Progress In Electromagnetics Research, PIER 35, 53–81, 2002

DYADIC GREEN'S FUNCTIONS IN MULTILAYERED STRATIFIED GYROELECTRIC CHIRAL MEDIA

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Abstract—To characterize electromagnetic waves in complex media has been an important topic because of its useful applications and scientific significance of its physical mechanism. Dyadic Green's functions, as a mathematical kernel or a dielectric medium response, relate directly the radiated electromagnetic fields and the source distribution. In terms of the vector wave functions in cylindrical coordinates, dyadic Green's functions in a unbounded and a planar, multilayered gyroelectric chiral media are formulated. By use of the scattering superposition principle and taking the multiple reflections into account, a general representation of the Green's dyadics is obtained. Furthermore, the scattering coefficients of the Green's dyadics are determined from the boundary conditions at each interface and are expressed in a greatly compact form of recurrence matrices. In the formulation of the Green's dyadics and their scattering coefficients, three cases are considered, i.e., the current source is impressed in (1) the first, (2) the intermediate, and (3) the last regions, respectively. Although the dyadic Green's functions for a unbounded gyroelectric chiral medium has been reported in the literature, some of the results are incorrect. As compared to the existing results, the current work basically contributes (1) a correct form of dyadic Green's function for a unbounded gyroelectric chiral medium, (2) the general representation of the dyadic Green's functions for a multi-layered gyroelectric chiral medium, and (3) a convincible and direct derivation of the irrotational Green's dyadic.

1 Introduction

2 General Formulation For Unbounded Gyroelectric Chiral Medium

1. INTRODUCTION

*D*yadic *G*reen's *f*unctions (DGFs) [1], as a mathematical kernel or a dielectric medium response, relate directly the radiated electromagnetic fields and the source distribution. Nowadays, the dyadic Green's function technique has been an important method employed [2] elsewhere for boundary value problems, such as in Method of Moments and Boundary Element Method. In formulating the DGFs, there are usually two approaches, one is the Fourier transform technique and the other is the vector wave function expansion technique. The former looks simpler and is efficient for the Cartesian coordinates [3]; but it may not be applicable in all the nine fundamental coordinate systems. The latter provides a systematic approach in electromagnetic theory for interpreting various electromagnetic representations [4]; most importantly, it is applicable in almost all the nine fundamental coordinate systems. Even in the planar structure to be considered in detail in this paper, the eigenfunction expansion technique can provide an explicit form of the dyadic Green's functions in cylindrical Bessel functions, so that it becomes easy and convenient when the source distribution is independent from the azimuth directions or when the far-zone fields are computed.

Looking backwards, we can easily find that the dyadic Green's functions in isotropic media have been well-documented by Tai [1], Collin [2], Chew [5], Cavalcante [6], Pathak [7], Pearson [8], Li *et al.* [9–16] using the vector wave functions. For anisotropic media, Kong [17, 18], Ali and Mahmoud [19], Lee and Kong [20–22], Krowne [23, 24], Monzon [25], Oldano [26], Habashy *et al.* [27], Kaklamani and Uzunogla [28], Ren [29], Weiglhofer and Lindell [30], Lindell [31], and Cheng and Ren [32] have derived various formulas of dyadic Green's functions using (1) the Fourier transform technique, (2) the method of angular spectrum expansion, and (3) the transmission matrix method. As for gyroelectric media, the DGFs and fields have also been formulated by Uzunoglu *et al.* [33], Barkeshli [34, 35], Weiglhofer [36], and Cheng [37, 38]. This paper will consider a more general case where (1) each layer can be a gyroelectric chiral medium, (2) an arbitrarily-multiple stratified medium is considered, and (3) either the transmitter or the receiver can be located in any region of the layered structure. Although some results for bianisotropic media are available nowadays, but are basically limited to unbounded media only. So they cannot be directly employed here.

As the eigenfunction expansion technique will be used in the formulation of dyadic Green's functions for a unbounded and a multi-layered gyroelectric chiral media, some ideas introduced in [14, 39] will be used in this paper and will be generalized in most cases. Although the result for a unbounded gyroelectric chiral medium has been published by Cheng [38], it is realized [40] that both the idea introduced there and the result obtained inside in [38] are not correct. Although Cheng [32, 41–45] developed the dyadic Green's functions for a bit more general class of media, some of the results published there are, however, in certain senses that (1) the irrotational DGF was not obtained, (2) there was a very obvious mistake of wrong eigenvalues obtained at the beginning in [41] so that subsequent problem may raise, and (3) all the work in [32, 41–45] started with the Fourier transform; and the Green's dyadics in vector wave function forms are indirectly obtained from the transformation in terms of the plane-wave expression to cylindrical-wave expansion or spherical-wave expansion. In this paper, we propose a direct formulation of the vector wave functions. Therefore, we intend to derive a completely new set of formulas for the representations of the dyadic Green's functions. Furthermore, a more generalized case, i.e., the planar-multilayered structure, will be considered in the formulation as well.

This paper is organized as follows. In Section 2, the eigenfunction expansion of the dyadic Greens' functions in cylindrical coordinates is proposed for the unbounded gyroelectric chiral medium. New formulas of the DGFs including the irrotational part are found using the direct expansion of vector wave functions and, at the mean time some mistakes occurring in the publications are pointed out. To include the effects of the multiple interfaces, the principle of scattering superposition is use in Section 3 to obtain the scattering dyadic Greens' functions. As expected, various wave modes, such as direct waves and multiple reflected waves (associated with the planar interfaces) that propagate in different wave numbers, are included in the formulation. In Section 4, By evaluating the scattering dyadics with , the scattering coefficients of the dyadic Green's functions are obtained using the boundary conditions on each planar interfaces and represented by a set of recurrence matrices. In the derivation of these coefficients, three cases, that is, sources located in the first, the intermediate and the last layers, are considered. The results for the multilayered gyroelectric chiral medium can be proved to be reducible to, but do not resemble, those of the multilayered isotropic medium. Throughout the paper, a time dependence $e^{-i\omega t}$ is always suppressed.

2. GENERAL FORMULATION FOR UNBOUNDED GYROELECTRIC CHIRAL MEDIUM

A homogeneous gyroelectric chiral medium can be characterized by a set of constitutive relations [38] for the time harmonic excitation,

$$\boldsymbol{D} = \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} + i\xi_c \boldsymbol{B}, \tag{1a}$$

$$\boldsymbol{H} = i\xi_c \boldsymbol{E} + \boldsymbol{B}/\mu, \tag{1b}$$

where

$$\overline{\boldsymbol{\epsilon}} = \begin{bmatrix} \epsilon & -ig & 0 \\ ig & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{2}$$

This relation has been widely used in the previously published work such as those in [33], [34, 35], [36], and [37, 38]. Experimentally, there might be a problem of fabricating some materials of the constitutive relations for generalized bianisotropy. However, it does not mean it may not be produced for ever. Also, theoretical physics sometimes goes ahead, relative to the experimental physics. In this work, we just concentrate on the previously used material for our discussion.

Substituting (1a) and (1b) into the source incorporated Maxwell's equations leads to

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} - 2\omega\mu\xi_c \boldsymbol{\nabla} \times \boldsymbol{E} - \omega^2 \mu \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} = i\omega\mu \boldsymbol{J}. \tag{3}$$

2.1. General Formulation of DGFs

The electric field can thus be expressed in terms of the DGF and electric source distribution as follows:

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega\mu \int_{V'} \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}')\, dV', \tag{4}$$

where V' denotes the volume occupied by the exciting current source. Again, substituting (4) into (3) leads to

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r}') - 2\omega\mu\xi_c \boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r}') - \omega^2 \mu \overline{\boldsymbol{\epsilon}} \cdot \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r}') = \overline{\boldsymbol{I}}\delta(\boldsymbol{r} - \boldsymbol{r}'), \tag{5}$$

where $\overline{\boldsymbol{I}}$ and $\delta(\boldsymbol{r} - \boldsymbol{r}')$ denotes the unit dyadic and Dirac δ function, respectively.

To formulate the dyadic Green's functions, we basically use the Ohm-Rayleigh method and the vector wave function expansion. For completeness and self-contained content of the discussion, we still follow the standard procedure given by Tai [1] in the first a few steps. However, the latter part of the discussion in this section can never be found in the existing work elsewhere.

According to the well-known Ohm-Rayleigh method, the source term in (5) can be expanded in terms of the solenoidal and non-solenoidal cylindrical vector wave functions in cylindrical coordinate system. Thus, we have

$$\begin{aligned} \overline{\boldsymbol{I}}\delta(\boldsymbol{r} - \boldsymbol{r}') &= \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_{n\lambda}(h) \boldsymbol{A}_{n\lambda}(h) \right. \\ &\quad \left. + \boldsymbol{N}_{n\lambda}(h) \boldsymbol{B}_n(h, \lambda) + \boldsymbol{L}_n(h, \lambda) \boldsymbol{C}_n(h, \lambda) \right], \end{aligned} \tag{6}$$

where $\boldsymbol{M}_n(h, \lambda)$ & $\boldsymbol{N}_n(h, \lambda)$ are the solenoidal, and $\boldsymbol{L}_{n\lambda}(h)$ is the irrotational, cylindrical vector wave functions while λ and h are the spectral longitudinal and radial wave numbers, respectively. The solenoidal and non-solenoidal cylindrical vector wave functions are defined as [38]

$$\boldsymbol{M}_n(h, \lambda) = \boldsymbol{\nabla} \times \left[\Psi_n(h, \lambda) \hat{\boldsymbol{z}} \right], \tag{7a}$$

$$\boldsymbol{N}_n(h, \lambda) = \frac{1}{k_\lambda} \boldsymbol{\nabla} \times \boldsymbol{M}_n(h, \lambda), \tag{7b}$$

$$\boldsymbol{L}_n(h, \lambda) = \boldsymbol{\nabla} \left[\Psi_n(h, \lambda) \right], \tag{7c}$$

where $k_\lambda = \sqrt{\lambda^2 + h^2}$, and the generating function is given by

$$\Psi_n(h, \lambda) = J_n(\lambda\rho) e^{i(n\phi + hz)}. \tag{8}$$

The vector expansion coefficients $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$ in (6) are to be determined from the orthogonality relationships among the cylindrical vector wave functions given by:

$$\begin{aligned}&\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{M}_n(h,\lambda)\cdot \boldsymbol{M}_{-n'}(-h',-\lambda')\\ &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{N}_n(h,\lambda)\cdot \boldsymbol{N}_{-n'}(-h',-\lambda')\\ &= 4\pi^2\lambda\delta(\lambda-\lambda')\delta(h-h')\delta_{nn'},\end{aligned} \tag{9a}$$

$$\begin{aligned}&\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{L}_n(h,\lambda)\cdot \boldsymbol{L}_{-n'}(-h',-\lambda')\\ &= 4\pi^2\frac{(\lambda^2+h^2)}{\lambda}\delta(\lambda-\lambda')\delta(h-h')\delta_{nn'},\end{aligned} \tag{9b}$$

and

$$\begin{aligned}&\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{M}_n(h,\lambda)\cdot \boldsymbol{N}_{-n'}(-h',-\lambda')\\ &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{N}_n(h,\lambda)\cdot \boldsymbol{L}_{-n'}(-h',-\lambda')\\ &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{L}_n(h,\lambda)\cdot \boldsymbol{M}_{-n'}(-h',-\lambda')\\ &= 0.\end{aligned} \tag{9c}$$

Therefore, by taking the scalar product of (6) with $\boldsymbol{M}_{-n'}(-h',-\lambda')$, $\boldsymbol{N}_{-n'}(-h',-\lambda')$ and $\boldsymbol{L}_{-n'}(-h',-\lambda')$ each at a time, the vector expansion coefficients are given by:

$$\boldsymbol{A}_n(h,\lambda) = \frac{1}{4\pi^2\lambda}\boldsymbol{M}'_{-n}(-h,-\lambda), \tag{10a}$$

$$\boldsymbol{B}_n(h,\lambda) = \frac{1}{4\pi^2\lambda}\boldsymbol{N}'_{-n}(-h,-\lambda), \tag{10b}$$

$$\boldsymbol{C}_n(h,\lambda) = \frac{\lambda}{4\pi^2(\lambda^2+h^2)}\boldsymbol{L}'_{-n}(-h,-\lambda), \tag{10c}$$

where the prime notation of the cylindrical vector wave functions denotes the evaluation at the source $\boldsymbol{r}'$.

So far, we have provided the fundamental formulation. From now on, we will discuss the formulation of the dyadic Green's functions in a different way from those in the literature such as in [38].

In a similar fashion to the dyadic form of the identity matrix, the dyadic Green's function can thus be expanded as follows:

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') &= \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_n(h,\lambda)\,\boldsymbol{a}_n(h,\lambda)\right.\\ &\left.+\boldsymbol{N}_n(h,\lambda)\,\boldsymbol{b}_n(h,\lambda)+\boldsymbol{L}_n(h,\lambda)\,\boldsymbol{c}_n(h,\lambda)\right], \qquad (11)\end{aligned}$$

where the vector expansion coefficients $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$ and $\boldsymbol{c}_n(h,\lambda)$ are unknown vectors to be determined from the orthogonality and permittivity tensor properties.

To obtain these unknown vectors, we substitute (11) and (6) into (5) which the dyadic Green's function must satisfy. Noting the instinct properties of the vector wave functions,

$$\boldsymbol{\nabla}\times\boldsymbol{N}_n(h,\lambda) = k_\lambda \boldsymbol{M}_n(h,\lambda), \qquad (12a)$$

$$\boldsymbol{\nabla}\times\boldsymbol{M}_n(h,\lambda) = k_\lambda \boldsymbol{N}_n(h,\lambda), \qquad (12b)$$

$$\boldsymbol{\nabla}\times\boldsymbol{L}_n(h,\lambda) = 0, \qquad (12c)$$

we can then end up with

$$\begin{aligned}&\int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \Big\{\Big[k_\lambda^2\overline{\boldsymbol{I}}-\omega^2\mu\overline{\boldsymbol{\epsilon}}\Big]\\ &\cdot\left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda)+\boldsymbol{N}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda)\right]\\ &-2k\omega\mu\xi_c\left[\boldsymbol{N}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda)+\boldsymbol{M}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda)\right]\\ &-\omega^2\mu\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{L}_n(h,\lambda)\boldsymbol{c}_n(h,\lambda)\Big\}\\ =\;&\int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{A}_n(h,\lambda)\right.\\ &\left.+\boldsymbol{N}_n(h,\lambda)\boldsymbol{B}_n(h,\lambda)+\boldsymbol{L}_n(h,\lambda)\boldsymbol{C}_n(h,\lambda)\right]. \qquad (13)\end{aligned}$$

By taking respectively the anterior scalar product of (13) with the vector wave equations and performing the integration over the entire space, we can formulate the equations obtained in a matrix form as given below:

$$[\Omega][X] = [\Theta], \qquad (14)$$

where $[\Omega]$ is a 3×3 coefficient matrix given by

$$[\Omega] = \left[\begin{array}{cc} k_\lambda^2-\omega^2\mu\epsilon & -\omega\mu\left(2\xi_c k_\lambda+\omega g\frac{h}{k_\lambda}\right) \\ -\omega\mu\left(2\xi_c k_\lambda+\omega g\frac{h}{k_\lambda}\right) & k_\lambda^2-\frac{\omega^2\mu}{k_\lambda^2}(h^2\epsilon+\lambda^2\epsilon_z) \\ -i\omega^2\mu g\frac{\lambda^2}{k_\lambda^2} & -\frac{ih\lambda^2}{k_\lambda^3}\omega^2\mu(\epsilon-\epsilon_z) \end{array}\right.$$

$$\begin{matrix} i\omega^2\mu g \\ \frac{ih}{k_\lambda}\omega^2\mu(\epsilon-\epsilon_z) \\ -\frac{\omega^2\mu}{k_\lambda}\left(\lambda^2\epsilon+h^2\epsilon_z\right) \end{matrix}\right], \tag{15a}$$

and $[X]$ and $[\Theta]$ are known and parameter column vectors given, respectively, by

$$[X]=\begin{bmatrix} \boldsymbol{a}_n(h,\lambda) \\ \boldsymbol{b}_n(h,\lambda) \\ \boldsymbol{c}_n(h,\lambda) \end{bmatrix}, \text{ and } \quad [\Theta]=\begin{bmatrix} \boldsymbol{A}_n(h,\lambda) \\ \boldsymbol{B}_n(h,\lambda) \\ \boldsymbol{C}_n(h,\lambda) \end{bmatrix}. \tag{15b}$$

Solving (14), we have the solutions for $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$ and $\boldsymbol{c}_n(h,\lambda)$ as

$$\boldsymbol{a}_n(h,\lambda) = \frac{1}{\Gamma}\left[\alpha_1\boldsymbol{A}_n(h,\lambda)+\beta_1\boldsymbol{B}_n(h,\lambda)+\gamma_1\boldsymbol{C}_n(h,\lambda)\right], \tag{16a}$$

$$\boldsymbol{b}_n(h,\lambda) = \frac{1}{\Gamma}\left[\alpha_2\boldsymbol{A}_n(h,\lambda)+\beta_2\boldsymbol{B}_n(h,\lambda)+\gamma_2\boldsymbol{C}_n(h,\lambda)\right], \tag{16b}$$

$$\boldsymbol{c}_n(h,\lambda) = \frac{1}{\Gamma}\left[\alpha_3\boldsymbol{A}_n(h,\lambda)+\beta_3\boldsymbol{B}_n(h,\lambda)+\gamma_3\boldsymbol{C}_n(h,\lambda)\right], \tag{16c}$$

where

$$\begin{aligned}\Gamma &= k_\lambda^2(h^2\epsilon_z+\epsilon\lambda^2)-\mu\omega^2\Big[2h^2\epsilon\epsilon_z-\lambda^2(g^2-\epsilon^2-\epsilon\epsilon_z) \\ &\quad +4\mu\xi^2(h^2\epsilon_z+\epsilon\lambda^2)\Big]-4gh\epsilon_z\mu^2\xi_c\omega^3+\epsilon_z\mu^2\omega^4(\epsilon^2-g^2)\end{aligned} \tag{17}$$

and the coupling coefficients are

$$\alpha_1 = h^2\epsilon_z+\lambda^2\epsilon-\omega^2\mu\epsilon\epsilon_z, \tag{18a}$$

$$\beta_1 = \alpha_2=\frac{\omega\mu}{k_\lambda}\Big[gh\epsilon_z\omega+2\xi_c(h^2\epsilon_z+\epsilon\lambda^2)\Big], \tag{18b}$$

$$\gamma_1 = -\frac{k_\lambda^2}{\lambda^2}\alpha_3=i\Big[2\omega\mu h\xi_c(\epsilon-\epsilon_z)+g(k_\lambda^2-\omega^2\mu\epsilon_z)\Big], \tag{18c}$$

$$\beta_2 = \frac{1}{k_\lambda^2}\Big[(k_\lambda^2-\omega^2\mu\epsilon)(h^2\epsilon_z+\lambda^2\epsilon)+\omega^2\mu g^2\lambda^2\Big], \tag{18d}$$

$$\gamma_2 = -\frac{k_\lambda^2}{\lambda^2}\beta_3=i\frac{1}{k_\lambda}\Big[h(k_\lambda^2-\mu\omega^2\epsilon)(\epsilon-\epsilon_z)+\omega\mu g(2k_\lambda^2\xi_c+gh\omega\Big], \tag{18e}$$

$$\begin{aligned}\gamma_3 &= \frac{1}{\omega^2\mu}\Bigg\{-k_\lambda^4+\omega^2\mu\Big[2h^2\epsilon+\lambda^2(\epsilon+\epsilon_z)+4k_\lambda^2\mu\xi_c^2\Big] \\ &\quad +4gh\xi_c\mu^2\omega^3+\frac{\omega^4\mu^2}{k_\lambda^2}\Big[h^2(g^2-\epsilon^2)-\epsilon\epsilon_z\lambda^2\Big]\Bigg\},\end{aligned} \tag{18f}$$

Now, it is clear that physically, both the coupling of the TE and TM waves and the coupling of the non-solenoidal and solenoidal wave functions are present. Mathematically, the presence of these couplings are actually due to the lack of orthogonalities when a permittivity tensor ϵ is inserted into the middle of the vector wave functions $\boldsymbol{M}$ and $\boldsymbol{N}$ in a form of scalar-product. In [38], this lack of the orthogonalities is ignored so that the solutions obtained are actually incorrect.

It should be noted that substituting (11) and (6) into (5) gives (16) after complicated mathematical manipulations. This substitution is based upon the condition that one can interchange the summation on n and the integrals on h and λ. This condition can be justified because the terms in the square brackets of (6) and (11) are continuous with respect to h and λ, simultaneously.

Hence, the unbounded dyadic Green's function can be written as

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = \int_{-\infty}^{\infty} dh \int_0^{\infty} d\lambda \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda\Gamma} \Bigg\{ & \boldsymbol{M}_n(h,\lambda)\Bigg[\alpha_1 \boldsymbol{M}'_{-n}(-h,-\lambda) \\ & + \beta_1 \boldsymbol{N}'_{-n}(-h,-\lambda) + \frac{\lambda^2}{k_\lambda^2}\gamma_1 \boldsymbol{L}'_{-n}(-h,-\lambda)\Bigg] \\ & + \boldsymbol{N}_n(h,\lambda)\Bigg[\beta_1 \boldsymbol{M}_{-n}(-h,-\lambda) \\ & + \beta_2 \boldsymbol{N}_{-n}(-h,-\lambda) + \frac{\lambda^2}{k_\lambda^2}\gamma_2 \boldsymbol{L}_{-n}(-h,-\lambda)\Bigg] \\ & + \frac{\lambda^2}{k_\lambda^2}\boldsymbol{L}_n(h,\lambda)\left[-\gamma_1 \boldsymbol{M}_{-n}(-h,-\lambda)\right. \\ & \left. - \gamma_2 \boldsymbol{N}_{-n}(-h,-\lambda) + \gamma_3 \boldsymbol{L}_{-n}(-h,-\lambda)\right] \Bigg\}. \end{aligned} \tag{19}$$

In this way, the dyadic Green's function in a unbounded gyroelectric chiral medium has been explicitly represented so far in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions, as given in (19). Comparing this form with that in [38], we immediately realize that the mutual coupling of the vector wave functions are, although exists, not included in the formulation of the unbounded DGF in [38]. This is a serious mistake and the further consideration in [38] will no longer meaningful.

The comparison can also be made between the results here and the formulas of DGFs published in [32, 41–45]. It is found that the form of DGF here is quite similar to those of DGFs in [32, 41–45]. However, they are quite different because the form of DGFs given in

[32, 41–45] is the one after the integration with respect to h, but the current form here is not.

Certainly, the expression of the DGF, as shown above, is a form in a pre-integration domain. To actually make use of it for practical problems, we need to integrate the DGF in the pre-integration domain using the contour integration. In order to apply the residue theorem to (19), however, we must first extract the part in (19) which does not satisfy the Jordan lemma [1]. To do so, we write

$$\boldsymbol{L}_n(h,\lambda) = \boldsymbol{L}_{nt}(h,\lambda) + \boldsymbol{L}_{nz}(h,\lambda), \tag{20a}$$

$$\boldsymbol{L}'_{-n}(-h,-\lambda) = \boldsymbol{L}'_{-nt}(-h,-\lambda) + \boldsymbol{L}'_{-nz}(-h,-\lambda), \tag{20b}$$

$$\boldsymbol{N}_n(h,\lambda) = \boldsymbol{N}_{nt}(h,\lambda) + \boldsymbol{N}_{nz}(h,\lambda), \tag{20c}$$

$$\boldsymbol{N}'_{-n}(-h,-\lambda) = \boldsymbol{N}'_{-nt}(-h,-\lambda) + \boldsymbol{N}'_{-nz}(-h,-\lambda), \tag{20d}$$

where the subscript t and z denote respectively the transverse vector components and the z-vector components of the two functions $\boldsymbol{L}_n(h,\lambda)$ and $\boldsymbol{N}_n(h,\lambda)$.

In terms of these functions, (19) can be rewritten in the form

$$\begin{aligned}
\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = & \int_{-\infty}^{\infty} dh \int_0^{\infty} d\lambda \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda\Gamma} \\
& \times\Big\{(h^2\epsilon_z + \lambda^2\epsilon - \omega^2\mu\epsilon\epsilon_z)\boldsymbol{M}_n(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda) \\
& +\frac{k_\lambda}{h}\left[g(\omega^2\mu\epsilon_z - \lambda^2) + 2h\epsilon_z\omega\mu\xi_c\right]\left[\boldsymbol{M}_n(h,\lambda)\right. \\
& \boldsymbol{N}'_{-nt}(-h,-\lambda) + \left.\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)\right] \\
& +k_\lambda(gh + 2\epsilon\omega\mu\xi_c)\left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\right. \\
& +\left.\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)\right] + \frac{k_\lambda^2}{h^2\omega^2\mu} \\
& \times\left[\lambda^2(\omega^2\mu\epsilon + 4\omega^2\mu^2\xi_c^2 - k_\lambda^2) + \omega^2\mu\epsilon_z(k_\lambda^2 - \epsilon\omega^2\mu)\right] \\
& \times\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda) + \frac{k_\lambda^2}{h\omega^2\mu} \\
& \times\left[h(k_\lambda^2 - \omega^2\mu\epsilon) - 2\omega^2\mu^2\xi_c(2h\xi_c + g\omega)\right]\left[\boldsymbol{N}_{nt}(h,\lambda)\right. \\
& \boldsymbol{N}'_{-nz}(-h,-\lambda) + \left.\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)\right] \\
& +\frac{k_\lambda^2}{\omega^2\mu\lambda^2}\left[-h^2k_\lambda^2 + \omega^2\mu\epsilon(2h^2 + \lambda^2)\right. \\
& \left.+4h\omega^2\mu^2\xi_c(h\xi_c + g\omega) + \omega^4\mu^2(g^2 - \epsilon^2)\right]
\end{aligned}$$

$$\times \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)\Bigg\}, \tag{21}$$

where we have expressed $\boldsymbol{L}_{nt}(h,\lambda)$, $\boldsymbol{L}_{nz}(h,\lambda)$, and their primed functions in terms of $\boldsymbol{N}_{nt}(h,\lambda)$, $\boldsymbol{N}_{nz}(h,\lambda)$, and their corresponding primed functions, namely

$$\boldsymbol{L}_{nt}(h,\lambda) = \frac{-ik_\lambda}{h}\boldsymbol{N}_{nt}(h,\lambda), \tag{22a}$$

$$\boldsymbol{L'}_{-nt}(-h,-\lambda) = \frac{ik_\lambda}{h}\boldsymbol{N'}_{-nt}(-h,-\lambda); \tag{22b}$$

and

$$\boldsymbol{L}_{nz}(h,\lambda) = \frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda), \tag{22c}$$

$$\boldsymbol{L'}_{-nz}(-h,-\lambda) = \frac{-ihk_\lambda}{\lambda^2}\boldsymbol{N'}_{-nz}(-h,-\lambda). \tag{22d}$$

2.2. Analytical Evaluation Of The h Integral

In this section, we will analytically evaluate the h integrals for the dyadic Green's function arisen in (11). This effort is intended to make the results applicable in solving the source-incorporated boundary value problems of planar, multilayered structures consisting of gyroelectric chiral media.

By applying the idea given by Tai [1] to obtain an exact expression of the irrotational dyadic Green's function, we have from (6)

$$\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\delta(\boldsymbol{r}-\boldsymbol{r'}) = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda}\frac{k_\lambda^2}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda). \tag{23}$$

Thus the singular term of the unbounded DGF in (21) is contained in the dyadic of $\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)$.

To simplify the expression in (17), we rewrite Γ into the following form in order to perform the h integration,

$$\Gamma = ph^4 + qh^2 + sh + t, \tag{24}$$

where

$$\begin{aligned}
p &= \epsilon_z,\\
q &= \lambda^2(\epsilon+\epsilon_z) - 2\epsilon_z\omega^2\mu(2\mu\xi_c^2+\epsilon),\\
s &= -4g\epsilon_z\omega^3\mu^2\xi_c,\\
t &= \epsilon\lambda^4 + \lambda^2\omega^2\mu\left[g^2 - \epsilon(\epsilon+\epsilon_z+4\mu\xi_c^2)\right] + \omega^4\mu^2\epsilon_z(\epsilon^2-g^2).
\end{aligned}$$

With appropriate algebraic manipulation, we can split (21) into

$$\begin{aligned}
\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') =& -\int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda} \frac{k_\lambda^2}{\omega^2\mu\epsilon_z\lambda^2} \\
&\times \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) + \int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda} \\
&\times \frac{1}{\epsilon_z(h-h_1)(h-h_2)(h-h_3)(h-h_4)} \\
&\times \Bigg\{ (h^2\epsilon_z + \lambda^2\epsilon - \omega^2\mu\epsilon\epsilon_z)\boldsymbol{M}_n(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda) + \frac{k_\lambda}{h} \\
&\times \left[g(\omega^2\mu\epsilon_z - \lambda^2) + 2h\epsilon_z\omega\mu\xi_c \right] (\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda) \\
&+ \boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)) + k_\lambda(gh + 2\epsilon\omega\mu\xi_c) \\
&\times (\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) + \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)) \\
&+ \frac{k_\lambda^2}{h^2\omega^2\mu} \left[\lambda^2(\omega^2\mu\epsilon + 4\omega^2\mu^2\xi_c^2 - k_\lambda^2) + \omega^2\mu\epsilon_z(k_\lambda^2 - \epsilon\omega^2\mu) \right] \\
&\times \boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda) + \frac{k_\lambda^2}{h\omega^2\mu} \Big[h(k_\lambda^2 - \omega^2\mu\epsilon) \\
&- 2\omega^2\mu^2\xi_c(2h\xi_c + g\omega) \Big] [\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) \\
&+ \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)] + \frac{k_\lambda^2}{\epsilon_z\omega^2\mu} \Big[\epsilon(k_\lambda^2 - \omega^2\mu\xi_c) \\
&+ \omega^2\mu(g^2 - 4\epsilon\mu\xi_c^2) \Big] \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) \Bigg\},
\end{aligned} \tag{25}$$

where h_j ($j = 1, 2, 3,$ and 4) is found by solving the fourth-order polynomial equation $\Gamma = \epsilon_z(h-h_1)(h-h_2)(h-h_3)(h-h_4) = 0$. The general solution of the equation can be obtained using the syntax, Solve[p hˆ4 + q hˆ2 + s h + t == 0, h], in Mathematica package, however, is too tedious to be shown here. An important point is that $\Gamma = 0$ gives four sets of solutions corresponding to four different waves of wave numbers h_i ($i = 1, 2, 3,$ and 4).

In view of (23), the first integration term in (25) is equal to

$$-\frac{1}{\omega^2\mu\epsilon_z}\,\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{26}$$

which is the contribution from the non-solenoidal vector wave functions. This term of the DGF expressions was never been obtained in the existing publications such as [32, 41–45] where the irrotational part of the DGF were all missing.

The second integration term can be evaluated by making use of the residue theorem in h-plane (appendix). This term contributes from the solenoidal vector wave functions. After some mathematical manipulations for simplicity, hence, we arrived at the final representation of the dyadic Green's function for a unbounded gyroelectric chiral medium. Since the integration has already been carried out, it is thus suitable for further direct analysis of wave characteristics in a planar, multilayered structure. For $z > z'$, the DGF is given by

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{R},\boldsymbol{R}') &= -\frac{1}{\omega^2\mu\epsilon_z}\,\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}') \\ &+\frac{i}{2\pi}\int_0^\infty d\lambda\sum_{n=-\infty}^{\infty}\frac{1}{\lambda(h_1-h_2)}\sum_{j=1}^{2}\frac{(-1)^{j+1}}{(h_j-h_3)(h_j-h_4)} \\ &\times\Bigg\{\boldsymbol{M}_{n,\lambda}(h_j)\boldsymbol{P'}_{-n,-\lambda}(-h_j)+\frac{k_{\lambda j}}{\epsilon_z}\boldsymbol{Q}_{n,\lambda}(h_j)\boldsymbol{M'}_{-n,-\lambda}(-h_j)+\frac{k_{\lambda j}^2}{h_j^2\omega^2\mu\epsilon_z} \\ &\times\boldsymbol{U}_{n,\lambda}(h_j)\boldsymbol{N'}_{-nt,-\lambda}(-h_j)+\frac{k_{\lambda j}^2}{\omega^2\mu\epsilon_z}\boldsymbol{V}_{n,\lambda}(h_j)\boldsymbol{N'}_{-nz,-\lambda}(-h_j)\Bigg\}. \end{aligned} \tag{27}$$

For $z < z'$, only the portion of

$$\frac{1}{\lambda(h_1-h_2)}\sum_{j=1}^{2}\frac{(-1)^{j+1}}{(h_j-h_3)(h_j-h_4)}$$

is replaced by

$$\frac{1}{\lambda(h_3-h_4)}\sum_{j=3}^{4}\frac{(-1)^{j+1}}{(h_1-h_j)(h_2-h_j)}.$$

The vector wave functions $\boldsymbol{P'}_{-n,-\lambda}(-h_j)$, $\boldsymbol{Q}_{n,\lambda}(h_j)$, $\boldsymbol{U}_{n,\lambda}(h_j)$ and $\boldsymbol{V}_{n,\lambda}(h_j)$, as defined above, are given respectively by

$$\begin{aligned}\boldsymbol{P'}_{-n,-\lambda}(-h_j) &= (h_j^2+\frac{\epsilon}{\epsilon_z}\lambda^2-\epsilon\,\omega^2\mu)\boldsymbol{M'}_{-n,-\lambda}(-h_j) \\ &+\frac{k_{\lambda j}}{\epsilon_z}\left[\frac{g}{h_j}(\epsilon_z\omega^2\mu-\lambda^2)+2\epsilon_z\omega\mu\xi_c\right]\boldsymbol{N'}_{-nt,-\lambda}(-h_j) \\ &+\frac{k_{\lambda j}}{\epsilon_z}(gh_j+2\epsilon\,\omega\mu\xi_c)\boldsymbol{N'}_{-nz,-\lambda}(-h_j), \end{aligned} \tag{28a}$$

$$\boldsymbol{Q}_{n,\lambda}(h_j) = \left[\frac{g}{h_j}(\epsilon_z\omega^2\mu - \lambda^2) + 2\epsilon_z\omega\mu\xi_c\right]\boldsymbol{N}_{nt,\lambda}(h_j) + (gh_j + 2\epsilon\,\omega\mu\xi_c)\boldsymbol{N}_{nz,\lambda}(h_j), \tag{28b}$$

$$\boldsymbol{U}_{n,\lambda}(h_j) = \left[(k_{\lambda j}^2 - \epsilon\,\omega^2\mu)(\epsilon_z\omega^2\mu - \lambda^2) + 4\lambda^2\omega^2\mu^2\xi_c^2\right]\boldsymbol{N}_{nt,\lambda}(h_j) + h_j\left[h_j(k_j^2 - \epsilon\,\omega^2\mu) - 2\omega^2\mu^2\xi_c(2h_j\xi_c + g\omega)\right]\boldsymbol{N}_{nz,\lambda}(h_j), \tag{28c}$$

$$\boldsymbol{V}_{n,\lambda}(h_j) = \left[(k_{\lambda j}^2 - \epsilon\,\omega^2\mu) - \frac{2\omega^2\mu^2\xi_c}{h_j}(2h_j\xi_c + g\omega)\right]\boldsymbol{N}_{nt,\lambda}(h_j) + \frac{1}{\epsilon_z}\left[\epsilon(k_{\lambda j}^2 - \epsilon\,\omega^2\mu) + \omega^2\mu(g^2 - 4\epsilon\mu\xi_c^2)\right]\boldsymbol{N}_{nz,\lambda}(h_j). \tag{28d}$$

So far, we have obtained a complete representation of the dyadic Green's function for a unbounded gyroelectric chiral medium. It is observed that (1) the dyadic Green's function contains a singularity term which is contributed by the solenoidal vector wave function and is reducible to those of anisotropic media and isotropic media, (2) regardless of source and field positions, the Green's function consists of two different wave modes denoted respectively by two wave numbers, and (3) there exist no pure TE or TM modes in the unbounded gyroelectric chiral medium and the coupling between each other is present.

It is claimed that the form of dyadic Green's functions in terms of vector wave eigenfunctions are obtained for the first time, and have never been given elsewhere in the publications. Although the results for the same problem were published in the aforementioned literature [38], they have been proved in [40] to be incorrect. It is believed that the current formulation gives the correct answer. After the integration with respect to h, the current form is also found to be quite different from the results given in [32, 41–45]. The current form is believed to be more reasonable since firstly we have extracted the singular term (of the representation of the dyadic Green's functions) which does not satisfy the Jordan lemma [1] and secondly we have considered all the possible modes and possible couplings in the formulation.

3. GENERAL FORMULATION FOR PLANAR, LAYERED GYROELECTRIC CHIRAL MEDIA

The motivation of this paper is not just restricted to the above discussion. Furthermore, we would extend our theoretical analysis

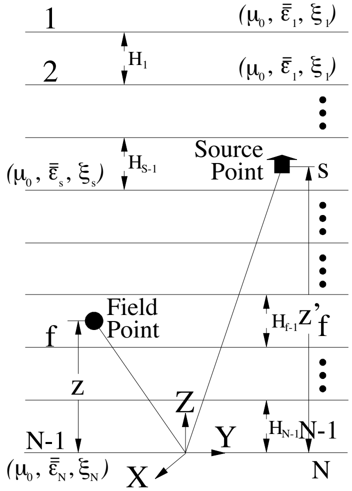

Figure 1. Geometry of a multilayered stratified gyroelectric chiral medium.

to the multilayered planar structure where the number of multiple layers is arbitrary, the location of either the source or the field is also arbitrary, and each layer can be a gyroelectric chiral, an anisotropic ($\xi_c = 0$), a chiral ($g = 0$ and $\epsilon = \epsilon_z$), or just simply an isotropic medium ($\xi_c = 0$, $g = 0$, and $\epsilon = \epsilon_z$), as shown in Fig. 1.

To take the dielectric parameters into account, we consider the permittivity tensor given by

$$\overline{\boldsymbol{\epsilon}}^f = \begin{bmatrix} \epsilon^f & ig^f & 0 \\ -ig^f & \epsilon^f & 0 \\ 0 & 0 & \epsilon_z^f \end{bmatrix},$$

and correspondingly the chiral parameter ξ^f for the layer f. Thus, the relation governing the wavenumber and eigenvalues is now re-written as

$$k_\lambda^f = \sqrt{\lambda^2 + (h^f)^2}.$$

3.1. Scattering Dyadic Green's Functions

Based on the principle of scattering superposition, we have

$$\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') = \overline{\boldsymbol{G}}_0(\boldsymbol{r}, \boldsymbol{r}')\delta_f^s + \overline{\boldsymbol{G}}_s^{(fs)}(\boldsymbol{r}, \boldsymbol{r}'), \tag{29}$$

where the representation of the scattered dyadic Green's function is given by

$$\overline{\boldsymbol{G}}_s^{(fs)}(\boldsymbol{r},\boldsymbol{r'}) = \sum_{j=1}^{4}\overline{\boldsymbol{G_j}}. \tag{30}$$

Considering the representations of each term, we realized that it is better to combine two terms as follows:

$$\begin{aligned}
\overline{\boldsymbol{G_1}}+\overline{\boldsymbol{G_2}} &= \frac{i}{2\pi}\int_0^\infty d\lambda \sum_{n=-\infty}^{\infty}\frac{1}{\lambda(h_{1s}-h_{2s})} \\
&\times\sum_{j=1}^{2}\frac{1}{(h_{js}-h_{3s})(h_{js}-h_{4s})}\Big\{(1-\delta_f^N)\boldsymbol{M}_{n,\lambda}(h_j^f) \\
&\times\Big[(1-\delta_s^1)A_{Mj}^{fs}\boldsymbol{P'}_{-n,-\lambda}(-h_j^s)+(1-\delta_s^N)B_{Mj}^{fs} \\
&\times\boldsymbol{P'}_{-n,-\lambda}(-h_{j+2}^s)\Big]+(1-\delta_f^N)\frac{k_{\lambda js}}{\epsilon_{zs}}\boldsymbol{Q}_{n,\lambda}(h_j^f) \\
&\Big[(1-\delta_s^1)A_{Qj}^{fs}\boldsymbol{M'}_{-n,-\lambda}(-h_j^s)+(1-\delta_s^N)B_{Qj}^{fs} \\
&\times\boldsymbol{M'}_{-n,-\lambda}(-h_{j+2}^s)\Big]+(1-\delta_f^N)\frac{k_{\lambda js}^2}{\epsilon_{zs}\omega^2\mu_{os}h_{js}^2}\boldsymbol{U}_{n,\lambda}(h_j^f) \\
&\times\Big[(1-\delta_s^1)A_{Uj}^{fs}\boldsymbol{N'}_{-nt,-\lambda}(-h_j^s)+(1-\delta_s^N)B_{Uj}^{fs} \\
&\times\boldsymbol{N'}_{-nt,-\lambda}(-h_{j+2}^s)\Big]+(1-\delta_f^N)\frac{k_{\lambda js}^2}{\epsilon_{zs}\omega^2\mu_{os}}\boldsymbol{V}_{n,\lambda}(h_j^f) \\
&\Big[(1-\delta_s^1)A_{Vj}^{fs}\boldsymbol{N'}_{-nz,-\lambda}(-h_j^s)+(1-\delta_s^N)B_{Vj}^{fs} \\
&\times\boldsymbol{N'}_{-nz,-\lambda}(-h_{j+2}^s)\Big]\Big\},
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
\overline{\boldsymbol{G_3}}+\overline{\boldsymbol{G_4}} &= -\frac{i}{2\pi}\int_0^\infty d\lambda \sum_{n=-\infty}^{\infty}\frac{1}{\lambda(h_{3s}-h_{4s})} \\
&\times\sum_{j=3}^{4}\frac{1}{(h_{1s}-h_{js})(h_{2s}-h_{js})}\Big\{(1-\delta_f^N)\boldsymbol{M}_{n,\lambda}(h_j^f) \\
&\times\Big[(1-\delta_s^1)A_{Mj}^{fs}\boldsymbol{P'}_{-n,-\lambda}(-h_{j-2}^s)+(1-\delta_s^N)B_{Mj}^{fs} \\
&\times\boldsymbol{P'}_{-n,-\lambda}(-h_j^s)\Big]+(1-\delta_f^N)\frac{k_{\lambda js}}{\epsilon_{zs}}\boldsymbol{Q}_{n,\lambda}(h_j^f) \\
&\times\Big[(1-\delta_s^1)A_{Qj}^{fs}\boldsymbol{M'}_{-n,-\lambda}(-h_{j-2}^s)+(1-\delta_s^N)B_{Qj}^{fs}
\end{aligned}$$

$$\times \boldsymbol{M'}_{-n,-\lambda}(-h_j^s)\Big] + (1-\delta_f^N)\frac{k_{\lambda js}^2}{\epsilon_{zs}\omega^2\mu_{os}h_{js}^2}\boldsymbol{U}_{n,\lambda}(h_j^f)$$
$$\times \Big[(1-\delta_s^1)A_{Uj}^{fs}\boldsymbol{N'}_{-nt,-\lambda}(-h_{j-2}^s) + (1-\delta_s^N)B_{Uj}^{fs}$$
$$\times \boldsymbol{N'}_{-nt,-\lambda}(-h_j^s)\Big] + (1-\delta_f^N)\frac{k_{\lambda js}^2}{\epsilon_{zs}\omega^2\mu_{os}}\boldsymbol{V}_{n,\lambda}(h_j^f)$$
$$\times \Big[(1-\delta_s^1)A_{Vj}^{fs}\boldsymbol{N'}_{-nz,-\lambda}(-h_{j-2}^s) + (1-\delta_s^N)B_{Vj}^{fs}$$
$$\times \boldsymbol{N'}_{-nz,-\lambda}(-h_j^s)\Big]\Big\}. \tag{32}$$

The combination of the two terms for the above two equations is due to the fact that each term has a static contribution to the dyadic Green's function because of the integration associated with the pole point $\lambda = 0$. It is also due to the fact that the two terms have the same wave modes (please refer to the derived wave numbers in the expressions). It is actually realized that the combination of the two terms leads to a cancellation of the static waves. To avoid introducing more intermediates, therefore, we did no write each term $\overline{\boldsymbol{G_j}}$ ($j = 1, 2, 3$ and 4) out separately.

It should be pointed out that the multiple reflection and transmission effects have been included in the formulation of the scattering dyadic Green's functions. Also, the Sommerfeld radiation condition has been taken into account in the construction of DGFs. The contributions from various wave modes to the DGFs have been considered as well.

4. DETERMINATION OF THE DGFS' SCATTERING COEFFICIENTS

To formulate the scattering coefficients of the DGFs, the boundary conditions have to be utilized. From the field expression with Green's dyadic, the boundary conditions satisfied by the dyadic Greens' function at the interface $z = z_j$, $(j = 1, 2, ..., N-1)$ are shown as follow:

$$\hat{\boldsymbol{z}} \times \overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') = \hat{\boldsymbol{z}} \times \overline{\boldsymbol{G}}_e^{[(f+1)s]}(\boldsymbol{r}, \boldsymbol{r}'), \tag{33a}$$

$$\hat{\boldsymbol{z}} \times \left[\frac{1}{\mu_f}\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') - \omega\xi_{cf}\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')\right]$$
$$= \hat{\boldsymbol{z}} \times \left[\frac{1}{\mu_{f+1}}\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_e^{[(f+1)s]}(\boldsymbol{r}, \boldsymbol{r}') - \omega\xi_{c(f+1)}\overline{\boldsymbol{G}}_e^{[(f+1)s]}(\boldsymbol{r}, \boldsymbol{r}')\right]. \tag{33b}$$

To simplify the derivation of the general solution of the coefficients, we rewrite the boundary conditions (33a) and (33b) into the following matrix form. Now, it is clear that the equations obtained here for the layered gyroelectric chiral medium is different from those for both the layered chiral media [46] and the layered isotropic media [11]. Similar to those for the anisotropic medium, the weighting factors of the transverse and perpendicular components for the TM waves in the layered gyroelectric chiral medium have to be considered here in the formulation of the scattering coefficients of DGFs.

4.1. Recurrence Formulae Of DGFs' Scattering Coefficients

By using the boundary conditions, a set of linear equations of the coefficients which can be replaced by a series of compact matrices [14] is given. The following compact recurrent equations:

$$\left[F_{lj'(f+1)}\right]\cdot\left\{\left[\Upsilon_{lj'(f+1)s}\right]+\delta^s_{f+1}\left[U_{(f+1)}\right]\right\}=\left[F_{lj'f}\right]\cdot\left\{\left[\Upsilon_{lj'fs}\right]+\delta^s_f\left[D_f\right]\right\}, \tag{34}$$

where $j'=1,2$ and $l=M,Q,U$ and V. These matrices are given by

$$[F_{M1f}]=\begin{bmatrix}\dfrac{e^{ih_{1f}z_f}}{(h_{1s}-h_{2s})(h_{1s}-h_{4s})} & \dfrac{e^{ih_{3f}z_f}}{(h_{3s}-h_{4s})(h_{2s}-h_{3s})}\\ \dfrac{(h_{1f}-\omega\mu_{of}\xi_{cf})e^{ih_{1f}z_f}}{\mu_{of}(h_{1s}-h_{2s})(h_{1s}-h_{4s})} & \dfrac{(h_{3f}-\omega\mu_{of}\xi_{cf})e^{ih_{3f}z_f}}{\mu_{of}(h_{3s}-h_{4s})(h_{2s}-h_{3s})}\end{bmatrix}, \tag{35a}$$

$$[F_{Q1f}]=\begin{bmatrix}\dfrac{k_{\lambda1s}h_{1f}e^{ih_{1f}z_f}}{k_{\lambda1f}(h_{1s}-h_{2s})(h_{1s}-h_{4s})} & \dfrac{k_{\lambda1s}\left[(w_{qt1}-w_{qz1})h_{1f}^2+w_{qz1}k_{\lambda1f}^2-w_{qt1}\omega\mu_{of}\xi_{cf}h_{1f}\right]e^{ih_{1f}z_f}}{\mu_{of}k_{\lambda1f}(h_{1s}-h_{2s})(h_{1s}-h_{4s})}\\ \dfrac{k_{\lambda3s}h_{3f}e^{ih_{3f}z_f}}{k_{\lambda3f}(h_{3s}-h_{4s})(h_{2s}-h_{3s})} & \dfrac{k_{\lambda3s}\left[(w_{qt3}-w_{qz3})h_{3f}^2+w_{qz3}k_{\lambda3f}^2-w_{qt3}\omega\mu_{of}\xi_{cf}h_{3f}\right]e^{ih_{3f}z_f}}{\mu_{of}k_{\lambda3f}(h_{3s}-h_{4s})(h_{2s}-h_{3s})}\end{bmatrix}^T \tag{35b}$$

$$[F_{U1f}]=\begin{bmatrix}\dfrac{k_{\lambda1s}^2h_{1f}e^{ih_{1f}z_f}}{h_{1s}^2k_{\lambda1f}(h_{1s}-h_{2s})(h_{1s}-h_{4s})} & \dfrac{k_{\lambda1s}^2\left[(w_{ut1}-w_{uz1})h_{1f}^2+w_{uz1}k_{\lambda1f}^2-w_{ut1}\omega\mu_{of}\xi_{cf}h_{1f}\right]e^{ih_{1f}z_f}}{\mu_{of}h_{1s}^2k_{\lambda1f}(h_{1s}-h_{2s})(h_{1s}-h_{4s})}\\ \dfrac{k_{\lambda3s}^2h_{3f}e^{ih_{3f}z_f}}{h_{3s}^2k_{\lambda3f}(h_{3s}-h_{4s})(h_{2s}-h_{3s})} & \dfrac{k_{\lambda3s}^2\left[(w_{ut3}-w_{uz3})h_{3f}^2+w_{uz3}k_{\lambda3f}^2-w_{ut3}\omega\mu_{of}\xi_{cf}h_{3f}\right]e^{ih_{3f}z_f}}{\mu_{of}h_{3f}^2k_{\lambda3f}(h_{3s}-h_{4s})(h_{2s}-h_{3s})}\end{bmatrix}^T \tag{35c}$$

$$[F_{V1f}] = \left[\begin{array}{c} \frac{k_{\lambda 1s}^2 h_{1f} e^{ih_{1f}z_f}}{k_{\lambda 1f}(h_{1s}-h_{2s})(h_{1s}-h_{4s})} \\ \frac{k_{\lambda 3s}^2 h_{3f} e^{ih_{3f}z_f}}{k_{\lambda 3f}(h_{3s}-h_{4s})(h_{2s}-h_{3s})} \end{array} \right. \left. \begin{array}{c} \frac{k_{\lambda 1s}^2 \left[(w_{vt1}-w_{vz1})h_{1f}^2+w_{vz1}k_{\lambda 1f}^2-w_{vt1}\omega\mu_{of}\xi_{cf}h_{1f}\right] e^{ih_{1f}z_f}}{\mu_{of}k_{\lambda 1f}(h_{1s}-h_{2s})(h_{1s}-h_{4s})} \\ \frac{k_{\lambda 3s}^2 \left[(w_{vt3}-w_{vz3})h_{3f}^2+w_{vz3}k_{\lambda 3f}^2-w_{vt3}\omega\mu_{of}\xi_{cf}h_{3f}\right] e^{ih_{3f}z_f}}{\mu_{of}k_{\lambda 3f}(h_{3s}-h_{4s})(h_{2s}-h_{3s})} \end{array} \right]^T \quad (35d)$$

where the superscript T denotes the transpose of the matrices. The matrices $[F_{lj'f}]$ remains the same form for $j' = 2$ or 4 except the subscript 1 is changed to 2 and the subscript 3 is changed to 4. Furthermore, the denominator which contains the term $(h_{1s} - h_{4s})$ is changed to $(h_{2s} - h_{3s})$ and vice versa. The terms w_{ltj} and w_{lzj} are the weighting factors associated with the scattering coefficients A_{lj}^{fs} and B_{lj}^{fs}. They are expressed as

$$w_{qtj} = \frac{g_s}{h_{js}}(\epsilon_{zs}\omega^2\mu_{os} - \lambda^2) + 2\epsilon_{zs}\omega\mu_{os}\xi_{cs}, \quad (36a)$$

$$w_{qzj} = g_s h_{js} + 2\epsilon_s\omega\mu_{os}\xi_{cs}, \quad (36b)$$

$$w_{utj} = (k_{\lambda js}^2 - \epsilon_s\omega^2\mu_{os})(\epsilon_{zs}\omega^2\mu_{os} - \lambda^2) + 4\lambda^2\omega^2\mu_{os}^2\xi_{cs}^2, \quad (36c)$$

$$w_{uzj} = h_{js}\left[h_{js}(k_\lambda^2 - \epsilon_s\omega^2\mu_{os}) - 2\omega^2\mu_{os}^2\xi_{cs}(2h_{js}\xi_{cs} + g_s\omega)\right], \quad (36d)$$

$$w_{vtj} = k_{\lambda js}^2 - \epsilon_s\omega^2\mu_{os} - \frac{2\omega^2\mu_{os}^2\xi_{cs}}{h_{js}}(2h_{js}\xi_{cs} + g_s\omega), \quad (36e)$$

$$w_{vzj} = \frac{1}{\epsilon_{zs}}\left[\epsilon_s(k_{\lambda js}^2 - \epsilon_s\omega^2\mu_{os}) + \omega^2\mu_{os}(g_s^2 - 4\epsilon_s\mu_{os}\xi_{cs}^2)\right]. \quad (36f)$$

The following matrices are also used in the formulation:

$$[\Upsilon_{lj'fs}] = \begin{bmatrix} A_{lj'}^{fs} & B_{lj'}^{fs} \\ A_{l,j'+2}^{fs} & B_{l,j'+2}^{fs} \end{bmatrix}, \quad (37a)$$

$$[U_f] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad (37b)$$

$$[D_f] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \quad (37c)$$

Defining the following transmission T-matrix:

$$[T_{lj'f}] = \left[F_{lj'(f+1)f}\right]^{-1} \cdot [F_{lj'ff}], \quad (38)$$

where $\left[F_{lj'(f+1)f}\right]^{-1}$ is the inverse matrix of $\left[F_{lj'(f+1)f}\right]$. We rewrite the linear equation into the following form:

$$\left[\Upsilon_{lj'(f+1)s}\right] = \left[T_{lj'f}\right] \cdot \left\{\left[\Upsilon_{lj'fs}\right] + \delta_f^s \left[D_f\right]\right\} - \delta_{f+1}^s \left[U_{(f+1)}\right]. \quad (39)$$

To shorten the expression, we also introduce:

$$\begin{aligned}\left[T_{lj'}^K\right]_{2\times 2} &= \left[T_{lj',N-1}\right]\left[T_{lj',N-2}\right]\cdots\left[T_{lj',K+1}\right]\left[T_{lj',K}\right] \\ &= \begin{bmatrix} T_{lj',11}^K & T_{lj',12}^K \\ T_{lj',21}^K & T_{lj',22}^K \end{bmatrix}. \end{aligned} \quad (40)$$

It should be noted that the coefficients matrices of the first and the last layers have the following relations:

$$\left[\Upsilon_{lj'1s}\right] = \begin{bmatrix} A_{lj'}^{1s} & B_{lj'}^{1s} \\ 0 & 0 \end{bmatrix}, \quad (41a)$$

$$\left[\Upsilon_{lj'Ns}\right] = \begin{bmatrix} 0 & 0 \\ A_{l,j'+2}^{Ns} & B_{l,j'+2}^{Ns} \end{bmatrix}. \quad (41b)$$

4.2. Specific Applications: Three Cases

The above discussion is based on the general formulation of the scattering coefficients of DGFs. Actually, it is realized from our current exercise of symbolic computation of the dyadic Green's functions for layered media using Mathematica that (1) without the aforementioned general formulation in the matrix form for the scattering coefficients, the Mathematica does not give the direct and compact solution, (2) the Mathematica can quite often run out of memory as the number of layers becomes larger so that a large number of symbolic derivations need to be involved, and (3) the direct implementation of the general formulation does not give the desired solutions in terms of the compact transmission matrices as expected for various source locations, even the above general formulas are implemented. Therefore, to gain insight into the specific mathematical expressions of the physical quantities such as the transmission and reflections coefficient matrices, the following three cases are specifically considered subsequently.

4.2.1. Source in the First Layer

When the current source is located in the first layer (i.e., $s = 1$), the first term containing $(1-\delta_s^1)$ in (31) vanishes. These will further reduce the coefficient matrices in (37a) and (4.1) to:

$$[\Upsilon_{lj',11}] = \begin{bmatrix} 0 & B_{lj'}^{11} \\ 0 & 0 \end{bmatrix}, \tag{42a}$$

$$[\Upsilon_{lj',m1}] = \begin{bmatrix} 0 & B_{lj'}^{m1} \\ 0 & B_{l,j'+2}^{m1} \end{bmatrix}, \tag{42b}$$

$$[\Upsilon_{lj',N1}] = \begin{bmatrix} 0 & 0 \\ 0 & B_{l,j'+2}^{N1} \end{bmatrix}, \tag{42c}$$

where $m = 2, 3, \cdots, N-1$. It can be seen that only four coefficients for the first layer and the last layer, but 8 coefficients for each of the remaining layers, need to be solved for. By following (39), the recurrence relations in the f^{th} layer become:

$$[\Upsilon_{lj',f1}] = [T_{lj',f-1}] \cdots [T_{lj',1}] \left\{ [\Upsilon_{lj',11}] + [D_1] \right\}. \tag{43}$$

With $f = N$ in (43), a matrix equation satisfied by the coefficient matrices in (42) can be obtained. The coefficients for the first layer where the source is (i.e. $s = 1$) is given by:

$$B_{lj'}^{11} = -\frac{T_{lj',12}^{(1)}}{T_{lj',11}^{(1)}}. \tag{44}$$

The coefficients for the last layer can be derived in terms of the coefficients for the first layer given by:

$$B_{l,j'+2}^{N1} = T_{lj',21}^{(1)} B_{lj'}^{11} + T_{lj',22}^{(1)}. \tag{45}$$

The coefficients for the intermediate layers can be then obtained by substituting the coefficients for the first layer in (44) to (43). Thus, all the coefficients can be obtained by these procedures.

4.2.2. Source in the Intermediate Layers

When the current source is located in an intermediate layer, (i.e., $s \neq 1, N$), only the terms containing $(1 - \delta_f^1)$ for the first layer and $(1 - \delta_f^N)$ for the last layer vanishes in (31). We thus have:

$$[\Upsilon_{lj',1s}] = \begin{bmatrix} A_{lj'}^{1s} & B_{lj'}^{1s} \\ 0 & 0 \end{bmatrix}, \tag{46a}$$

$$[\Upsilon_{lj',ms}] = \begin{bmatrix} A_{lj'}^{ms} & B_{lj'}^{ms} \\ A_{l,j'+2}^{ms} & B_{l,j'+2}^{ms} \end{bmatrix}, \tag{46b}$$

$$[\Upsilon_{lj',Ns}] = \begin{bmatrix} 0 & 0 \\ A_{l,j'+2}^{Ns} & B_{l,j'+2}^{Ns} \end{bmatrix}. \tag{46c}$$

From (39), the recurrence equation becomes:

$$\begin{aligned} [\Upsilon_{lj',fs}] &= [T_{lj',f-1}] \cdots [T_{lj',s}] \\ &\quad \cdot \{ [T_{lj',s-1}] \cdots [T_{lj',1}] [\Upsilon_{lj',1s}] \\ &\quad + u(f-s-1) [D_s] - u(f-s) [U_s] \}, \end{aligned} \tag{47}$$

where $u(x-x_0)$ is the unit step function. For $f = N$, the coefficients for the first layer are given by:

$$A_{lj'}^{1s} = \frac{T_{lj',11}^{(s)}}{T_{lj',11}^{(1)}}, \tag{48a}$$

$$B_{lj'}^{1s} = -\frac{T_{lj',12}^{(s)}}{T_{lj',11}^{(1)}}. \tag{48b}$$

For the last layer,

$$A_{l,j'+2}^{Ns} = T_{lj',21}^{(1)} A_{lj'}^{1s} - T_{lj',21}^{(s)}, \tag{49a}$$

$$B_{l,j'+2}^{Ns} = T_{lj',21}^{(1)} B_{lj'}^{(s)} + T_{lj',22}^{(s)}. \tag{49b}$$

Substituting (48) into (47), the rest of the coefficients can be obtained for the dyadic Green's function.

4.2.3. Source in the Last Layer

For the source to be located in the last layer (i.e., $S = N$), the coefficients are:

$$[\Upsilon_{lj',1N}] = \begin{bmatrix} A_{lj'}^{1N} & 0 \\ 0 & 0 \end{bmatrix}, \tag{50a}$$

$$[\Upsilon_{lj',mN}] = \begin{bmatrix} A_{lj'}^{mN} & 0 \\ A_{l,j'+2}^{mN} & 0 \end{bmatrix}, \tag{50b}$$

$$[\Upsilon_{lj',NN}] = \begin{bmatrix} 0 & 0 \\ A_{l,j'+2}^{NN} & 0 \end{bmatrix}. \tag{50c}$$

From the recurrence equation (39), similarly we have,

$$[\Upsilon_{lj',fN}] = [T_{lj',f-1}] \cdots [T_{lj',1}] [\Upsilon_{lj',1N}] - u(f-N) [U_N]. \tag{51}$$

By letting $f = N$, the coefficient for the first region is

$$A_{lj'}^{1N} = \frac{1}{T_{lj',11}^{(1)}}. \tag{52}$$

And for the last layer, it is found that

$$A_{l,j'+2}^{NN} = T_{lj',21}^{(1)} A_{lj'}^{1N}. \tag{53}$$

Similarly, we can obtain the rest of the coefficients by substituting (53) into (50) and (51).

We have now obtained a complete set of the dyadic Green's function in terms of the cylindrical vector wave functions for a gyroelectric chiral medium and their scattering coefficients in terms of compact matrices. Reduction can be made for formulating the dyadic Green's function in a less complex medium of specific planar geometries, e.g., an anisotropic medium where $\xi_c = 0$, a chiral medium where $g = 0$ and $\epsilon_z = \epsilon$, and an isotropic medium where $\xi_c = 0$, $g = 0$, and $\epsilon_z = \epsilon$.

5. CONCLUSION

In the present work, a correct form of dyadic Green's function in a unbounded gyroelectric chiral medium has been given in terms of the eigenfunction expansion of planar vector wave functions. The new form, which is not available elsewhere in the literature to the authors' knowledge, corrects the wrong ideas implemented inside the previous publication [38] which obviously employed an incorrect orthogonality and, therefore, ignored the coupling of the TE and TM modes as well as the coupling of solenoidal and non-solenoidal vector wave functions. As compared with those obtained using the Fourier transform technique for the same problem developed by Ali and Mahmoud [19], Lee and Kong [20–22], Krowne [23, 24], Monzon [25], and Habashy *et al.* [27], the dyadic Greens' function representation using the current expansion technique is easier and more direct to use for a source of isotropic independent azimuth direction and for obtaining far-zone field patterns. It is worth pointing out that the current form of dyadic Green's function for the unbounded gyroelectric chiral medium includes the irrotational dyadic which is contributed by the non-solenoidal vector wave functions, but were all missing in [32, 41–45]. This irrotational part has to be extracted out from the dyadic Green's function expression because the integrand function involved does not satisfy the Jordan lemma.

Moreover, the current work further develops a generalized dyadic Green's functions for a multilayered stratified gyroelectric chiral medium. The principle of scattering superposition is also utilized here for simplicity of the formulation. The formulation is carefully carried out by (1) decomposing the two cylindrical vector wave functions, namely $\boldsymbol{N}_n(h,\lambda)$ and $\boldsymbol{L}_{n,\lambda}(h)$, into the transverse and perpendicular directions, respectively; and (2) separating various modes corresponding to different wave numbers h_1, h_2, h_3, and h_4. The full eigenfunction expansion of the dyadic Green's functions in a multilayered stratified gyroelectric chiral medium is then proposed according to the multiple transmissions and reflections associated with these stratified interfaces. Applying the boundary conditions at each interface, the scattering coefficients of the dyadic Green's functions are obtained and represented in the form of compact recurrence matrices. As symbolic manipulations using Mathematica package can fail (basically run out of memory) due to the large number of layers involved, three specific cases are considered in the formulation of the scattering coefficients of DGFs, i.e., the source excitation located in the first, the intermediate and the last layer respectively.

As expected and mentioned before, the general representation of the dyadic Green's functions in the gyroelectric chiral medium can be reduced directly to those in a less complex medium of specific planar geometries, e.g., an anisotropic medium where $\xi_c = 0$, a chiral medium where $g = 0$ and $\epsilon_z = \epsilon$, and an isotropic medium where $\xi_c = 0$, $g = 0$, and $\epsilon_z = \epsilon$.

ACKNOWLEDGMENT

This work was supported in part by a Academic Research Fund grant RP3981676 from the Ministry of Education, Singapore.

APPENDIX A. INTEGRATION OF H

By performing the dh integral first,

$$I_1 = \int_{-\infty}^{\infty} dh \frac{f(h)e^{ih(z-z')}}{(h-h_1)(h-h_2)(h-h_3)(h-h_4)}. \tag{A1}$$

At this point, the integral in (A1) is undefined because of the existence of poles at $h = h_1$, h_2, h_3, and h_4. By introducing some loss, the integral is then well-defined. Consequently, if $z > z'$ and $f(h) \to 0$ when $h \to \infty$, the contour of the integration can be deformed from the real axis to the upper-half plane, thereby picking up a residue contribution of the poles located at $h = h_1$, and h_2.

Note that the integral on the deformed path (Figure A1) vanishes by virtue of Jordan's lemma. Therefore, by residue theorem calculus,

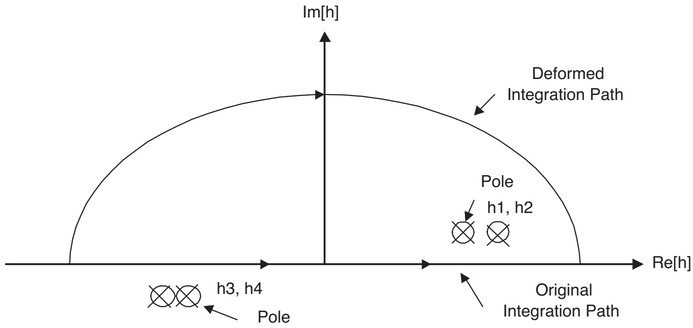

Figure A1. The original path of integration and the deformed path of integration on the complex h plane.

(A1) becomes:

$$I_1 = 2\pi i \sum_{j=1}^{2} (-1)^{j+1} \frac{f(h_j)}{(h_1 - h_2)(h_j - h_3)(h_j - h_4)} e^{ih_j(z-z')}, \; z > z' \quad \text{(A2)}$$

where $h_j^2 = k_{\lambda j}^2 - \lambda^2$. By the same token, a similar operation can be performed when $z < z'$. Therefore, (A1) is then given by

$$I_1 = 2\pi i \sum_{j=3}^{4} (-1)^{j+1} \frac{f(h_j)}{(h_3 - h_4)(h_1 - h_j)(h_2 - h_j)} e^{-ih_j(z-z')}, \; z < z' \quad \text{(A3)}$$

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