Progress In Electromagnetics Research C, Vol. 29, 109–122, 2012

AN IMPROVED L_1-SVD ALGORITHM BASED ON NOISE SUBSPACE FOR DOA ESTIMATION

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Abstract—In this paper, an improved L_1-SVD algorithm based on noise subspace is presented for direction of arrival (DOA) estimation using reweighted L_1 norm constraint minimization. In the proposed method, the weighted vector is obtained by utilizing the orthogonality between noise subspace and signal subspace spanned by the array manifold matrix. The presented algorithm banishes the nonzero entries whose indices are inside of the row support of the jointly sparse signals by smaller weights and the other entries whose indices are more likely to be outside of the row support of the jointly sparse signals by larger weights. Therefore, the sparsity at the real signal locations can be enhanced by using the presented method. The proposed approach offers a good deal of merits over other DOA techniques. It not only increases robustness to noise, but also enhances resolution in DOA estimation. Furthermore, it is not very sensitive to the incorrect determination of the number of signals and can primely suppress spurious peak in DOA estimation. Simulation results are shown that the presented algorithm has better performance than the existing algorithms, such as MUSIC, L_1-SVD algorithm.

1. INTRODUCTION

Direction-of-arrival (DOA) estimation of far-field narrowband signals has been of interest in the past few decades [1], which plays a fundamental role in many applications involving electromagnetic, acoustic, seismic sensing, etc. An important goal for source localization

Received 12 February 2012, Accepted 28 April 2012, Scheduled 12 May 2012

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methods is to be able to locate closely spaced signals in the presence of considerable noise. Many advanced methods [2, 3] for the localization of point signals attain super-resolution by exploiting the presence of a small number of signals. The most well-known existing nonparametric methods include beamforming [4] and its relevant algorithms [5, 6], Capon's method [7], and subspace based methods such as MUSIC [8]. Some additional methods (Root-MUSIC and ESPRIT) [9] are under the assumption that the array of sensors is linear. Beamforming spectrum suffers from the Rayleigh resolution limit, which is independent of the SNR. Capon's method and MUSIC are able to resolve signals within a Rayleigh cell (i.e., achieve super-resolution), provided that the SNR is reasonably high, the signals are not highly correlated, and the number of snapshots is sufficient.

The theme of sparse signal representation has evolved very rapidly in the last decade, finding application in all kinds of problems. There has also been some emerging research of these ideas in the context of spectrum estimation and array processing [10–13]. Sacchi et al. [10] exploit a Cauchy-prior to achieve sparsity in spectrum estimation and work out the resulting optimization problem by iterative approaches. Jeffs [11] makes use of an L_p-norm penalty with $p \leq 1$ to enforce sparsity for a plenty of applications, including sparse antenna array design. Gorodnitsky and Rao [12] use a recursive weighted minimum-norm algorithm called focal under-determined system solver (FOCUSS) to enforce sparsity in the problem of DOA estimation. It was later shown [14] that the algorithm is related to the optimization of L_p penalties with $p \leq 1$. The work of Fuchs [13] is involved in signal localization in the beamspace domain, under the assumption that the signals are uncorrelated, and the number of snapshots is abundant. The method tries to represent the vector of beamformer outputs to unknown signals as a sparse linear combination of vectors from a basis of beamformer outputs to isolated unit power signals. It makes use of the L_1 penalty for sparsity and the L_2 penalty for noise. Prior research has established sparse signal representation as a valuable tool for signal processing, but its application to signal localization is very limited in some scenarios. Recently, a new sparse-representation-based DOA estimation method, such as the L_1-SVD [15], provides another interpretation of array data by sparsely representing array data in an overcomplete basis, which emphasizes the fact that DOAs of incoming signals are usually very sparse relative to the whole spatial domain. In this way, the estimation problem is put in a model-fitting framework in which DOA estimation is attained by searching the sparsest representation of the received data. The L_1-SVD method [15] is of particular relevance to this work, as it converts DOA estimation problem into a sparse signal reconstruction one and exploits compressive sensing (CS) approach. It is carried out by L_1-norm constraint minimization due to it is a convex problem. However, L_1-norm constraint minimization has a drawback that larger coefficients of signal are punished more heavily than smaller coefficients, unlike the more impartial punishment of the L_0-norm constraint minimization [16]. This incurs the degradation of signal recovery performance based on regular L_1-norm constraint minimization [16]. To surmount this problem, the iterative reweighted L_1-norm constraint minimization is devised for the single measurement vector (SMV) problem, in which large weights are used to restraint nonzero entries in the recovered signal [16]. The convergence of the iterative reweighted L_1-norm constraint minimization is detailed in [17]. The iterative reweighted L_1-norm constraint minimization can improve not only recoverable sparsity thresholds upon the regular L_1-norm constraint minimization but also the recovery accuracy in the noisy case [18]. The substance of the iterative reweighted L_1-norm constraint minimization algorithm is that large weights could be used to punish the entries whose indices are more likely to be outside of the signal support, which facilitates sparsity at the right locations [16–18].

In this paper, the methodology of the iterative reweighted L_1-norm constraint minimization is expanded from the SMV case to the multiple measurement vectors (MMV) case for DOA estimation. Making use of the orthogonality between noise subspace and signal subspace spanned by the array manifold matrix, the objective of weighted L_1-norm constraint minimization can be achieved, namely, the nonzero entries whose indices correspond to the row support of the jointly sparse signals are punished by smaller weights and the other entries whose indices are more likely to be outside of the row support are punished by larger weights.

This paper is organized as follows. Section 2 briefly represents the problem of DOA estimation in the sparse signal framework. The proposed method is given in Section 3. Section 4 presents several simulation results to verify the performance of the proposed method. Section 5 provides a concluding remark to summarize the paper.

2. BACKGROUND

Assume that L far-field stationary and narrowband signals impinge on a uniform linear array (ULA) of $M(M > L)$ sensors from distinct direction angles $\{\theta_l, l = 1, \ldots, L\}$, which are corrupted by additive Gaussian white noise. The array output at time t can be expressed as

$$\mathbf{y}(t) = \mathbf{A}(\theta)\mathbf{u}(t) + \mathbf{n}(t), \qquad t = 1, 2, \ldots, T \tag{1}$$

where $\mathbf{u}(t) = [u_1(t), \ldots, u_L(t)]^T$ is a zero-mean signal vector; the superscript $(\cdot)^T$ stands for transpose operation; $\mathbf{n}(t) = [n_1(t), \ldots, n_M(t)]^T$ with $n_i(t)$ denoting the additive noise of the ith sensor, where $n_i(t)$ is a complex Gaussian random process with zero-mean and equal covariance $\sigma^2\mathbf{I}_M$; T is the number of data samples; $\mathbf{A}(\theta)$ is an $M \times L$ array manifold matrix, whose lth column is the lth signal array steering vector as follows

$$\begin{aligned}\mathbf{a}(\theta_l) \;=\; & [1, \exp(-j2\pi f d_{21} \sin\theta_l/c), \ldots, \exp(-j2\pi f d_{m1} \sin\theta_l/c), \\ & \ldots, \exp(-j2\pi f d_{M1} \sin\theta_l/c)]^T\end{aligned}$$

where f is the carrier frequency of the signals, d_{m1} the distance between the mth sensor and the first sensor, and c the velocity of propagation.

Because the distribution of the actual signals is sparse in space domain, reference [15] has formulated the DOA estimation problem into a sparse signal reconstruction problem. Introducing an overcomplete representation $\mathbf{\Phi}$ in terms of all possible signal locations. Let $\{\varphi_1, \ldots, \varphi_K\}$ be a sampling grid of all signal locations of interest, e.g., from $-90°$ to $90°$ with $1°$ intervals. The number of potential signal locations K will typically be much greater than the number of signals L or even the number of sensors M. Constructing a matrix composed of steering vectors corresponding to each potential signal location as its columns: $\mathbf{\Phi} = [\mathbf{a}(\varphi_1), \ldots, \mathbf{a}(\varphi_K)] \in \mathbb{C}^{M\times K}$. In this framework, $\mathbf{\Phi}$ is known and does not depend on the actual signal locations. The data model (1) can be reformulated as

$$\mathbf{y}(t) = \mathbf{\Phi}\mathbf{s}(t) + \mathbf{n}(t), \qquad t = 1, 2, \ldots, T \tag{2}$$

where $\mathbf{s}(t) \in \mathbb{C}^{K\times 1}$ is the expanded snapshot of the arriving signals, whose ith entry is equal to the jth entry of $\mathbf{u}(t)$ if $\varphi_i = \theta_j$, otherwise is 0. Therefore, DOA information of the signals is converted into the positions of the non-zero entries in $\mathbf{s}(t)$.

Now recovering $\mathbf{s}(t)$ from the under-determined linear equation system (2) reduces to a sparse reconstruction problem similar to many treated in the CS application. Given sparsity of $\mathbf{s}(t)$ and various restrictions on $\mathbf{\Phi}$, theorems [19–21] claim that $\mathbf{s}(t)$ can be almost surely recovered through L_0-norm constraint minimization

$$(P_0) \quad \min_{\mathbf{s}(t)} \|\mathbf{s}(t)\|_0 \quad s.t. \quad \mathbf{y}(t) = \mathbf{\Phi}\mathbf{s}(t)$$

Unfortunately, (P_0) is an NP-hard problem. A remedy is to use the L_1-norm constraint minimization instead

$$(P_1) \quad \min_{\mathbf{s}(t)} \|\mathbf{s}(t)\|_1 \quad s.t. \quad \mathbf{y}(t) = \mathbf{\Phi}\mathbf{s}(t)$$

The convexity and effectiveness of (P_1) has made it very popular in CS and sparse signal reconstruction.

With multiple measurement vectors (MMV), the model (2) can be rewritten as [15]

$$\mathbf{Y} = \mathbf{\Phi S} + \mathbf{N}$$

where $\mathbf{Y} = [\mathbf{y}(1), \ldots, \mathbf{y}(T)] and \mathbf{N} = [\mathbf{n}(1), \ldots, \mathbf{n}(T)]$. T is the number of snapshots, and $\mathbf{S} = [\mathbf{s}(1), \ldots, \mathbf{s}(T)]$ is the expanded snapshots of the arriving signals.

The L_1-SVD method as presented in [15] consists of three steps: computing the singular value decomposition (SVD) of $\mathbf{Y} = \mathbf{U \Lambda V}^H$; taking the first L columns of $\mathbf{U}$ which is denoted by $\mathbf{Y}_{SV} = \mathbf{U \Lambda D_L} = \mathbf{YVD_L} \in \mathbb{C}^{M \times L}$, where $\mathbf{D_L} = [\mathbf{I_L}\ 0]^T$, here $\mathbf{I_L}$ is a $L \times L$ identity matrix, and 0 is a $L \times (T - L)$ matrix of zeros. In addition, let $\mathbf{S}_{SV} = \mathbf{SVD_L}$ and $\mathbf{N}_{SV} = \mathbf{NVD_L}$; solving the following optimization problem

$$\min_{S_{SV}} \|\tilde{\mathbf{s}}\|_1 \qquad s.t. \quad \|\mathbf{Y}_{SV} - \mathbf{\Phi S}_{SV}\|_f^2 \leq \beta^2$$

where $\mathbf{S}_{SV} \in \mathbb{C}^{K \times L}$ is the first L columns of $\mathbf{SV}$; $\tilde{\mathbf{s}} \in \mathbb{C}^{K \times 1}$ is the estimated spatial spectrum whose entries are defined to be the 2-norm of the corresponding rows of $\mathbf{SV}$; β is the pre-given regularization parameter [15].

3. THE PROPOSED METHOD

The L_1-SVD algorithm enforces sparsity by the regular L_1-norm constraint minimization [15]. However, the regular L_1-norm constraint minimization can not obtain exact recovery in signal recovery processing [16]. To solve this problem, Candes *et al.* designed an iterative reweighted formulation of L_1-norm constraint minimization that large weights are appointed to the entries of the recovered signal whose indices are outside of the signal support [16]. The iterative L_1 reweighted is given as

$$w_i^{(p+1)} = \left[x_i^{(p+1)} + \varepsilon\right]^{-1}$$

where x_i denotes the ith entry of the recovered signal; w_i is the corresponding weighted value; $\varepsilon > 0$ is an application-dependent parameter and must be carefully designed; p is the iteration count number.

Now, the idea of iterative reweighted L_1-norm constraint minimization is expanded from the SMV problem to the MMV problem. This idea can be achieved by utilizing the orthogonality between noise subspace and signal subspace spanned by the array manifold matrix. By taking advantage of the singular value decomposition (SVD) on the data matrix $\{\mathbf{y}(t)\}_{t=1}^{\infty}$, the following equation can be obtained

$$\{\mathbf{y}(t)\}_{t=1}^{\infty} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H = [\mathbf{U}_S \ \ \mathbf{U}_N]\boldsymbol{\Lambda}\mathbf{V}^H$$

where $\{\mathbf{y}(t)\}_{t=1}^{\infty}$ is the received data matrix; t is from 1 to ∞; $\mathbf{U}$ and $\mathbf{V}$ are the matrixes, which consist of the left singular vectors and the right singular vectors of $\{\mathbf{y}(t)\}_{t=1}^{\infty}$, respectively. $\boldsymbol{\Lambda}$ is a diagonal matrix which consists of the singular values of $\{\mathbf{y}(t)\}_{t=1}^{\infty}$, and the singular values are nonnegative and arranged in descending order. $\mathbf{U}_S$ is the signal subspace, which is the first L columns of $\mathbf{U}$ and $\mathbf{U}_N$ is the noise subspace, which is the last $M-L$ columns of $\mathbf{U}$. According to [8], it is easy to know that

$$\mathbf{A}^H\mathbf{U}_N = 0 \in \mathbb{C}^{L\times(M-L)} \tag{3}$$

Considering the relation between the overcomplete basis matrix $\boldsymbol{\Phi}$ and the array manifold matrix $\mathbf{A}$, $\boldsymbol{\Phi}$ can be rewritten as $\boldsymbol{\Phi} = [\mathbf{A} \ \ \mathbf{B}]$, where $\mathbf{B} \in \mathbb{C}^{M\times(K-L)}$.

Utilizing the property in (3), we have the following equation

$$\boldsymbol{\Phi}^H\mathbf{U}_N = \left[\mathbf{U}_N^H\mathbf{A} \ \ \mathbf{U}_N^H\mathbf{B}\right]^H = \left[0^H \ \ \mathbf{D}^H\right]^H$$

where $\mathbf{D}_i^{(l_2)} > 0$, $\mathbf{D}_i^{(l_2)}$ denotes the ith entry of $\mathbf{D}^{(l_2)}$, $\mathbf{D}^{(l_2)}$ is the column vector that denotes the L_2-norm of each row of $\mathbf{D}$. In actual application, we have to substitute the sample data matrix $\mathbf{Y}$ for $\{\mathbf{y}(t)\}_{t=1}^{\infty}$. Substituting $\hat{\mathbf{U}}_N$ for $\mathbf{U}_N$ yields that

$$\boldsymbol{\Phi}^H\hat{\mathbf{U}}_N = \left[\hat{\mathbf{U}}_N^H\mathbf{A} \ \ \hat{\mathbf{U}}_N^H\mathbf{B}\right]^H = \left[\mathbf{W}_A^H \ \ \mathbf{W}_B^H\right]^H = \mathbf{W}$$

The weighted vector can be obtained as follows

$$\mathbf{w}^{(l_2)} = \left[\mathbf{W}_A^{(l_2)T} \ \ \mathbf{W}_B^{(l_2)T}\right]^T$$

when the snapshot $T \to \infty$, then $\mathbf{W}_A^{(l_2)} \to 0^{(l_2)} \in \mathbb{R}^{L\times 1}$ and $\mathbf{W}_B^{(l_2)} \to \mathbf{D}^{(l_2)} \in \mathbb{R}^{(K-L)\times 1}$, and then the entries of $\mathbf{W}_A^{(l_2)}$ are smaller than those of $\mathbf{W}_B^{(l_2)}$.

Define

$$\mathbf{G} = \mathrm{diag}\{\mathbf{w}^{(l_2)}\} \tag{4}$$

Consequently, we can employ $\mathbf{G}$ as a weighted matrix to achieve the idea that the nonzero entries whose indices are inside of the row support of the jointly sparse signals are punished by smaller weights and the other entries whose indices are more likely to be outside of the row support of the jointly sparse signals are punished by larger weights. Lastly, we can formulate the noise subspace weighted L_1-norm constraint minimization for sparse signal reconstruction

$$\min_{S_{SV}} \|\mathbf{G}\tilde{\mathbf{s}}\|_1 \qquad s.t. \quad \|\mathbf{Y}_{SV} - \mathbf{\Phi}\mathbf{S}_{SV}\|_f^2 \leq \beta^2 \tag{5}$$

The equation (5) can be calculated by SOC programming software packages such as CVX. DOA estimation is then obtained by plotting $\tilde{\mathbf{s}}$, solved from (5).

The procedure of the proposed method is concluded as follows

(1) Collect received data $\mathbf{Y}$ and construct the overcomplete basis matrix $\mathbf{\Phi}$.

(2) Compute the SVD of $\mathbf{Y}, \mathbf{S}, \mathbf{N}$ and obtain $\mathbf{Y_{SV}}, \mathbf{S_{SV}}, \mathbf{N_{SV}}$, respectively.

(3) Acquire the regularization parameter β according to [15].

(4) Obtain the weighted matrix by (4).

(5) Estimate DOA by calculating (5).

4. SIMULATION

In this section, we will provide many simulations to testify the performance of the presented method. In the following simulations, we employ a uniform linear array (ULA) of $\mathbf{M} = 8$ sensors whose separation distances are half a wavelength.

4.1. DOA Estimation for Uncorrelated Signals

Consider four uncorrelated equal power signals that arrive from [$-30°$, $-25°$, $30°$, $87°$] impinging on the array. The direction grid is set to have 181 points sampled from $-90°$ to $90°$ with $1°$ intervals, and the number of snapshots is 64, the signal-to-noise-ratio (SNR) is 5 dB. Figure 1 shows the spatial spectra of MUSIC, L_1-SVD and proposed method. From Figure 1, we can find that when the angle spacing is large, all the algorithms can resolve the signal which is from the direction of $30°$. When the signal is near the edges of the angle sector [$-90°$, $90°$], for example $87°$, MUSIC can not acquire DOA estimation. When the separation distance between the signals is small, for example $5°$, MUSIC can not distinguish the two close signals, and L_1-SVD algorithm not only can not obtain the true signal localization accurately, but also has serious spurious peaks. The main reason is that: on one hand, we usually substitute L_1-norm constraint for L_0-norm constraint in the recovery processing, because the L_0-norm constraint optimization is an NP-hard problem; on the other hand, the problem we want to solve is not absolutely sparse when there exists noise in actual application; in addition, the coherence of the

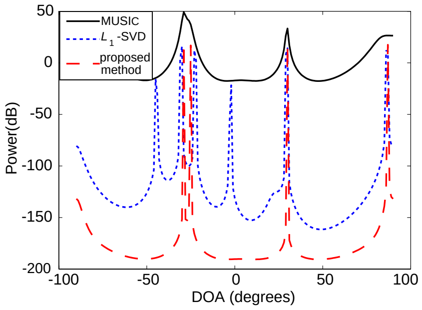

Figure 1. Spatial spectra of MUSIC, L_1-SVD and proposed method for uncorrelated signals.

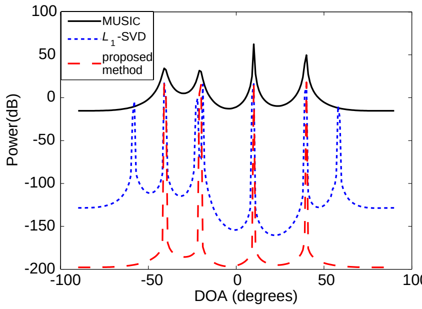

Figure 2. Spatial spectra of MUSIC, L_1-SVD and proposed method for coherent signals.

columns is usually so high in the overcomplete basis matrix that it is hard to satisfy the restricted isometry property (RIP), so there exists serious spurious peaks; while the proposed method not only has higher resolution and more sharp peaks, but also can suppress spurious peaks because of the weighted operation.

4.2. DOA Estimation for Correlated Signals

Suppose that there are four equal-power signals that arrive from [$-40°$, $-20°$, $10°$, $40°$]. The first signal is the same as the second one, and the third signal is the same as the fourth one. The second and third signals are uncorrelated. The SNR is set to 0 dB, and the number of snapshots is 64. In order to estimate correlated signals, a spatial smoothing [22] preprocessing scheme should be added to the proposed method. Then the noise subspace can be acquired to construct the weighted vector. The forward/backward spatial smoothing (see reference [22]) is applied to MUSIC to decorrelate the coherent signals, using a 6-element smoothing subarray. Figure 2 shows that the proposed method has higher resolution than that of MUSIC and no spurious peak compared with L_1-SVD.

4.3. Sensitivity of MUSIC and Proposed Method to the Assumed Number of Signals

The presented algorithm has a crucial advantage that it is not very sensitive to the incorrect determination of the number of signals. An illustration of this statement is given in Figure 3 and Figure 4. The actual number of signals is $L = 4$, which arrive from [$-30°$,

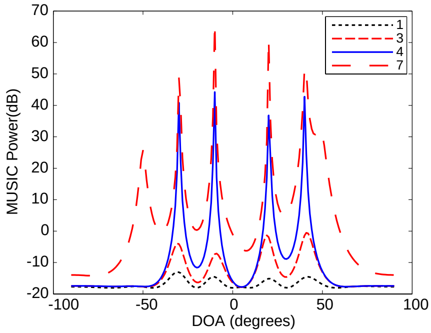

Figure 3. Sensitivity of MUSIC to the assumed number of signals. The correct number is 4.

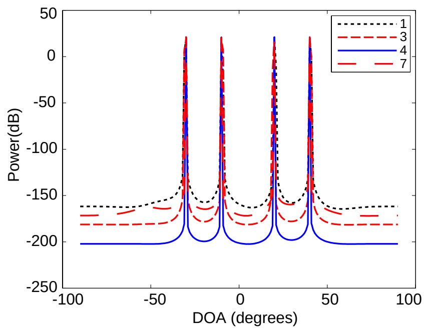

Figure 4. Sensitivity of proposed method to the assumed number of signals. The correct number is 4.

$-10°$, $20°$, $40°$] impinging on the array, and the SNR is 5 dB; the number of snapshots is 128. In Figure 3, we plot the spatial spectra acquired by MUSIC when we change the assumed number of signals. Underestimating the number of signals results in a strong deterioration of the quality of the spatial spectra, including widening and possible disappearance of some of the peaks. A large overestimate of the number of signals leads to the appearance of spurious peaks due to noise. In Figure 4, we plot the spatial spectra acquired using the proposed approach for the same assumed numbers of signals, and the alteration in the spatial spectra is very small. The importance of the low sensitivity of the proposed method to the assumed number of signals is twofold. First, the number of signals is usually unknown, and low sensitivity provides robustness against mistakes in estimating the number of signals. Second, even if the number of signals is known, low sensitivity may allow one to reduce the computational complexity of the presented method by taking a smaller number of singular vectors. Our formulation uses information about the number of signals L, but we empirically observe that incorrect determination of the number of signals in our approach has no catastrophic consequences (such as complete disappearance of some of the signals as may happen with MUSIC). The main reason is that the proposed method is not relying on the structural assumptions of the orthogonality of the signal and noise subspaces, but depending on sparse recovery theory for obtaining DOA estimation in signal recovery processing. Though the weighted vector in this framework is obtained by making use of the orthogonality between noise subspace and signal subspace, it is only used for enhancing sparsity, suppressing spurious peak, obtaining more accurate DOA estimation.

4.4. RMSE Curves of All the Algorithms Versus SNR

Figure 5 shows a comparison of root-mean-square-error (RMSE) of DOA estimation of the aforementioned methods versus SNR. In this simulation, we consider three uncorrelated sources at 20°, −10° and −45°. The number of snapshot is taken as $T = 128$, and all the results are averaged over 100 Monte Carlo runs for each SNR. The RMSE of DOA estimation is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{100L}\sum_{n=1}^{100}\sum_{l=1}^{L}\left(\hat{\theta}_l(n) - \theta_l\right)^2}$$

where $\hat{\theta}_l(n)$ is the estimation of θ_l for the nth Monte Carlo trial, and L is the number of signals. From Figure 5, we can see that the proposed method has better performance than MUSIC and L_1-SVD algorithm. Especially when SNR is low, the superiority is more apparent.

4.5. RMSE Curves of All the Algorithms Versus Angular Separation

A comparison of RMSE of DOA estimation for MUSIC, L_1-SVD and the proposed method versus angular separation is shown in Figure 6. Assume that two uncorrelated signals impinge on the array from $\theta_1 = -40°$ and $\theta_2 = -40° + \Delta\theta$, respectively, where $\Delta\theta$ is varied from 2° to 10° in 1° steps. The SNR is set to 5 dB, and the snapshot is 256.

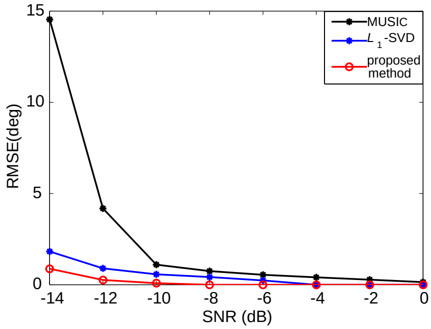

Figure 5. RMSE curves of MUSIC, L_1-SVD and proposed method versus SNR.

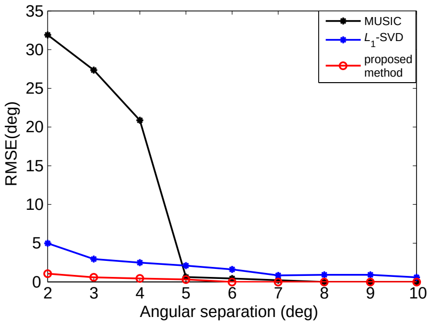

Figure 6. RMSE curves of MUSIC, L_1-SVD and proposed method versus angular separation.

Table 1. Elapsed CPU time of different algorithms with different signals.

Algorithm	Two Signals	Three Signals
MUSIC	0.1242s	0.1295s
L_1-SVD	2.2857s	2.3372s
Proposed Method	2.2971s	2.3505s

The RMSE versus angular separation is obtained via 100 independent Monte Carlo trials for each angle spacing in Figure 6. Under the test condition, the bias curves in Figure 6 show that the proposed algorithm tends to become unbiased when $\Delta\theta$ is greater than about $6°$, that the unbiased angle spacing is about $8°$ for MUSIC, and that L_1-SVD cannot become unbiased even the angle spacing is about $10°$. In addition, the proposed method and L_1-SVD has higher resolution than MUSIC when the angle spacing is less than $5°$. In other words, the proposed method shows better performance than the other two approaches in angular resolution.

4.6. Execution CPU Time

In this part, TIC and TOC are used to count the execution CPU time in Matlab. Then we chose ASUS K42JR as a convenient platform, which has a modest CPU (2.27 GHz Intel Core i3-350M) and a moderate memory space (2 GB RAM) for data processing. Assume that the number of snapshots is 64 and SNR 5 dB. We compute elapsed CPU time with 1000 repeating runs for MUSIC, L_1-SVD and the proposed method in Table 1. From Table 1, the computational complexity of the proposed method is higher than MUSIC and the same as L_1-SVD. However, the advantages of the proposed method outweigh the cost of additional computation: the proposed method has higher resolution than MUSIC and L_1-SVD, can suppress spurious peaks to obtain accurate DOA estimation, and is not very sensitive to the correct determination of the number of signals.

5. CONCLUSION

In this paper, an improved L_1-SVD algorithm based on noise subspace is developed for DOA estimation, in which the weighted vector is obtained by utilizing the orthogonality between noise subspace and signal subspace spanned the array manifold matrix. The proposed algorithm penalizes the nonzero entries whose indices correspond to the row support of the jointly sparse signals by smaller weights and the other entries whose indices are more likely to be outside of the row support of the jointly sparse signals by larger weights, and therefore it can encourage sparsity at the true signal locations. Through the above simulations, it can be demonstrated that the proposed method not only has supper-resolution in DOA estimation, but also can effectively suppress spurious peak. Furthermore, it is not very sensitive to the incorrect determination of the number of signals.

ACKNOWLEDGMENT

This work has been supported by the Fundamental Research Funds for the Central Universities under Grant No. N100423001, the National Natural Science Foundation of China under Grant Nos. 60874108 and 60904035, Science and Technology Support planning project of Hebei Province, China, under Grant No. 11213502D, the China Postdoctoral Science Foundation under Grant No. 20100471369, the Natural Science Foundation of Liaoning Province, China, under Grant No. 20102064, and Directive Plan of Science Research from the Bureau of Education of Hebei Province, China, under Grant No. Z2009105.

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