.

These results can be attributed to the fact that the IAL0 algorithm uses the continuous function to approximate  $l_0$  norm constraint and the online DL method to dynamically adjust the dictionary for matching the changes of radio signals. In the meantime, we can find that the localization performance of the proposed algorithm is also better than the traditional LS algorithm with the improvement in terms of the mean error and RMSE about 0.58 m and 1.27 m, respectively.

The complexity is also compared in terms of the average CPU running time. From Table 1, it is shown that the average running times of the LS algorithm and the GMP algorithm are almost identical and have the fastest running speed among four algorithms, while the  $l_1$  algorithm based on convex optimization requires the longest running time. Due to adding the DL step in our algorithm, the running speed of the proposed method is also slower than the LS and GMP algorithms, although its execution time is far less than the  $l_1$  algorithm. However, since the online DL method can avoid performing operations on large matrices, the increment of execution time is not too much. In addition, we exploit the FR algorithm to solve the optimization problem in Eq. (9), instead of the steepest descent method in [20], which also can reduce execution time. Although the running time of the proposed method is larger than the LS and GMP algorithms, this slight growth of complexity is totally acceptable considering the large performance gain that the proposed method achieves.

The effects of the number of APs on the localization performance are studied in Fig. 4. It can be observed that as an overall trend, the localization errors of all CS-based algorithms decreases with increasing number of APs, since more measurement information from more APs can be used to realize sparse reconstruction and thus higher accuracy is achieved. However, even though the number of APs is sufficient, the localization errors of the  $l_1$  and GMP methods are still more than 1.8 m due to the effects of RSS fluctuations. By comparison, the localization error of the proposed algorithm is smallest because of exploiting the online DL technique to mitigate the effects of RSS variations and directly utilizing the  $l_0$  norm penalty instead of  $l_1$  norm penalty to find the sparsest solution in the reconstruction procedure.



Figure 4. The localization errors with respect to the number of APs.



Figure 5. The localization errors with respect to the number of targets.

These results confirm that the proposed algorithm is suitable for indoor positioning by using only a small set of noisy measurements, even if the fluctuations of RSS variations are serious. Different from the CS-based localization methods, the localization performance of non-CS based LS algorithm is almost not improved by the number of APs, and even the accuracy decreases slightly when the number of APs increases to seven or higher.

Figure 5 illustrates the location error with respect to the number of targets. With the increase in the number of targets, the accuracy of the GMP and LS algorithms decreases quickly due to the high sensitivity to the estimated number of targets. On the contrary, the variations of accuracy in the  $l_1$  and proposed algorithms are very small. The importance of the low sensitivity of our algorithm to the number of targets is twofold. First, the number of sources is usually unknown, and low sensitivity provides robustness against mistakes in estimating the number of targets. In addition, even if the number of sources is known, low sensitivity may allow one to reduce the computational complexity.

# 6. CONCLUSION

RSS-based indoor localizations have attracted considerable attention due to their simplicity and low cost. To mitigate the effects of RSS variations, a novel adaptive RSS-based sparse localization algorithm is proposed to adjust both the dictionary and the sparse solution online so that location estimates can better adapt to dynamic nature of indoor environments. At the same time, this algorithm can perform well without prior knowledge about the environments and without time-consuming off-line surveys. In addition, we propose an improved approximate  $l_0$  norm minimization algorithm to enhance reconstruction performance for sparse signals, which has its capability for signal reconstruction without prior information of the sparsity, i.e., the number of targets in this paper. The effectiveness of the proposed scheme for indoor localization has been demonstrated by experimental results where the proposed algorithm can get results more robust against noises and sensitivities of RSS measurements, and substantial improvement for localization performance is achieved.

# ACKNOWLEDGMENT

This work was supported by the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20133207120007), and the Open Research Fund of Jiangsu Key Laboratory of Meteorological Observation and Information Processing (Grant No. KDXS1408).

# REFERENCES

- Gonzalo, S. G., A. L. Jose, J. B. David, and L. R. Gustavo, "Challenges in indoor global navigation satellite systems," *IEEE Signal Process. Mag.*, Vol. 29, No. 2, 108–131, 2012.
- Mitilineos, S. A. and S. C. A. Thomopoulos, "Positioning accuracy enhancement using error modeling via a polynomial approximation approach," *Progress In Electromagnetics Research*, Vol. 102, 49–64, 2010.
- Dianu, M. D., J. Riihijarvi, and M. Petrova, "Measurement-based study of the performance of IEEE 802.11ac in an indoor environment," *Proc. IEEE International Conference on Communications*, 5771–5776, Sydney, Australia, Jun. 2014.
- Kaemarungsi, K. and P. Krishnamurthy, "Properties of indoor received signal strength for Wi-Fi location fingerprinting," Proc. IEEE International Conference on Mobile and Ubiquitous System, 14–23, Boston, USA, Aug. 2004.
- Golmie, N., "Interference in the 2.4 GHz band," Proc. International Conference on Applications and Services in Wireless Networks, 2540–2545, Helsinki, Finland, Jue. 2001.
- 6. Liu, H., H. Darabi, H. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Trans. Systems, Man, and Cybernetics Part C*, Vol. 37, No. 6, 1067–1080, 2007.
- Cevher, V., M. F. Duarte, and R. G. Baraniuk., "Distributed target localization via spatial sparsity," *Proc. 16th European Signal Processing Conference*, 15–19, Lausanne, Switzerland, Aug. 2008.

- 8. Feng, C., W. S. A. Au, S. Valaee, and Z. Tan, "Received signal strength based indoor positioning using compressive sensing," *IEEE Trans. Mobile Computing*, Vol. 11, No. 12, 1983–1993, 2012.
- 9. Zhang, L. and Z. Tan, "Study on compressive sensing in the application of wireless localization," J. Internet Technol., Vol. 11, No. 1, 129–134, 2010.
- Zhang, B., X. Cheng, N. Zhang, Y. Cui, Y. Li, and Q. Liang, "Sparse target counting and localization in sensor networks based on compressive sensing," *Proc. IEEE INFOCOM*, 2255–2263, Shanghai, China, Apr. 2011.
- 11. Feng, C., W. S. A. Au, S. Valaee, and Z. Tan, "Orientation-aware indoor localization using affinity propagation and compressive sensing," *Proc. IEEE International Workshop on Computational Advances in Multi-sensor Adaptive Processing*, 261–264, Aruba, Netherlands, Dec. 2009.
- 12. Ke, W. and L. Wu, "Indoor localization in the presence of RSS variations via sparse solution finding and dictionary learning," *Progress In Electromagnetics Research B*, Vol. 45, 353–368, 2012.
- 13. Chang, N., R. Rashidzadeh, and M. Ahmadi, "Robust indoor positioning using differential Wi-Fi access points," *IEEE Trans. Consumer Electron.*, Vol. 56, No. 3, 1860–1867, Aug. 2010.
- Lim, H., L. C. Kung, and J. C. Hou, "Zero-configuration indoor localization over IEEE 802.11 wireless infrastructure," Wireless Netw, Vol. 16, No. 2, 405–420, 2010.
- Honkavirta, V., T. Perala, S. A. Loytty, and R. Piche, "A comparative survey of Wi-Fi location fingerprinting methods," *Proc. 6th Workshop on Positioning, Navigation and Communication*, 243– 251, Hannover, Germany, Mar. 2009.
- Fang, S. H., Y. T. Hsu, and W. H. Kuo, "Dynamic fingerprinting combination for improved mobile localization," *IEEE Trans. Wireless Comm.*, Vol. 10, No. 12, 4018–4022, Dec. 2011.
- Rappaport, T. S. Wireless Communication: Principles and Practice, Prentice-Hall, Englewood Cliffs, NJ, 1999.
- Tosic, I. and P. Frossard, "Dictionary learning," *IEEE Signal Process. Mag.*, Vol. 28, No. 2, 27–38, 2011.
- 19. Candes, E. J., M. B. Wakin, and S. P. Boyd, "Enhancing sparsity by reweighted l<sub>1</sub> minimization," *J. Fourier Anal. Applicat.*, Vol. 14, No. 5–6, 877–905, Jun. 2008.
- 20. Mohimani, H., M. B. Zadeh, and C. Jutten, "A fast approach for overcomplete sparse decomposition based on smoothed  $l_0$  norm," *IEEE Trans. Signal Process.*, Vol. 57, No. 1, 289–301, 2009.
- 21. Antoniou, A. and W.-S. Lu, *Practical Optimization: Algorithms and Engineering Applications*, Springer, 2006.
- 22. Rubinstein, R., A. M. Bruckstein, and M. Elad, "Dictionaries for sparse representation modeling," *Proc. IEEE*, Vol. 98, No. 6, 1045–1057, 2010.
- Mairal, J., F. Bach, J. Ponce, and G. Sapiro, "Online learning for matrix factorization and sparse coding," J. Mach. Learn. Res., Vol. 11, No. 3, 19–60, Jan. 2010.
- Cheung, K. W., H. C. So, W.-K. Ma, and Y. T. Chan, "A constrained least squares approach to mobile positioning: Algorithms and optimality," *EURASIP J. Applied Signal Process.*, Vol. 2006, 123, 2006.