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Metamaterials Photonic Filter Based on Electromagnetically Induced Transparency Resonance

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ABSTRACT: In this paper, we give an analytical demonstration of electromagnetic induced transparency (EIT) resonance by a simple photonic device consisting of two grafted resonators (metamaterials of type Epsilon Negative Gauchy (ENG)) of lengths d_2 and d_3. Then, we study theoretically the transmission spectrum and the dispersion relation of periodic photonic comb-like waveguides system built of periodic segments of length d_1 (of right-handed material). The electrical permittivity, ε, of the two asymmetric resonators with lengths d_2 and d_3, depends on the frequency of the incident waves (ENG material). The presence of geometrical (ENG resonators) defects inside the perfect structure creates the defect modes inside the band gaps. Consequently, we demonstrate the existence of two filtered frequencies. This structure can be used as a new photonic filter in the microwave range with an important quality factor and a high transmission rate.

1. INTRODUCTION

Science and technology have a tendency to search for better and more efficient materials, capable of pushing the limits of what is possible. In the electromagnetic field, precisely the microwave field, "metamaterial" is a kind of artificial structure with properties that do not exist in any natural material. The main areas in which metamaterials are expected to develop are information and communication technology. The presence of metamaterials of different types namely μ-negative (MNG), ε-negative (ENG), and double-negative (DNG) in comb-like waveguides (either at the segments or resonators) has been studied by several researchers due to their unusual and extraordinary properties by comparing them with right-handed materials (RHMs). The first theory about their electromagnetic properties was introduced by the Russian researcher Veselago in 1968 [1], and in 2000, a first practical realization was proposed by the American researchers Smith et al. [2]. Since the practical appearance of metamaterials of various types, researchers have exploited them, with the aim of making more compact and miniaturized filters [3–11]. Researchers have been interested in metamaterials because they enable novel microwave applications and optimizations as well as the introduction of new physical properties [12, 13].

For a long time, researchers have studied the band structure and transmission coefficient of one-dimensional photonic crystals (multilayer structures) containing metamarials [8–13]. Bria et al. proposed an omnidirectional light reflector for TE and TM polarizations with a multilayer structure composed of conventional RHM and metamaterials [14]. In another work, Essadqui et al. observed new dispersion curves that do not exist in the usual superlattices composed only of conventional RHMs [15]. The presence of metamaterials in guided microwave comb-like waveguide structures has become very important in different fields, because of their capacity to filter electromagnetic waves [16–19]. This metamaterial periodic comb-like waveguides system can create large gaps in which the propagation of the electromagnetic waves is forbidden [20, 21]. The existence of metamaterial defects in the comb-like waveguides creates very narrow defect modes in the band gaps with higher quality factor and important transmission rate [16].

Several authors have studied the electromagnetic filter based on Fano and EIT (electromagnetically induced transparency) resonances [22]. The coupling of photonic waveguides with one or more resonators has been studied and applied to obtain Fano and EIT resonances [23–29]. In transmission spectra, the Fano profile appears as a maximum transmission peak near a transmission zero [14, 15]. When the Fano resonance falls between two anti-resonances (two transmission zeros), it becomes an EIT resonance.

In general, to create this type of resonance in classical systems, we need two or more resonators directly or indirectly connected to a waveguide. This system is called a cross shaped structure.

The objective of this work is to study the presence of different types of metamaterials in one-dimensional photonic comb-like waveguides containing defects. The system in question is com-

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posed of the periodicity of segments made of double positive (DPS) material and N' grafted resonators (made of metamaterials) in N equidistant sites with $N' = N_1' + N_2'$ (N_1' and N_2' are the numbers of asymmetric grafted resonators of lengths d_2 and d_3, respectively). We study the presence of metamaterials defects in one-dimensional photonic comb-like waveguides system using the Green function method.

2. MODEL AND FORMALISM

2.1. EIT Resonance Based on Two Grafted Resonators at the Same Site

Our theoretical analysis is performed using Green's function method in order to calculate the transmission rate. The structure proposed in this part is composed of two grafted resonators at the same site located between two semi-infinite segments (Fig. 1(a)).

The principle of this method consists of building the inverse of the Green's function $g^{-1}(0,0)$ of the ensemble system from a superposition of the different constituents, which are two semi-infinite segments and two resonators of lengths d_2 and d_3.

The inverse of the Green's function of a semi-infinite segment that constitutes the input and output is given by [22–24]:

$g_0^{-1}(0,0) = -F_s = \frac{-j\omega}{\sqrt{\frac{\mu}{\varepsilon}}}$ with: $\varepsilon = \varepsilon_2 = \varepsilon_3 = 1 - \frac{1.33^2}{\Omega^2}$.

The inverse of the Green's functions of resonators (of lengths d_2 and d_3) is given by [22–24]:

$$g_2^{-1}(0,d_2) = \frac{-S_2F_2}{C_2} \quad \text{and} \quad g_3^{-1}(0,d_3) = \frac{-S_3F_3}{C_3} \tag{1}$$

with: $C_i = \mathrm{Cosh}(\alpha_i d_i) = \mathrm{Cosh}\,(j\alpha'_i d_i) = \mathrm{Cos}\,(\alpha'_i d_i)$, $S_i = \mathrm{Sinh}(\alpha_i d_i) = \mathrm{Sinh}(j\alpha'_i d_i) = j\mathrm{Sin}(\alpha'_i d_i)$, $\tanh(\alpha_i d_i) = \frac{\mathrm{Sinh}(\alpha_i d_i)}{\mathrm{Cosh}(\alpha_i d_i)} = \frac{j\mathrm{Sin}(\alpha'_i d_i)}{\mathrm{Cos}(\alpha'_i d_i)} = j\tan(\alpha'_i d_i)$, $\alpha_i = j\frac{\omega}{c}\sqrt{\varepsilon_i\mu_i} = j\alpha'_i$ with $j = \sqrt{-1}$.

We assume that: $F_s = F_i = F$; $2 < i < 3$.

By superposing the inverse elements of the Green's functions of the different constituents, we obtain the inverse interface element of the structure (Fig. 1(a)) at the site 0:

$$\begin{aligned} g^{-1}(0,0) &= -2F - \frac{FS_2}{C_2} - \frac{FS_3}{C_3} \\ &= -2F - jF(\tan(\alpha'_2 d_2) + \tan(\alpha'_3 d_3)) \end{aligned} \tag{2}$$

The transmission coefficient through the structure is given by the following expression:

$$\begin{aligned} t &= -2Fg(0,0) \\ &= \frac{2}{2 + j(\tan(\alpha'_2 d_2) + \tan(\alpha'_3 d_3))} \\ &= \left[\frac{4}{4 + ((\tan(\alpha'_2 d_2) + \tan(\alpha'_3 d_3)))^2}\right] \\ &\quad -j\left[\frac{2(\tan(\alpha'_2 d_2) + \tan(\alpha'_3 d_3))}{4 + ((\tan(\alpha'_2 d_2) + \tan(\alpha'_3 d_3)))^2}\right] \\ &= |t|\, e^{j\varphi} \end{aligned} \tag{3a}$$

The transmission rate T through the structure is given by the following relation:

$$T = |t|^2 = \frac{4}{4 + ((\tan(\alpha'_2 d_2) + \tan(\alpha'_3 d_3)))^2} \tag{3b}$$

2.2. Dispersion Relation of the Infinite Periodic System

In this part, we calculate the dispersion relation of the infinite comb-like waveguides structure.

The Green's function of a segment of length d_1 is written in the following form [22]:

$$\overleftrightarrow{g}_1^{-1}(0,d_1) = \begin{pmatrix} \frac{-F_1C_1}{S_1} & \frac{F_1}{S_1} \\ \frac{F_1}{S_1} & \frac{-F_1C_1}{S_1} \end{pmatrix} \tag{4}$$

with: $C_1 = \mathrm{Cosh}(\alpha_1 d_1)$; $S_1 = \mathrm{Sinh}(\alpha_1 d_1)$ and $\alpha_1 = j\frac{\omega}{c}\sqrt{\varepsilon_1\mu_1}$.

The Green's functions of the grafted lateral branches (resonators) of lengths d_2 and d_3 are written in this form:

$$g_2^{-1}(0,d_2) = \frac{-S_2F_2}{C_2} \quad \text{and} \quad g_3^{-1}(0,d_3) = \frac{-S_3F_3}{C_3} \tag{5}$$

with: $C_i = \mathrm{Cosh}(\alpha_i d_i)$; $S_i = \mathrm{Sinh}(\alpha_i d_i)$ and $\alpha_i = j\frac{\omega}{c}\sqrt{\varepsilon_i\mu_i}$; $2 < i < 3$.

In the infinite photonic comb-like waveguides structure, the inverse of the Green's function $\overleftrightarrow{g}_\infty^{-1}(MM)$ is an infinite tridiagonal matrix formed by the superposition of the elements g_i^{-1} in the interfaces space $\{M\}$. This matrix is written as follows:

$$\overleftrightarrow{g}_\infty^{-1}(MM) = \begin{pmatrix} \ddots & \ddots & \ddots & & & \\ & A & B & A & & \\ & & A & B & A & \\ & & & A & B & A \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \tag{6}$$

with: $A = \frac{F_1}{S_1}$ and $B = -2F_1\frac{C_1}{S_1} - \frac{S_2F_2}{C_2} - \frac{S_3F_3}{C_3}$.

We deduce the dispersion relation in the form:

$$\cos(Kd_1) = C_1 + \frac{1}{2}\frac{S_1}{F_1}\left(\frac{S_2F_2}{C_2} + \frac{S_3F_3}{C_3}\right) \tag{7}$$

where K is the Bloch vector.

The system is periodic in the x direction, and the Fourier transform $\overleftrightarrow{g}^{-1}[(K;MM)]$ is written as follows:

$$\overleftrightarrow{g}^{-1}[(K;MM)]$$

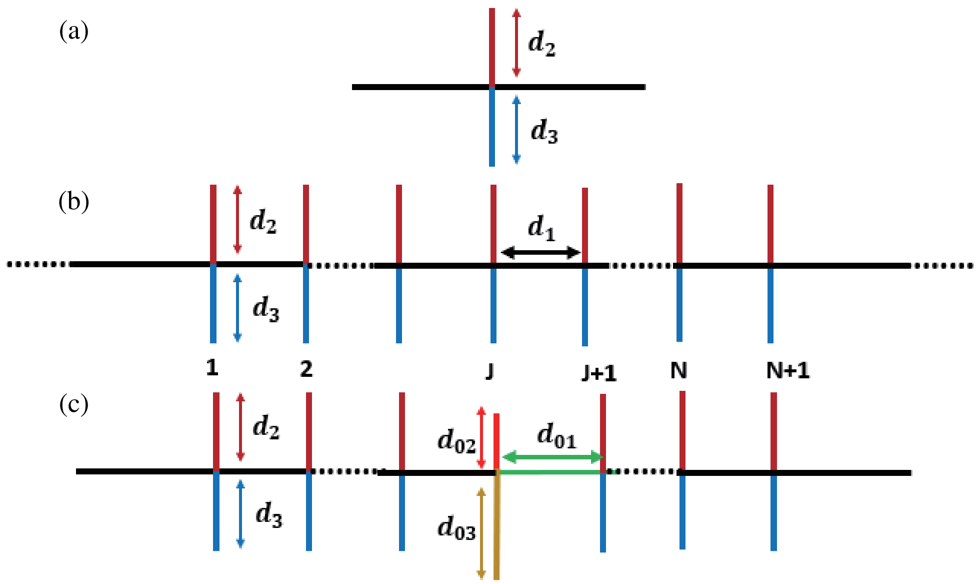

FIGURE 1. (a) One-dimensional electromagnetic waveguides with two grafted metamaterials resonators of lengths d_2 and d_3 at the same site. (b) One-dimensional comb-like waveguides structure composed of the periodicity of segments of length d_1 and two grafted resonators of lengths d_2 and d_3. (c) Comb-like waveguides structure containing three defects at the segment of length d_{01} and resonators of lengths d_{02} and d_{03}.

$$= -2F_1\frac{C_1}{S_1} - \frac{S_2F_2}{C_2} - \frac{S_3F_3}{C_3} + \frac{F_1}{S_1}\left(e^{jKd_1} + e^{-jKd_1}\right) \tag{8a}$$

From where:

$$\overleftrightarrow{g}^{-1}[(K;MM)] = 2\frac{F_1}{S_1}[-\delta + \cos(Kd_1)] \tag{8b}$$

where: $\delta = C_1 + \frac{1}{2F_1}S_1\left(\frac{S_2F_2}{C_2} + \frac{S_3F_3}{C_3}\right)$.

The bulk bands of the comb-like waveguides structure are obtained from the poles of the Green's function by the following relation:

$$\cos(Kd_1) = \delta \tag{9}$$

2.3. Transmission Rate of Defects Photonic System

In this section, we study the transmission properties of finite comb-like waveguides containing the defects. This structure (Fig. 1(c)) is constructed as follows: a finite system containing N cells is cut from the infinite periodic system and connected at its extremities by two semi-infinite segments. The defects are created inside the structure. The first defect consists in changing a resonator of length d_2 located in the site J by another resonator of length d_{02}; the second defect consists in changing a resonator of length d_3 located in the site J by another resonator of length d_{03}; and the third defect consists in changing a segment of length d_1 located between the sites J and $J+1$ by another segment of length d_{01}. Therefore, the disturbed states are: $Ms = \{-1, 0, J, J+1, N, N+1\}$ [25–29].

The cleavage operator $\overleftrightarrow{V}(M_sM_s) = \overleftrightarrow{g}_t^{-1}(MM) - \overleftrightarrow{g}_\infty^{-1}(MM)$ is the matrix (6×6) defined in the interfaces domain consisting of sites $n = -1, 0, J, J+1, N, N+1$.

$$\overleftrightarrow{V}(M_sM_s) = \begin{pmatrix} \frac{F_1C_1}{S_1} & -\frac{F_1}{S_1} & 0 & 0 & 0 & 0 \\ -\frac{F_1}{S_1} & \frac{F_1C_1}{S_1} + \left(\frac{F_2S_2}{C_2} + \frac{F_3S_3}{C_3}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & \left(\frac{F_1}{S_1} - \frac{F_{01}}{S_{01}}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{F_1}{S_1} - \frac{F_{01}}{S_{01}}\right) & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{F_1C_1}{S_1} & -\frac{F_1}{S_1} \\ 0 & 0 & 0 & 0 & -\frac{F_1}{S_1} & \frac{F_1C_1}{S_1} \end{pmatrix} \tag{10}$$

with: $\beta = \frac{F_2S_2}{C_2} + \frac{F_3S_3}{C_3} - \left(\frac{F_{02}S_{02}}{C_{02}} + \frac{F_{03}S_{03}}{C_{03}}\right)$.

$\overleftrightarrow{g}_\infty^{-1}(MM)$ is the inverse of Green's function of the infinite system, and $\overleftrightarrow{g}_t^{-1}(MM)$ is the inverse of Green's function of the finite structure containing the defects.

The knowledge of the elements of the Green's inverse function in the infinite star waveguides structure $\overleftrightarrow{g}_\infty^{-1}(MM)$ and those of the cleavage operator $\overleftrightarrow{V}(M_sM_s)$ makes it possible to deduce the necessary elements of the response function of the finite structure for the calculation of the transmission rate.

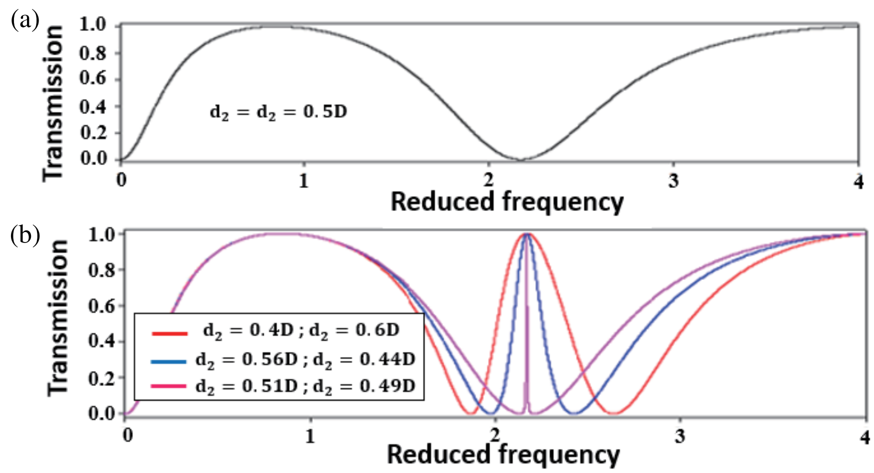

FIGURE 2. Transmission rate as a function of the reduced frequency Ω for different values of d_2 and d_3.

The interfaces response operator $\overleftrightarrow{A}(M_s M_s)$ is written as follows:

$$\overleftrightarrow{A}(M_s M_s) = \sum_{M_s} \overleftrightarrow{V}(M_s M_s)\, \overleftrightarrow{g}(M_s M_s) \tag{11}$$

The operator $\overleftrightarrow{\Delta}(M_s M_s)$ is given by the following relation:

$$\overleftrightarrow{\Delta}(M_s M_s) = \overleftrightarrow{I}(M_s M_s) + \overleftrightarrow{A}(M_s M_s) \tag{12}$$

After calculating the operator $\overleftrightarrow{\Delta}(M_s M_s)$, we write it in the interfaces space $M_0 = \{0, J, J+1, N\}$.

The Green's function $\overleftrightarrow{d}(M_0 M_0)$ for finite star waveguides structure is defined in the interfaces space M_0 by the following equation:

$$\overleftrightarrow{d}(M_0 M_0) = \overleftrightarrow{g}(M_0 M_0)\, \overleftrightarrow{\Delta}^{-1}(M_0 M_0) \tag{13}$$

with $\overleftrightarrow{\Delta}^{-1}(M_0 M_0)$ being the inverse of the operator $\overleftrightarrow{\Delta}(M_0 M_0)$.

We deduce the truncated matrix $\overleftrightarrow{d}_{tr}^{-1}(M'_0 M'_0)$ in the interfaces space $M'_0 = \{0, N\}$. The inverse of this matrix is written as:

$$\overleftrightarrow{d}_{tr}^{-1}(M'_0 M'_0) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \tag{14}$$

where d_{11}, d_{12}, d_{21}, and d_{22} are the functions of the elements (1, 1), (1, 4), (4, 1), and (4, 4) of the matrix $\overleftrightarrow{d}_{tr}^{-1}(M'_0 M'_0)$.

Finally, the Green's function of finite photonic star waveguides $\overleftrightarrow{d}_h(M'_0 M'_0)$ located between two semi-infinite segments $(-F_1)$:

$$\overleftrightarrow{d}_h(M'_0 M'_0) = \frac{1}{(A_{22}-F_1)(A_{11}-F_1)-A_{12}A_{21}}$$

$$\begin{bmatrix} d_{22}-F_1 & -d_{21} \\ -d_{12} & d_{11}-F_1 \end{bmatrix} \tag{15}$$

The transmission rate T through the structure is given by the following relation:

$$T = \left| \frac{2F_1}{(A_{22}-F_1)(A_{11}-F_1)-A_{12}A_{21}} A_{12} \right|^2 \tag{16}$$

3. RESULTS AND DISCUSSIONS

3.1. EIT Resonance Corresponds to Grafted Resonators

Cross shaped resonators with a segment lead to obtain an EIT resonance. All curves are given with the reduced frequency $\Omega = \frac{\omega\sqrt{\varepsilon_1\mu_1}D}{\mathbf{c}}$, with **c** being the velocity of electromagnetic waves in vacuum and ω the pulsation (s^{-1}).

Similar to ordinary materials (RHM) [22–24], EIT resonance is also present in metamaterials. In general, to create this type of resonance in conventional systems (RHM), two or more resonators are grafted at the same site (cross structure) or two different sites (U shape structure).

Our proposed metamaterials system (Fig. 1(a)) consists of a segment with two grafted resonators (ENG) at the same site, the two resonators are transmission lines made of the same materials ($\varepsilon_2 = \varepsilon_3 = 1 - \frac{1.33^2}{\Omega^2}$; $\mu_2 = \mu_3 = 1$) and characterized by the different lengths d_2 and d_3.

In this part, we examine the effect of changing the lengths d_2 and d_3 on the transmission spectrum. Fig. 2 shows the variation of the transmission rate versus the reduced frequency. We notice the appearance of an EIT resonance since the maximum transmission peak is wedged between two transmissions zero. This resonance is a consequence of constructive interferences, while the transmission zero is a consequence of destructive interferences. The full width at half maximum of the EIT resonance can be adjusted by adapting the length variation

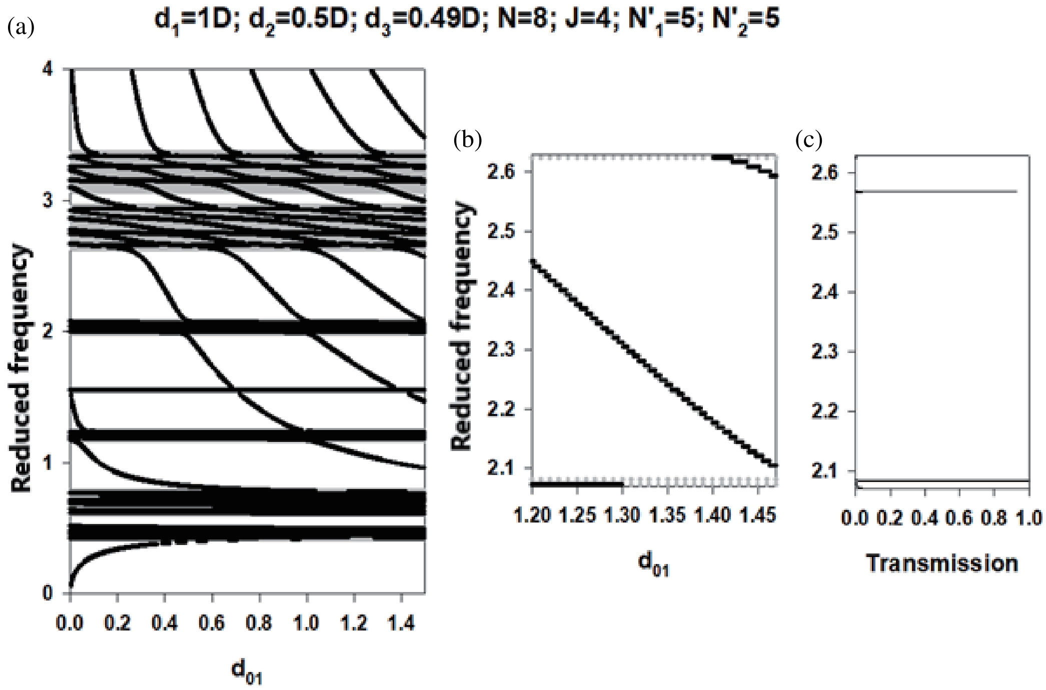

FIGURE 3. (a) Variation of the reduced frequency Ω as a function of the defect length d_{01}. (b) Zoom of the sixth band gap located between $\Omega = 2.1$ and $\Omega = 2.7$. (c) Transmission rate versus the reduced frequency for $d_{01} = 0.47D$.

($\Delta = d_3 - d_2$) between two resonators. In particular, the full width at half maximum of the EIT resonance decreases with the decrease of the parameter Δ and disappears in the case where $\Delta = 0$ (Fig. 2(a)). These results are in good agreement with those found by Mouadili et al. and Noual et al. in photonics and plasmonic structures [23, 24, 30–33].

3.2. Effect of the Defect Segment Length d_{01}

In this paragraph, we show that the presence of a defect segment between the sites J and $J + 1$ gives the existence of defect modes in the band gaps. Fig. 3 represents the variation of the reduced frequency Ω versus the defect length d_{01} with $N'1 = N'2 = 1$ (two ENG resonators in each site), $N = 8$ and $J = 4$. The grey areas represent the passbands, and the white areas indicate the band gaps. In the passbands, we observe that there exist some structure branches which are independent of d_{01}, while the others are decreasing in frequency. However, two behaviors of the defect modes inside the band gaps are observed: For the first gap located between $\Omega = 0$ and $\Omega = 0.2$ (Fig. 3(a)), we notice that the defect mode frequencies increase with the length d_{01} until it becomes a mode of the perfect structure. For the third gap located between $\Omega = 0.8$ and $\Omega = 1.2$ (Fig. 3(a)), the defect modes decrease with variation of the length d_{01}, but other defect modes are absent in the second gap. We conclude that our proposed defective structure creates defect modes in the band gaps, and the frequencies of these modes decrease or increase with the variation of the defect length d_{01}. Also, we study, in Fig. 3(c), the variation of the transmission rate for two defect modes shown in Fig. 3(b) versus the reduced frequency with $d_{01} = 0.47D$. The two defect modes located at two reduced frequency values $\Omega = 2.07$ and $\Omega = 2.56$ have a maximum transmission rate and a high-quality factor (very narrow defect modes). Therefore, when a defective segment of length d_{01} is inserted, two defect modes appear in the band gaps with a maximum transmission rate and a good quality factor. This structure can be found in many applications such as frequency demultiplexing and selective filters. From a general point of view, to realize a photonic filter, it is necessary to design a structure in which the transmission rate has well-defined characteristics to allow filter over a sufficiently wide frequency range.

In order to consider a new one-dimensional frequency-filtering device, we propose a comb-like waveguides structure with two geometrical defects at the resonators of lengths d_{02} and d_{03}.

3.3. Effect of the ENG Defect Resonators of Length d_{02} and d_{03}

In this part, we study, in Fig. 4(a), the evolution of the transmission rate of a perfect photonic comb-like waveguide structure versus the reduced frequency with $N' = 2$ ($N'_1 = N'_2 = 1$) and $N = 7$. We find the creation of five passbands separated by four band gaps. However, the presence of defects of lengths $d_{02} = 1.2D$ and $d_{03} = 1.4D$ at the resonators levels allows the decrease of the amplitude in the passbands and the creation of one or two defect modes in the band gaps (Figs. 4(b)–(d)). We conclude that the introduction of ENG defect resonators gives rise to defect modes in the band gaps, and these defect modes have maximum transmission rates and very high-quality factors.

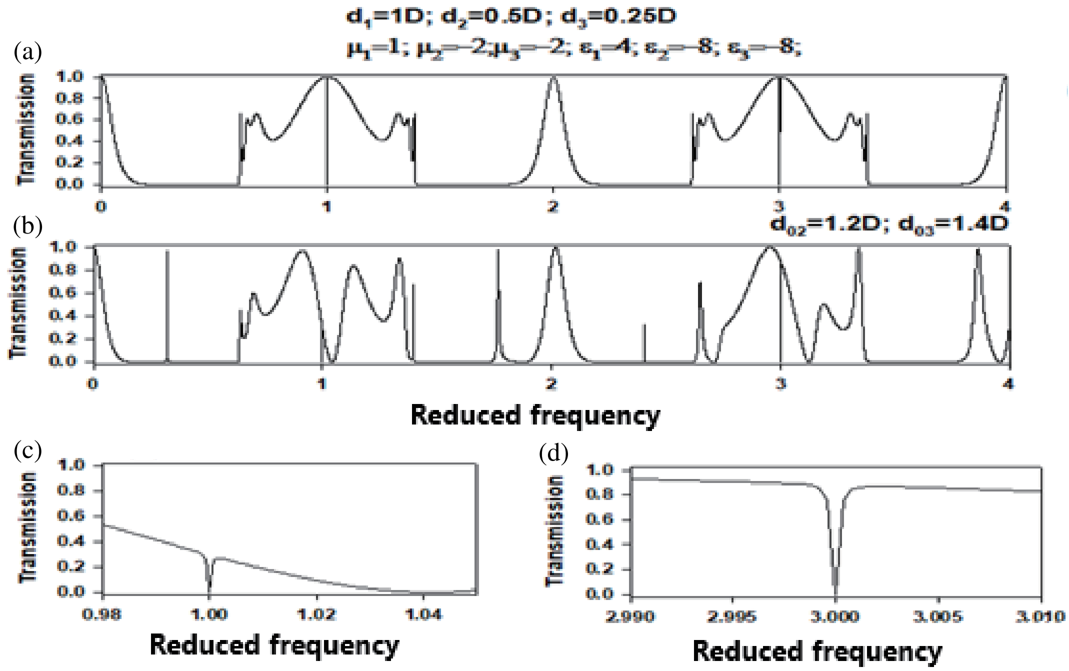

FIGURE 4. (a) Variation of the transmission rate as a function of the reduced frequency for a perfect periodic structure with $N' = 2$. (b) Transmission rate versus the reduced frequency in the case of presence of two defects resonators of lengths $d_{02} = 1.2D$ and $d_{03} = 1.4D$.

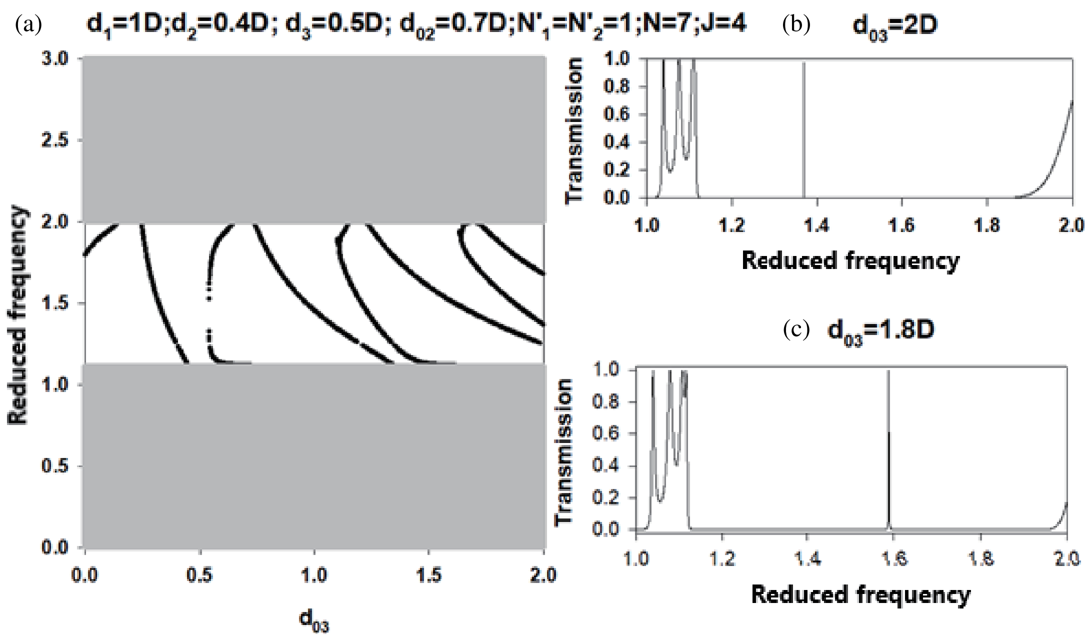

FIGURE 5. (a) Variation of the reduced frequency Ω as a function of the defect length d_{03} for $N = 7$, $J = 4$, $d_1 = 1D$, $d_2 = 0.4D$, $d_3 = 0.5D$ and $d_{02} = 0.7D$. (b)–(c) Transmission rate versus the reduced frequency for $d_{03} = 2D$ and $d_{03} = 1.8D$, respectively.

3.4. Effect of the Defect Length d_{03} on the Transmission Spectrum and the Band Structure

For a better understanding the existence and behavior of the resonators defect modes, we study, in Fig. 5(a), the evolution of the reduced frequency of the finite system as a function of defect resonator of length d_{03} with $d_{02} = 0.7D$, and we show that the defect modes frequencies decrease with increasing the defect length d_{03}. According to this figure, we can obtain three defect modes when the defect length d_{03} varies between 1.6 and 2. Finally, we study the variation of the transmission rate versus the reduced frequency for two values of the defect length d_{03} (Figs. 5(b)–(c)). We note that there exists one defect mode with a high transmission rate. This expected result allows to design a photonic filter with very high performance (higher transmission rate and quality factor).

4. CONCLUSION

In this work, we study the band structure of a perfect comb-like waveguides system and the transmission rate for an asymmetric comb-like structure. This structure is composed by the periodicity of segments (RHM)) of length d_1 grafted on each site by a finite number of asymmetric resonators (ENG) [34, 35] of lengths d_2 and d_3. Firstly, we have studied a simple case when we consider that two asymmetric resonators located between two semi-infinite segments. In this last situation, we have observed that the EIT resonance appears with a maximum transmission, and this EIT resonance is wedged between two transmissions of zero. The quality factor of the EIT resonance is very sensitive to the lengths of the asymmetric resonators. Secondly, we consider the asymmetric defects resonators (ENG) in the system, and we have shown that the defect modes appear in the band gaps. These defects modes frequencies decrease with the variation of the defects lengths. This metamaterials system can be used as filtering of the electromagnetic waves in the microwave range with an important quality factor and a high transmission rate. We use the properties of the EIT resonance in designing a photonic demultiplexer, filter, and coupler applications [31–33].

REFERENCES

[1] Veselago, V. G., "Electrodynamics of substances with simultaneously negative electrical and magnetic permeabilities," *Physics Uspekhi*, Vol. 10, No. 4, 504–509, 1968.

[2] Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Physical Review Letters*, Vol. 84, No. 18, 4184, May 2000.

[3] Liang, L., B. Jin, J. Wu, Y. Huang, Z. Ye, X. Huang, D. Zhou, G. Wang, X. Jia, H. Lu, L. Kang, W. Xu, J. Chen, and P. Wu, "A flexible wideband bandpass terahertz filter using multi-layer metamaterials," *Applied Physics B*, Vol. 113, No. 2, 285–290, 2013.

[4] Zhu, Z., X. Zhang, J. Gu, R. Singh, Z. Tian, J. Han, and W. Zhang, "A metamaterial-based terahertz low-pass filter with low insertion loss and sharp rejection," *IEEE Transactions on Terahertz Science and Technology*, Vol. 3, No. 6, 832–837, Nov. 2013.

[5] Horestani, A. K., W. Withayachumnankul, A. Chahadih, A. Ghaddar, M. Zehar, D. Abbott, C. Fumeaux, and T. Akalin, "Metamaterial-inspired bandpass filters for terahertz surface waves on Goubau lines," *IEEE Transactions on Terahertz Science and Technology*, Vol. 3, No. 6, 851–858, Nov. 2013.

[6] Lin, Y.-S., Y. Qian, F. Ma, Z. Liu, P. Kropelnicki, and C. Lee, "Development of stress-induced curved actuators for a tunable thz filter based on double split-ring resonators," *Applied Physics Letters*, Vol. 102, No. 11, 111908, Mar. 2013.

[7] Palikaras, G. and T. Kallos, "U.S. Patent No. 10,698,143. Washington, DC: U.S. Patent and Trademark Office," Tech. Rep., 2020.

[8] Rathore, V., S. Awasthi, and A. Biswas, "Design of compact dual-band bandpass filter using frequency transformation and its implementation with split ring resonator dual-band bandpass filter using SRR," in *2014 44th European Microwave Conference*, 949–952, Rome, Italy, Oct. 2014.

[9] Huang, T.-Y. and T.-J. Yen, "A high-ratio bandwidth square-wave-like bandpass filter by two-handed metamaterials and its application in 60 GHz wireless communication," *Progress In Electromagnetics Research Letters*, Vol. 21, 19–29, 2011.

[10] Lu, M., W. Li, and E. R. Brown, "Second-order bandpass terahertz filter achieved by multilayer complementary metamaterial structures," *Optics Letters*, Vol. 36, No. 7, 1071–1073, Apr. 2011.

[11] Jindal, S. and J. Sharma, "Review of metamaterials in microstrip technology for filter applications," *International Journal of Computer Applications*, Vol. 54, No. 3, 48–54, 2012.

[12] Jiang, H., H. Chen, H. Li, Y. Zhang, and S. Zhu, "Omnidirectional gap and defect mode of one-dimensional photonic crystals containing negative-index materials," *Applied Physics Letters*, Vol. 83, No. 26, 5386–5388, Dec. 2003.

[13] Houck, A. A., J. B. Brock, and I. L. Chuang, "Experimental observations of a left-handed material that obeys Snell's law," *Physical Review Letters*, Vol. 90, No. 13, 137401, Apr. 2003.

[14] Bria, D., B. Djafari-Rouhani, A. Akjouj, L. Dobrzynski, J. P. Vigneron, E. H. E. Boudouti, and A. Nougaoui, "Band structure and omnidirectional photonic band gap in lamellar structures with left-handed materials," *Physical Review E*, Vol. 69, No. 6, 066613, Jun. 2004.

[15] Essadqui, A., J. Ben-Ali, D. Bria, B. Djafari-Rouhani, and A. Nougaoui, "Photonic band structure of 1d periodic composite system with left handed and right handed materials by green function approach," *Progress In Electromagnetics Research B*, Vol. 23, 229–249, 2010.

[16] Ben-Ali, Y., Z. Tahri, and D. Bria, "Electromagnetic filters based on a single negative photonic comb-like structure," *Progress In Electromagnetics Research C*, Vol. 92, 41–56, 2019.

[17] Zhang, L., Z. Wang, H. Chen, H. Li, and Y. Zhang, "Experimental study of quasi-one-dimensional comb-like photonic crystals containing left-handed material," *Optics Communications*, Vol. 281, No. 14, 3681–3685, 2008.

[18] Cocoletzi, G. H., L. Dobrzynski, B. Djafari-Rouhani, H. Al-Wahsh, and D. Bria, "Electromagnetic wave propagation in quasi-one-dimensional comb-like structures made up of dissipative negative-phase-velocity materials," *Journal of Physics: Condensed Matter*, Vol. 18, No. 15, 3683, Apr. 2006.

[19] Yin, C.-P. and H.-Z. Wang, "Narrow transmission bands of quasi-1D comb-like photonic waveguides containing negative index materials," *Physics Letters A*, Vol. 373, No. 11, 1093–1096, Mar. 2009.

[20] Tan, W., Z. Wang, and H. Chen, "Complete tunneling of light through mu-negative media," *Progress In Electromagnetics Research M*, Vol. 8, 27–37, 2009.

[21] Vu, T.-M., "Contribution à la mise en oeuvre de fonctions accordables à MEMs RF en bade millimétrique sur silicium," Ph.D. dissertation, Doctoral dissertation, Université Paul Sabatier-Toulouse III, 2010.

[22] Ben-Ali, Y., A. Ghadban, Z. Tahri, K. Ghoumid, and D. Bria, "Accordable filters by defect modes in single and double negative star waveguides grafted dedicated to electromagnetic communications applications," *Journal of Electromagnetic Waves and Applications*, Vol. 34, No. 4, 539–558, 2020.

[23] Mouadili, A., E. H. E. Boudouti, A. Soltani, A. Talbi, A. Akjouj, and B. Djafari-Rouhani, "Theoretical and experimental evidence of fano-like resonances in simple monomode photonic circuits," *Journal of Applied Physics*, Vol. 113, No. 16, 164101, Apr. 2013.

[24] Mouadili, A., E. H. E. Boudouti, A. Soltani, A. Talbi, B. Djafari-Rouhani, A. Akjouj, and K. Haddadi, "Electromagnetically induced absorption in detuned stub waveguides: A simple analytical and experimental model," *Journal of Physics: Condensed Matter*, Vol. 26, No. 50, 505901, 2014.

[25] Errouas, Y., Y. Ben-Ali, I. E. Kadmiri, Z. Tahri, and D. Bria, "Propagation of electromagnetic waves in one dimensional symmetric and asymmetric comb-like photonic structure containing defects," *Materials Today: Proceedings*, Vol. 31, S16–S23, 2020.

[26] Errouas, Y., Y. Ben-Ali, A. Ouariach, Z. Tahri, and D. Bria, "Propagation of the electromagnetic waves in one-dimensional asymmetric photonic comb-like structure based on Fibonacci chains of grafted resonators," *Materials Today: Proceedings*, Vol. 27, No. 4, 3058–3064, 2020.

[27] Ben-Ali, Y., I. E. Kadmiri, E. Younes, A. Ahaitouf, and D. Bria, "Four frequencies filtering by the one-dimensional photonic defectives star waveguides structure," *Optical Memory and Neural Networks*, Vol. 31, No. 2, 163–178, Jun. 2022.

[28] Errouas, Y., Y. Ben-ali, A. Ouariach, Z. Tahri, and D. Bria, "Propagation of the electromagnetic waves in asymmetric defective one-dimensional photonic comb-like structure," in *Distributed Sensing and Intelligent Systems*, 607–617, Springer, Cham, 2022.

[29] Errouas, Y., Z. Tahri, Y. Ben-Ali, and D. Bria, "Band structure and photonic band gap in comb-like structure asymmetric with left-handed materials," in *2019 International Conference on Intelligent Systems and Advanced Computing Sciences (ISACS)*, 1–5, IEEE, 2019.

[30] Noual, A., O. E. Abouti, E. H. E. Boudouti, A. Akjouj, Y. Pennec, and B. Djafari-Rouhani, "Plasmonic-induced transparency in a MIM waveguide with two side-coupled cavities," *Applied Physics A*, Vol. 123, 1–7, 2017.

[31] Khattou, S., M. Amrani, A. Mouadili, E. H. E. Boudouti, A. Talbi, A. Akjouj, and B. Djafari-Rouhani, "Three port photonic and plasmonic demultiplexers based on Cross and U-shaped stub structures: Application for filtering and sensing," *Journal of Applied Physics*, Vol. 131, No. 15, 153102, Apr. 2022.

[32] Amrani, M., S. Khattou, Y. Rezzouk, A. Mouadili, A. Noual, E. H. E. Boudouti, and B. Djafari-Rouhani, "Analytical and numerical study of T-shaped plasmonic demultiplexer based on Fano and induced transparency resonances," *Journal of Physics D: Applied Physics*, Vol. 55, No. 7, 075106, 2021.

[33] El-Aouni, M., Y. Ben-Ali, I. E. Kadmiri, and D. Bria, "Y-shaped: ICEERE 2022, 20-22 May 2022, Saidia, Morocco (pp. 193-202)," Singapore: Springer Nature Singapore.

[34] Fernandes, P. and M. Raffetto, "Existence, uniqueness and finite element approximation of the solution of time-harmonic electromagnetic boundary value problems involving metamaterials," *Compel — The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. 24, No. 4, 1450–1469, 2005.

[35] Wicharn, S. and P. Buranasiri, "Quasibirefringent phase-matching technique for third-harmonic pulse generation from multilayered metamaterials," *Optical Engineering*, Vol. 57, No. 11, 111803, 2018.