

## Coupled Line Rat-Race Coupler with Wide Adjustable Power Dividing Ratio and Uncrossed Input/Output Ports

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**Abstract**—A novel rat-race coupler with wide adjustable range of power-dividing ratio and uncrossed input/output ports is presented by using coupling adjustable trans-directional (TRD) coupled lines, parallel coupled lines and a  $180^\circ$  phase shifting line. Wide adjustable range of power-dividing ratios is accomplished by varying the coupling of the TRD coupled lines. Moreover, with the combination of the TRD coupled lines and parallel coupled lines, the input and output ports of the rat-race coupler are uncrossed. The structure of the proposed rat-race coupler is analyzed, and the design equations are derived. As an example to validate the feature of the proposed rat-race coupler, a prototype operating at 1.6 GHz is devised, fabricated and measured. The measured results show that the designed coupler has a wide adjustable range ( $-7 \sim 15$  dB) of power dividing ratio with a controlled voltage range of 3.3 to 13.5 V.

### 1. INTRODUCTION

Rat-race couplers, which are essential in microwave circuits, perform important functions of in-phase and out-of-phase power splitting while maintaining perfectly matched ports [1]. Since the isolation between the input ports may be independent of the output termination impedances, the rat-race couplers are extensively used in balanced mixers [2], multipliers, power splitting/combining networks, and antenna feed networks [3].

With the advent of multi-standard frontends in modern communication systems, directional couplers are required to be frequency reconfigurable [4–7], coupling reconfigurable [8–12], and phase reconfigurable for smart antenna and software defined radio applications. In [13], a rat-race coupler with  $-9 \sim 6$  dB power dividing ratio is proposed. It offers excellent performances in both port isolation and return loss for all dividing ratios, as well as simple circuit implementation. Unfortunately, due to its large variation in insertion loss over frequency, the available bandwidth is less than 10%, which is impractically small. To broaden the bandwidth of the rat-race coupler, frequency compensating networks are adopted, and the fractional bandwidth is broadened to 40% [14]. However, since the rat-race couplers in [13] and [14] are based on the same design theory, the adjustable range of power-dividing ratios is small and need to be improved. Recently, a rat-race coupler with  $-10 \sim 13$  dB adjustable power dividing ratio and 40 ~ 70% size reduction is proposed [15]. But the center frequency of measured results shifts about 7.5% corresponding to the simulated ones.

In the paper, the design of a coupled line rat-race coupler with wide adjustable range of power dividing ratio is introduced. Moreover, with the combination of the proposed TRD coupled lines and parallel coupled lines, the rat-race coupler has the feature of uncrossed input/output ports [16–18], which is a new direction of improvements focusing on the rat-race coupler. Such a property can be beneficial when laying out complex microwave circuit.

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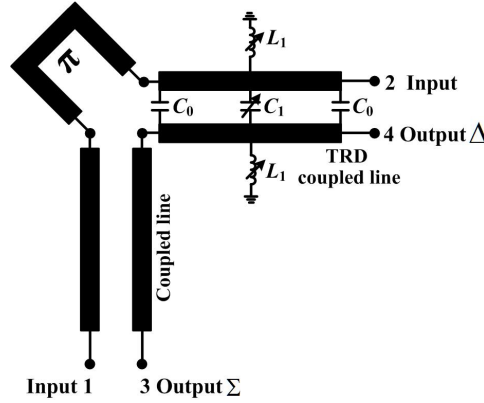
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## 2. CIRCUIT STRUCTURE AND DESIGN THEORY

Figure 1 shows the schematic diagram of the proposed rat-race coupler. It is composed of a pair of adjustable TRD coupled lines, a pair of parallel coupled lines, and a  $180^\circ$  phase shifting line. One of the parallel coupled lines is connected with the TRD coupled line via a  $180^\circ$  phase-shifting line, the other parallel coupled line is directly connected with the TRD coupled line. If the parallel coupled lines are assumed to have coupling factor of  $C_{L1}$  and transmission coefficient of  $T_{L1}$ , and if the coupling adjustable TRD coupled lines have a coupling factor of  $C_{T1}$  and a transmission coefficient of  $T_{T1}$ , the  $S$ -parameters of the rat-race coupler can be expressed in a matrix form [18], as shown below.

$$S_{rat-race} = \begin{bmatrix} 0 & 0 & \frac{C_{L1} + C_{T1}}{1 + C_{L1}C_{T1}} & \frac{-T_{L1}T_{T1}}{1 + C_{L1}C_{T1}} \\ 0 & 0 & \frac{T_{L1}T_{T1}}{1 + C_{L1}C_{T1}} & \frac{C_{L1} + C_{T1}}{1 + C_{L1}C_{T1}} \\ \frac{C_{L1} + C_{T1}}{1 + C_{L1}C_{T1}} & \frac{T_{L1}T_{T1}}{1 + C_{L1}C_{T1}} & 0 & 0 \\ \frac{-T_{L1}T_{T1}}{1 + C_{L1}C_{T1}} & \frac{C_{L1} + C_{T1}}{1 + C_{L1}C_{T1}} & 0 & 0 \end{bmatrix} \quad (1)$$

From Equation (1), it is observed that if the coupling factor of the parallel coupled lines is assigned to a fixed value, the power dividing ratio of the rat-race coupler is directly related to the coupling of the TRD coupled lines. In the following, the proposed coupling adjustable TRD coupled lines are analyzed, and the design equations will be derived. It is observed from Figure 1 that the proposed TRD coupled lines consist of capacitor loaded coupled lines and shunt inductors. The even- and odd-mode electrical lengths of the coupled line are defined as  $\theta_e$  and  $\theta_o$ , and the even- and odd-mode characteristic impedances of the coupled line are defined as  $Z_{ce}$  and  $Z_{co}$ , respectively. The two fixed capacitors  $C_0$  are placed between the two ends of the coupled line, separately. The inductors denoted as  $L_1$  are used as tunable elements to make the TRD coupled lines work with adjustable coupling. A varactor diode  $C_1$  is shunted between the middle of the TRD coupled line to keep perfect return loss and isolation when the coupling is varied.

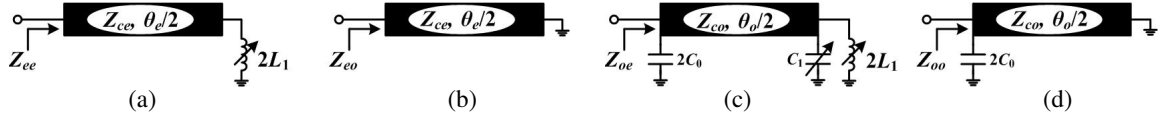


**Figure 1.** The schematic of the proposed rat-race coupler.

The even-odd mode analysis is employed to derive the closed-form equations of the coupling adjustable TRD coupled lines. Figure 2 gives the even- and odd-mode decomposition circuit. The input impedances of each mode are derived as follows:

$$Z_{ee} = -jZ_{ce} \left( 4\pi f L_1 + Z_{ce} \tan \left( \frac{\theta_e}{2} \right) \right) / \left( 4\pi f L_1 \tan \left( \frac{\theta_e}{2} \right) - Z_{ce} \right) \quad (2a)$$

$$Z_{eo} = jZ_{ce} \tan \left( \frac{\theta_e}{2} \right) \quad (2b)$$



**Figure 2.** Even-odd mode circuits of the proposed adjustable TRD coupler under different excitations. (a) Even-even mode. (b) Even-odd mode. (c) Odd-even mode. (d) Odd-odd mode.

$$Z_{oe} = 1/j4\pi f C_0 + j \left( 4\pi f L_1 \tan \left( \frac{\theta_o}{2} \right) - Z_{co} \left( 1 - 8(\pi f)^2 L_1 C_1 \right) \right) / \left\{ Z_{co} \left( 4\pi f L_1 + Z_{co} \tan \left( \frac{\theta_o}{2} \right) \left( 1 - 8(\pi f)^2 L_1 C_1 \right) \right) \right\} \quad (2c)$$

$$Z_{oo} = -j / \left[ 4\pi f C_0 - \cot \left( \frac{\theta_o}{2} \right) / Z_{co} \right] \quad (2d)$$

In the above equations,  $f$  is the center frequency of the TRD coupled lines. The  $S$ -parameters of the TRD coupled line can be determined by using the four port impedances as shown in Equation (2) [18]. To link the dual-mode input impedances and  $S$ -parameters, the mode of each parameter should be matched to single even-odd mode. After solving single-mode  $S$ -parameters and  $Z$ -parameters, the dual-mode impedance condition for matching and isolation can be derived in

$$Z_0^2 = Z_{ee} Z_{oe} \quad (3a)$$

$$Z_0^2 = Z_{eo} Z_{oo} \quad (3b)$$

Then, the coupling coefficient of the coupler can be expressed as

$$k = |S_{31}| = \left| \frac{Z_{ee} Z_{eo} - Z_0^2}{Z_{ee} Z_{eo} + Z_0^2 + Z_0 (Z_{ee} + Z_{eo})} \right| \quad (4)$$

From Equations (4), (2a) and (2b), it is observed that  $k$  increases with the decrease of inductor  $L_1$  if the value of  $Z_{ce}$  is fixed. In Equation (3b) (with the help of Equations (2b) and (2d)), it can be found that the variable  $L_1$  is not in Equation (3b). Thus the changing of  $L_1$  has no influence on Equation (3b) if the values of  $Z_{ce}$ ,  $Z_{co}$ , and  $C_0$  are unchanged. However, since Equation (3a) includes  $L_1$ , we should find the relationship between  $C_1$  and  $L_1$  to keep perfect return loss and isolation. After solving Equation (3a), we can get

$$C_1 = \frac{1}{2\omega^2 L_1} - \frac{4\pi f L_1 \left( \tan \left( \frac{\theta_o}{2} \right) - \frac{Z_{co}}{Z_0^2} \{ Z_{ce} [4\pi f L_1 + Z_{ce} \tan \left( \frac{\theta_e}{2} \right)] / [Z_{ce} - 4\pi f L_1 \tan \left( \frac{\theta_e}{2} \right)] - 4\pi f C_0 Z_0^2 \} \right)}{2\omega^2 L_1 \left( Z_{co} + \frac{Z_{co}}{Z_0^2} \tan \left( \frac{\theta_o}{2} \right) \{ Z_{ce} [4\pi f L_1 + Z_{ce} \tan \left( \frac{\theta_e}{2} \right)] / [Z_{ce} - 4\pi f L_1 \tan \left( \frac{\theta_e}{2} \right)] - 4\pi f C_0 Z_0^2 \} \right)} \quad (5)$$

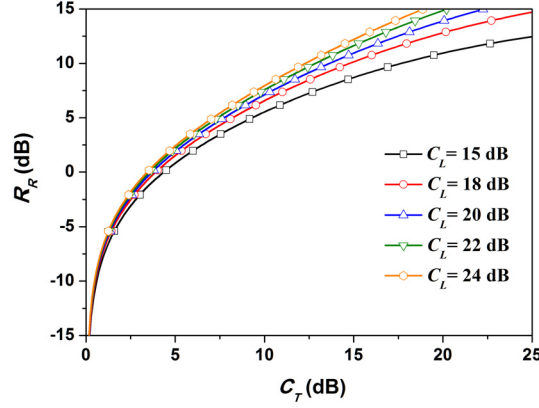
### 3. PARAMETERS ANALYSIS

To verify the proposed structure, a rat-race coupler with an operating frequency of 1.6 GHz is designed. According to Equation (1), the power dividing ratio of the rat-race coupler ( $R_R$ ) can be expressed as follows:

$$R_R = \frac{S_{41}}{S_{31}} = \frac{-T_{L1} T_{T1}}{C_{L1} + C_{T1}} \quad (6)$$

Figure 3 illustrates the curves of the power dividing ratio of the rat-race coupler ( $R_R$ ) versus the coupling of the TRD coupled lines ( $C_T$ ) with different coupling values of the parallel coupled lines ( $C_L$ ). It is observed that the power dividing ratio of the proposed rat-race coupler can be adjusted in the range of  $-15 \sim 15$  dB. Considering the fabrication constraints, the coupling of the parallel coupled lines is chosen as 20 dB. Then, the required  $C_T$  value ( $0.24 \sim 22.5$  dB) can be obtained from Figure 3.

For 20 dB coupling, the even and odd-mode impedances of the parallel coupled line coupler are  $55.3 \Omega$  and  $45.2 \Omega$ . In the following, the parameters of the TRD coupler will be obtained based on the



**Figure 3.** The curves of  $R_R$  versus  $C_T$  with different  $C_L$  values.

equations in Section 2. For simplification, the even-mode electrical length ( $\theta_e$ ) is assumed equal to the odd-mode electrical length ( $\theta_o$ ). In the first step, assume that the initial value of  $L_1$  is positive infinity, and the initial coupling of the TRD coupler is 22.5 dB, then the value of  $Z_{ce}$  is calculated to be  $54.8 \Omega$  according to Equation (4) with  $\theta_e = \theta_o = 60^\circ$ . In the next step, the initial values of  $Z_{co}$ ,  $C_0$  and  $C_1$  should be found. According to Equation (3), the relationship between  $Z_{co}$  and  $C_1$  is listed below while  $L_1$  is positive infinity.

$$Z_{co} = \frac{Z_{ce}(\tan(\frac{\theta_e}{2}) + \cot(\frac{\theta_e}{2})) + \sqrt{Z_{ce}^2 (\tan(\frac{\theta_e}{2}) + \cot(\frac{\theta_e}{2}))^2 + 8\pi f C_1 Z_{ce} Z_0^2 (\tan(\frac{\theta_o}{2})^2 + 1) (\tan(\frac{\theta_o}{2}) + \cot(\frac{\theta_o}{2}))}}{4\pi f C_1 Z_{ce} (\tan(\frac{\theta_o}{2})^2 + 1)} \quad (7)$$

With the above equations, the suitable values of  $Z_{co}$  can be chosen, and the corresponding value of  $C_1$  is obtained. Here,  $Z_{co} = 48.9 \Omega$ ,  $C_1 = 6.8 \text{ pF}$ . The value of  $C_0$  ( $C_0 = 2.39 \text{ pF}$ ) can be calculated using the following equation which is derived from Equation (3b).

$$C_0 = \cot\left(\frac{\theta_o}{2}\right) / 4\pi f \left(1/Z_{co} + Z_{ce} \tan\left(\frac{\theta_e}{2}\right)^2 / Z_0^2\right) \quad (8)$$

In the last step, decrease the value of  $L_1$  to change the coupling of the TRD coupled lines and

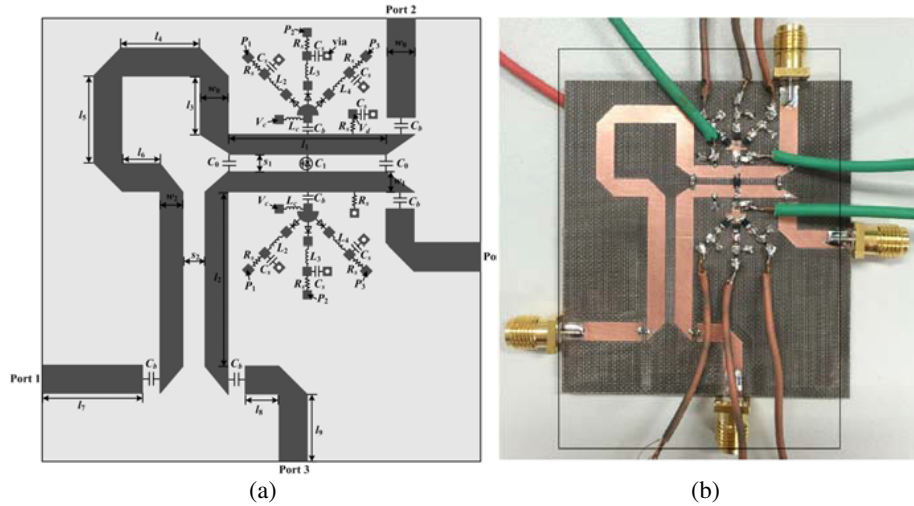
**Table 1.** Detailed parameters of  $L_1$ ,  $C_1$ , the coupling of the proposed coupled line, and the corresponding power dividing ratio of the rat-race coupler.

$L_1$ (nH)	$C_1$ (pF)	Coupling of the TRD coupled line (dB)	Power dividing ratio of the rat-race coupler (dB)
infinite	6.8	22.5	15.0
22	6.3	14.1	10.7
15	6.1	12.2	9.3
10	5.7	9.6	7.3
5.6	4.9	6.0	4.1
3.6	3.6	3.6	0.56
2.4	2.5	2.4	-1.6
1.8	1.2	1.5	-4.8
1.2	0.9	0.9	-7.3

recalculate the corresponding values of  $C_1$  with Equation (5) while the values of  $Z_{ce}$ ,  $Z_{co}$  and  $C_0$  are fixed. Thus, the rat-race coupler with adjustable power dividing ratio and uncrossed input/output ports is accomplished. Table 1 lists the detailed parameters of  $L_1$ ,  $C_1$ , the coupling of the TRD coupled line, and the corresponding power dividing ratio of the rat-race coupler. It is observed that the realizable power dividing ratio range of the rat-race coupler is  $-7.3 \sim 15.0$  dB.

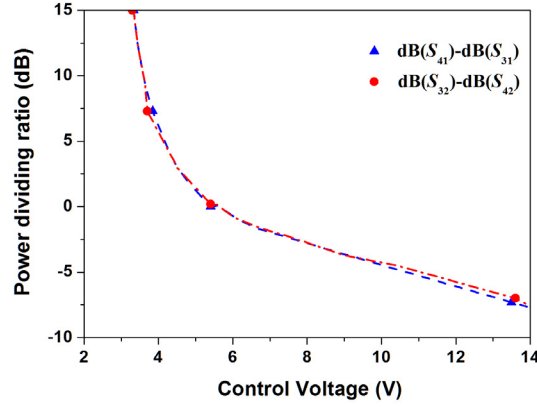
#### 4. EXPERIMENTAL RESULTS

To realize the proposed structure, a prototype has been optimized using advanced design system (ADS) and fabricated on an F4B substrate with a dielectric constant of 3.5,  $\tan \delta$  of 0.003, and thickness of 1.5 mm. Figure 4(a) shows the layout of the proposed TRD coupler. The optimized dimensions are:  $w_0 = 3.38$  mm,  $w_1 = 2.3$  mm,  $s_1 = 1.3$  mm,  $l_1 = 18$  mm,  $w_2 = 3.33$  mm,  $s_2 = 1.88$  mm,  $l_2 = 27$  mm,  $l_3 = 7$  mm,  $l_4 = 11.7$  mm,  $l_5 = 10.5$  mm,  $l_6 = 6.5$  mm,  $l_7 = 15$  mm,  $l_8 = 5$  mm, and  $l_9 = 10$  mm. The required  $C_0$  value was implemented by a Murata 2.4 pF capacitor which gives the closest result to simulation. The varactor diode (SMV1265) was utilized to replace  $C_1$ . The adjustable coupling of the TRD coupled lines is realized by connecting different values of inductors through RF switch diodes (SMP1340). In the paper, three inductors are used ( $L_2 = 10$  nH,  $L_3 = 3.6$  nH, and  $L_4 = 1.2$  nH) which means that four power dividing ratios (15 dB, 7.3 dB, 0.56 dB and  $-7.3$  dB) can be obtained. Simple  $RC$  networks ( $R_s = 10$  k $\Omega$ ,  $C_s = 1$   $\mu$ F) were adopted to bias the RF switch diodes and varactor diode. And they are grounded through vias. Additional biasing components such as dc blocking capacitors  $C_b$ , at all RF ports, and RF choke inductors  $L_c$  are used in order to establish the proper biasing conditions. The points  $V_c$ ,  $V_d$ ,  $P_1$ ,  $P_2$  and  $P_3$  are used to weld wires. When  $V_c$  and  $P_1$  are connected to the anode and cathode of the dc power supply, respectively, the path in which  $L_2 = 15$  nH is switched on. At the same time, adjusting the voltage on the point  $V_d$ , proper power dividing ratios will be realized. By this analogy, four different configurations can be achieved. A photograph of the proposed rat-race coupler is illustrated in Figure 4(b). The dimensions of the fabricated prototype are  $54 \text{ mm} \times 60 \text{ mm}$  ( $0.57\lambda_g \times 0.63\lambda_g$ ), which includes the rat-race coupler, dc blocking capacitors, biasing circuits and input/output 50  $\Omega$  transmission lines.

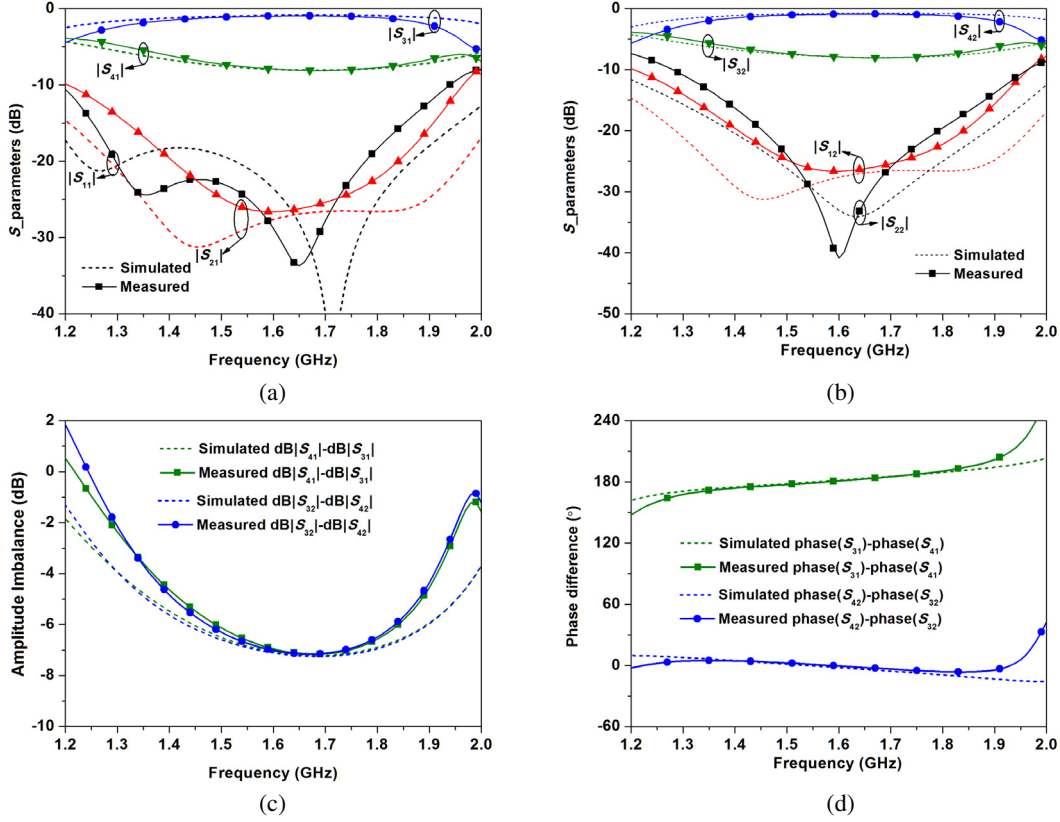


**Figure 4.** The proposed rat-race coupler. (a) Layout. (b) Photograph.

To validate the design, scattering parameter measurements were performed by using a 4-port network analyzer (Agilent N5230A) over the frequency range from 1.2 GHz to 2.0 GHz. By measurement (Figure 5), the power dividing ratio was found to vary from about  $-7$  dB to 15 dB with a control voltage  $V_d$  ranging from 13.5 to 3.3 V. Figure 6 gives the simulated and measured results of the proposed rat-race coupler at a control voltage of  $V_d = 13.5$  V. At the designed center frequency of 1.6 GHz, the measured power divisions  $|S_{41}|$ ,  $|S_{31}|$ ,  $|S_{32}|$ , and  $|S_{42}|$  are  $-7.97$  dB,  $-0.97$  dB,  $-7.91$  dB, and  $-0.9$  dB, respectively.



**Figure 5.** Measured power dividing ratio versus control voltage.



**Figure 6.** Simulated and measured results of the proposed rat-race coupler with  $V_d = 13.5$  V. (a)  $S$ -parameters of port1 excited. (b)  $S$ -parameters of port2 excited. (c) Amplitude imbalance. (d) Output phase differences.

Insertion losses of  $6.8 \pm 1.2$  dB ( $S_{41}$ ) and  $6.9 \pm 1.1$  dB ( $S_{32}$ ) were obtained over a fractional bandwidth of 30%. Under the criterion of return loss  $> 10$  dB ( $|S_{11}| = |S_{33}|$  and  $|S_{22}| = |S_{44}|$ ), the measured relative bandwidth is 42.5% (1.28–1.96 GHz), and the measured bandwidth for isolation better than 15 dB is from 1.32 GHz to 1.91 GHz (36.9%). The measured bandwidth for amplitude imbalance  $< 1$  dB is 360 MHz (22.5%). The measured output ports phase differences for input at port 1 is  $180^\circ \pm 10^\circ$  from 1.32 to 1.79 GHz (29.4%), and that for input at port 2 is  $0^\circ \pm 10^\circ$  from 1.2 to 1.96 GHz (47.5%). Figure 7 shows the simulated and measured results at a control voltage of  $V_d = 5.4$  V. The measured

power division  $|S_{41}|$ ,  $|S_{31}|$ ,  $|S_{32}|$ , and  $|S_{42}|$  are  $-3.43$  dB,  $-3.39$  dB,  $-3.22$  dB, and  $-3.34$  dB, respectively. The measured insertion loss is  $2.75 \pm 0.85$  dB ( $S_{41}$ ,  $S_{32}$ ) over the 30% fractional bandwidth. Under the criterion of return loss  $> 10$  dB ( $|S_{11}| = |S_{33}|$  and  $|S_{22}| = |S_{44}|$ ), the measured relative bandwidth is 38.7% (1.38–2.00 GHz), and the measured bandwidth for isolation better than 15 dB is from 1.5 GHz to 1.92 GHz (26.3%). In the range of 1.49–1.77 GHz, the measured amplitude imbalance is less than 1 dB. From 1.47 to 1.95 GHz (30%), the proposed prototype exhibits an in-phase phase difference of  $0^\circ \pm 10^\circ$  and an anti-phase phase difference of  $180^\circ \pm 10^\circ$ . Figure 8 illustrates the simulated and measured performances at a control voltage of  $V_d = 3.3$  V. The measured power divisions  $|S_{41}|$ ,  $|S_{31}|$ ,  $|S_{32}|$ , and  $|S_{42}|$  are  $-0.59$  dB,  $-14.69$  dB,  $-0.54$  dB, and  $-14.56$  dB, respectively. The measured 10-dB return loss bandwidth is from 1.26 to 1.8 GHz (33.8%), and the 15-dB isolation bandwidth is from 1.53 to 1.69 GHz (10%). From 1.53 GHz to 1.70 GHz (10.6 %), the measured amplitude imbalance is  $15 \pm 1$  dB, and the measured insertion losses are  $0.1 \pm 0.4$  dB ( $S_{41}$ ) and  $0.95 \pm 0.35$  dB ( $S_{32}$ ). The measured output ports phase difference for input at port 1 is  $180^\circ \pm 10^\circ$  from 1.50 to 1.67 GHz (10.6%, and that for input at port 2 is  $0^\circ \pm 10^\circ$  from 1.53 to 1.64 GHz (6.9%). The discrepancies between the simulated and measured results in Figures 6–8 are mainly due to the inaccurate values of the capacitors and the uncertain parasitic effect of the applied lumped elements and varactors.

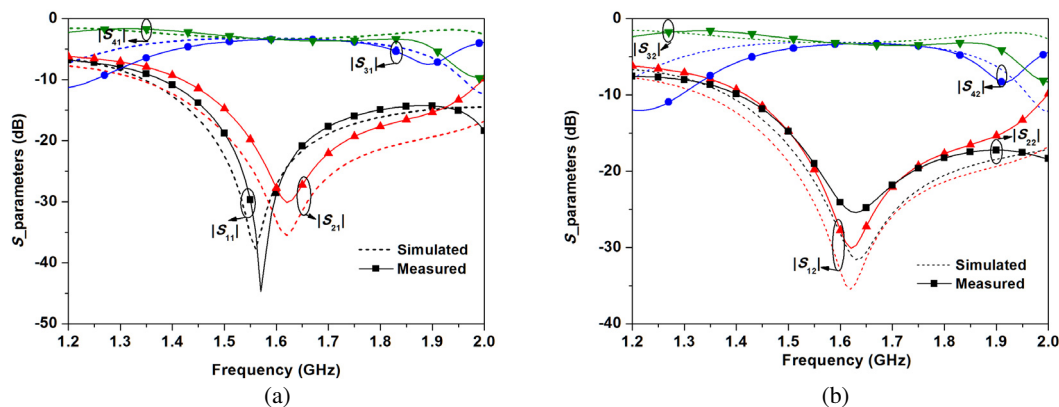
**Table 2.** Comparisons between the proposed rat-race coupler and the existing designs.

Reference	[13]	[14]	[15]	<b>This work</b>
Input/output port distribution	Crossed	Crossed	Crossed	<b>Uncrossed</b>
Power dividing ratio (dB)	$-9 \sim 6$	$-10 \sim 10$	$-10 \sim 13$	<b><math>-7 \sim 15</math></b>
Control voltage (V)	$2.5 \sim 12.5$	$1.0 \sim 15.0$	$3.0 \sim 12.0$	<b><math>3.3 \sim 13.5</math></b>
Insertion loss (dB) <sup>ab</sup>	$2.2 \sim 4.5$	$3.0 \sim 4.1$	$3.1 \sim 5.2$	<b><math>1.9 \sim 3.6</math></b>
Relative bandwidth (%) <sup>b</sup>	Return loss $> 10$ dB	20	50	<b>33.8</b>
	Isolation $> 15$ dB	–	–	<b>26.3</b>
	In-phase ( $0^\circ \pm 10^\circ$ )	–	–	<b>36.3</b>
	Anti-phase ( $180^\circ \pm 10^\circ$ )	–	–	<b>30.0</b>

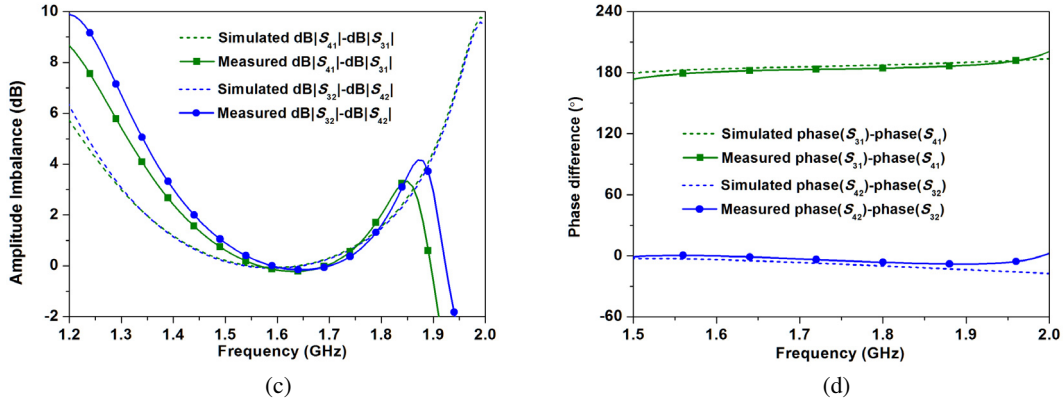
<sup>a</sup> Over a fractional bandwidth of 30%.

<sup>b</sup> For 0 dB power dividing ratio.

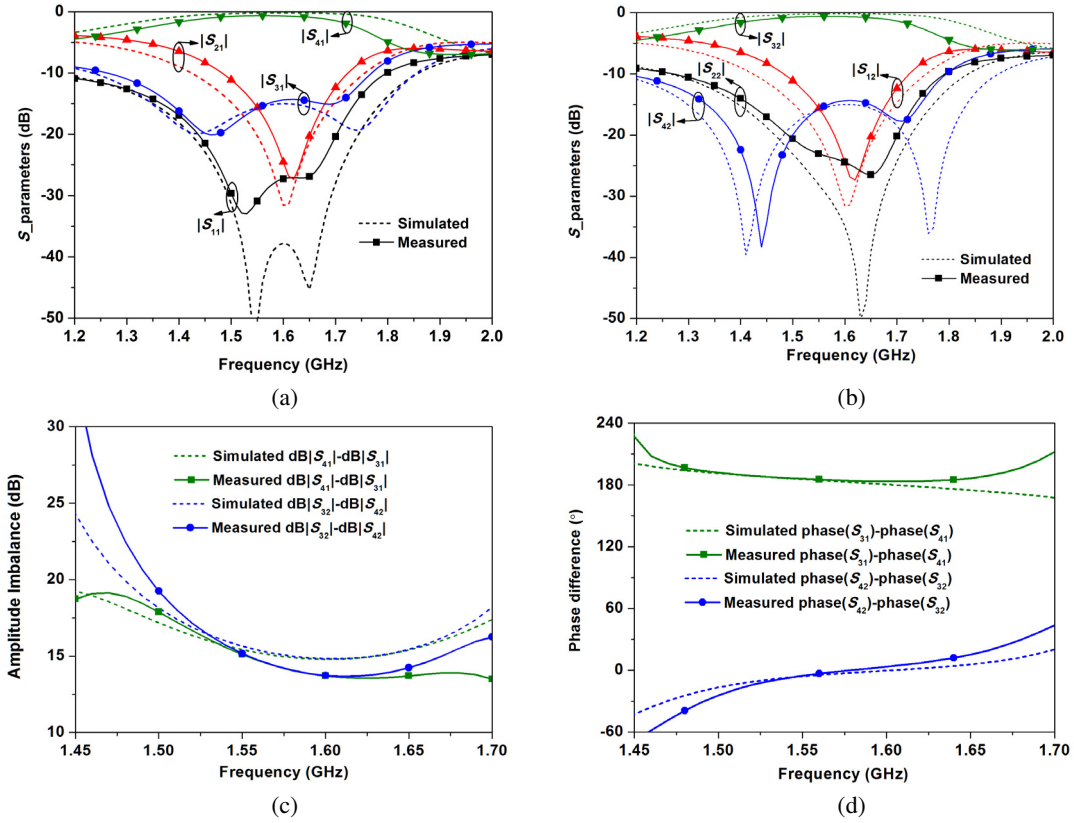
Table 2 illustrates the comparisons of the measured results between the proposed rat-race coupler and the existing designs [13–15]. The design concept of the existing rat-race couplers [13–15] is similar, which comes from the design theory of [13]. And the input and output ports of the existing couplers are crossed, same as the traditional rat-race coupler. This paper proposes a new structure for designing the rat-race coupler with adjustable power dividing ratio. By combining the TRD coupled lines and parallel coupled lines, the input and output ports are uncrossed. Such a property can be beneficial when laying out complex microwave circuit. From Table 2 it is observed that the range of the power dividing







**Figure 7.** Simulated and measured results of the proposed rat-race coupler with  $V_d = 5.4$  V. (a)  $S$ -parameters of port1 excited. (b)  $S$ -parameters of port2 excited. (c) Amplitude imbalance. (d) Output phase differences.



**Figure 8.** Simulated and measured results of the proposed rat-race coupler with  $V_d = 3.3$  V. (a)  $S$ -parameters of port1 excited. (b)  $S$ -parameters of port2 excited. (c) Amplitude imbalance. (d) Output phase differences.

ratio is wider than the couplers in [13, 14] and similar to the coupler in [15]. However, the relative bandwidth of the designed coupler under the criterion of 10-dB return loss is wider than that in [13] and [15]. In addition, the proposed prototype exhibits less insertion loss than the other designs over a fractional bandwidth of 30%. Thus, in a comprehensive view, the proposed rat-race coupler shows better performances and is suitable for microwave applications.



## 5. CONCLUSION

In this paper, a rat-race coupler with adjustable power dividing ratio is presented. The coupler has the feature of uncrossed input and output ports, which is beneficial when laying out complex microwave circuit. To obtain wide range of power dividing ratio, the design parameters are analyzed, and design equations are derived. Using the proposed method, a microstrip rat-race coupler with a power dividing ratio of  $-7 \sim 15$  dB is implemented. The fabricated coupler has uncrossed input and output ports with wide bandwidth of 10-dB return loss, 15 dB-isolation,  $0^\circ \pm 10^\circ$  in-phase phase difference, and  $180^\circ \pm 10^\circ$  anti-phase phase difference.

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