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EXCITATION AND RECEPTION OF ELECTROMAGNETIC, MAGNETOSTATIC AND SPIN WAVES IN FERRITE FILMS

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Abstract—The paper presents a new universal formulation of electromagnetic fields in microwave ferrite-dielectric waveguiding structures from the given magnetic field distribution of an external source. The solution is derived from the linearized Landau-Lifshits equation using an orthogonality condition for eigenwaves of a magnetization. The magnetization excited in ferrite is obtained in an electrodynamic, magnetostatic or dipole-exchange approximation, depending on the approximation used for eigenwaves. Results are applied for the formulation and the analytical solution of self-consistent electrodynamic problems of an excitation and a reception of waves in ferrite films by transmission lines of an arbitrary type. Numerical calculations are performed for filters and delay lines on the base of symmetric strip-lines using surface and forward volume magnetostatic waves in ferrite films.

1. INTRODUCTION

Magnetostatic waves are short and slow electromagnetic waves in ferrites whose propagation characteristics can be obtained on the base of magnetostatic equations, neglecting time derivatives in the Maxwell equations. Spin waves are short magnetostatic waves whose spectrum strongly depends on dipole-exchange interactions between electrons responsible for ferromagnetism. Different wave processes in ferrite-dielectric multi-layered structures, ferrite waveguides and ferrite films are of a great interest for practical applications and are intensively investigated during last years. Electromagnetic, magnetostatic and spin waves represent wave processes in ferrites having the same electromagnetic nature, however till the present time the problems of their excitation were considered independently (e.g., see [1–6]). This paper first presents a general theory of the excitation and the reception of waves of all mentioned types, which enables to obtain analytical closed form solutions for a wide class of complicated electrodynamic problems for ferrite-dielectric multi-layered structures.

The theory of the excitation of open waveguides is usually formulated for electric or equivalent magnetic currents with a given distribution. A well known solution of these electrodynamic problems is based on the modal expansion of fields into discrete and continuous spectra and on the application of the Lorentz's lemma [7]. However, in a number of cases a more preferable form of a solution would be the representation of the excited field through the known field of an external source. In the Section 2 we present such alternative formulation of the theory of an excitation, as applied to ferrite-dielectric waveguiding structures excited by a spatially localized magnetic field. Multi-layered structures with one-dimensional and two-dimensional cross-sections are considered for an arbitrary direction of a magnetizing field. The initial equations for the development of the theory are linearized Landau-Lifshits equations [1], introducing a relation between the excited magnetization in ferrite and the magnetic field, and also between the magnetization and the corresponding magnetic field for eigenwaves. The application of these equations instead of the Maxwell equations has certain advantages, since enables to obtain a more general solution and, in particular, to take into account an exchange interaction in final results by including the magnetic field of a dipole-exchange interaction into the effective magnetic field of eigenwaves. When constructing a solution, the eigenwaves of the magnetization are supposed to be known and should be obtained from an electrodynamic boundary problem for a multi-layered waveguiding structure in a corresponding approximation (electrodynamic, magnetostatic or dipole-exchange). Note that in the electromagnetic approximation dielectric properties of ferrite in this approach are taken into account indirectly, through the effective magnetic field.

The magnetization excited in a ferrite layer or in a waveguide is represented as an integral modal expansion in the two- or one-dimensional space of wave vectors. From the total magnetic field, the external field is excluded and the secondary field, induced by the magnetization, is represented as an integral expansion through the eigenwaves of the magnetization. By using an orthogonality condition for eigenwaves, the Landau-Lifshits equation for the total magnetic field is reduced to an infinite linear algebraic system for unknown coefficients of integral expansions. This system is then simply resolved in "the diagonal approximation", corresponding to the case of unbounded waves. Residues in the integral expansion for induced magnetic field in poles, located on the real axis, give propagating waves in ferrite, while the principal value of the integral, in the sum with the external field, comprises the near field of a source.

Results of the Section 2 hold for an arbitrary thickness of a ferrite layer and for an arbitrary cross section of a ferrite waveguide. The application of the proposed computation procedure to electrodynamic problems, for which solutions are known, demonstrates its efficiency and correctness. In particular, it gives very simple solutions, coinciding with known ones, for the problem of the excitation of a multi-layered ferrite-dielectric structure by the field of a thin wire with a given current distribution [8] (the electrodynamic approximation), and for the problem of the excitation of spin waves by a one-dimensional nonuniform high-frequency magnetic field in a ferrite film magnetized normally or tangentially [9] (the dipole-exchange approximation).

In the most of cases, the solution of the problem of the excitation of waves by given sources is only the first approximation of a more complicated electrodynamic problem. In fact, excited waves produce a "back" influence on a source (an exciting element) and its field can not be considered as given, i.e. an electrodynamic problem should be formulated and solved as self-consistent one. In practice, an exciting element can be usually treated as a section of a transmission line. In the Section 3, we demonstrate advantages of the new formulation of electromagnetic fields in ferrite waveguiding structures from the given magnetic field distribution of an external source, as applied to the analytical solution of electrodynamic problems of the excitation and the reception of waves in ferrite films by transmission lines of an arbitrary type. An initial electrodynamic problem of the excitation (reception) of waves is artificially decomposed into two problems, which primarily are considered to be independent ones. Thus, a self-consistent electrodynamic problem of the excitation (reception) of waves is formulated by a system of two singular integral equations. The first one comprises the representation of the magnetization induced by the given magnetic field of a transmission line, while the second equation, derived from the Lorentz's lemma, is the magnetic field of a transmission line excited by the magnetization of a ferrite film with the given distribution. This magnetic field is obtained in a one-mode approximation, using eigenwaves of a transmission line in the absence of a ferrite film. The proposed new technique, based on the artificial decomposition of the initial electrodynamic problem, enables to obtain analytical closed form solutions for self-consistent electrodynamic problems of the excitation and the reception of waves in ferrite films and to develop efficient models of devices using them. Results of the Section 3 hold for thin-ferrite-film structures with the cross section of a transmission line much larger, than that of a ferrite film. For a special case of surface magnetostatic waves this procedure leads to the earlier reported results [10].

The Section 4 presents the application of the developed unified approach for solving the problems of the excitation and the reception of different types of magnetostatic waves (MSW) in ferrite films (for classification of MSW see, e.g., [1, 2]). Numerical calculations are performed in the Subsection 4.1 for filters and delay lines on the base of symmetric strip-lines using surface magnetostatic waves (SMSW) and forward volume magnetostatic waves (FVMSW). Presented results give detailed understanding of the influence of peculiarities of the interaction of electromagnetic waves with MSW on the characteristics of devices and the dependence of their performance on different parameters of structures (in particular, the strip width, the distance from a ferrite film to the strip and to metal screens). Known models of MSW strip transducers [4] are limited by the case of their location directly on the surface of a ferrite film and do not take into account the influence of metal screens, that are usually used in practice.

In the Subsections 4.2 and 4.3 we give the analysis of the second order effects associated with the excitation of higher order electromagnetic modes in transmission lines and higher order magnetostatic modes in ferrite-thin-film waveguides. In the appendix there are presented analytical expressions for eigenwave functions of the magnetization for surface and forward volume magnetostatic waves, that were used in numerical calculations.

2. EXCITATION OF WAVES IN FERRITE LAYERS AND WAVEGUIDES BY HIGH-FREQUENCY MAGNETIC FIELD

Let a ferrite layer (a ferrite film) of the thickness d, magnetized to a saturation in an arbitrary direction by a uniform magnetic field, be excited by a magnetic field $\mathbf{h}$ of an external source with the frequency ω. High-frequency components of the excited magnetization $\mathbf{M}$ and of the total magnetic field $\mathbf{H}$ satisfy the linearized Landau-Lifshits equation [1]

$$\mathrm{M}_x = \chi \mathrm{H}_x + iv\mathrm{H}_y, \qquad \mathrm{M}_y = \chi \mathrm{H}_y - iv\mathrm{H}_x. \tag{1}$$

Components of the internal tensor of the magnetic susceptibility read

$$\chi = \frac{\Omega_H}{\Omega_H^2 - \Omega^2}, \qquad v = \frac{\Omega}{\Omega_H^2 - \Omega^2}, \tag{2}$$

where $\Omega_H = \omega_H/\omega_M$, $\Omega = \omega/\omega_M$, $\omega_H = \mu_0 \alpha H_i$, $\Omega_M = \mu_0 \alpha M_0$, μ_0 is the permeability of free space, α is the gyromagnetic ratio, H_i is the stationary magnetizing internal field in the direction of the z axis, M_0 is the saturation magnetization of ferrite.

Eigenwaves waves of the magnetization $\mathbf{m}_n$, which are determined by boundary conditions on the layer surfaces and are supposed to be known, and corresponding magnetic field also satisfy the relations

$$\mathrm{m}_{nx} = \chi_n \mathrm{h}_{nx} + iv_n \mathrm{h}_{ny}, \qquad \mathrm{m}_{ny} = \chi_n \mathrm{h}_{ny} - iv_n \mathrm{h}_{nx}, \tag{3}$$

where χ_n and v_n are parameters (2) taken at the eigenfrequencies of waves ω_n, which are determined by a dispersion equation for waves in the ferrite layer.

Let us find the magnetization excited in the layer as an integral modal expansion in the two-dimensional space of wave vectors in the plane of the layer

$$\mathbf{M} = \sum_n \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (c_n \mathbf{m}_n) d^2 k_n, \tag{4}$$

where $\mathbf{m}_n = \mathbf{m}_n^0(\eta) \exp(-i\mathbf{k}_n \mathbf{r})$, η is the axis normal to the layer with an origin on the bottom surface, $\mathbf{k}_n$ is a wave vector in the plane of the layer and $\mathbf{r}$ is a vector-radius. The sum is taken over the branches of the spectrum of waves (the sign of a sum in (4) can be omitted if there is only one propagating wave at the frequency of an external source).

The total high-frequency magnetic field in ferrite $\mathbf{H}$ is the sum of the given magnetic field of an external source and the field induced by the magnetization $\mathbf{M}$

$$\mathbf{H} = \mathbf{h} + \mathbf{h}_{\mathrm{ind}}. \tag{5}$$

Using eqns. (3) and taking into account a linearity of the electrodynamic problem, the induced field can be represented in the form

$$\begin{aligned} \mathrm{h}_{\mathrm{ind}\,x} &= \Omega_H \sum_n \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (c_n \mathrm{m}_{nx}) d^2\mathrm{k}_n - i \sum_n \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (c_n \Omega_n \mathrm{m}_{ny}) d^2\mathrm{k}_n, \\ \mathrm{h}_{\mathrm{ind}\,y} &= \Omega_H \sum_n \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (c_n \mathrm{m}_{ny}) d^2\mathrm{k}_n + i \sum_n \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (c_n \Omega_n \mathrm{m}_{nx}) d^2\mathrm{k}_n. \end{aligned} \tag{6}$$

Substituting eqns. (4)–(6) into (1), we multiply the first of eqns. (1) primarily by m_{py}^* and then by m_{px}^* and the second of eqns. (1) by m_{px}^* and m_{py}^*. Subtracting and adding up the resultant equations by twos, we then integrate them over an infinite volume V of the layer. In the result we come to a system of two integral equations, quadratic relatively components of the magnetization. By using the orthogonality conditions

$$\int_V \left(\mathrm{m}_n \mathrm{m}_p^*\right) dV = (2\pi)^2 \delta(\mathbf{k}_n - \mathbf{k}_p) \int_0^d \left(\mathrm{m}_n^0 \mathrm{m}_p^{0*}\right) d\eta, \tag{7}$$

which are valid for every component of the magnetization in eigenwaves and follow from the properties of the δ-function, we can reduce the system of integral equations to an infinite (in a general case) linear algebraic system for unknown coefficients c_n of the integral expansion

$$\sum_n c_n \Phi_{np}(\omega - \omega_n) = -i\omega_M \int\limits_V \left(\mathbf{h}\mathbf{m}_p^*\right) dV, \qquad (n, p = 1, 2, 3, \ldots), \quad (8)$$

where

$$\Phi_{np} = (2\pi)^2 \int\limits_0^d \left[\mathbf{m}_n^0 \times \mathbf{m}_p^{0*}\right]_z d\eta \qquad (9)$$

and all functions are taken at the wave vector $\mathbf{k}_p$. Note that the integrand in (9) represent a z-component of the cross product of vectors describing the variation of the magnetization in eigenwaves over a layer thickness. Neglecting non-resonant terms in the sums of eqns. (8), we can simply resolve this system in "the diagonal approximation" and obtain, after changing indexes p to n, the coefficients of the integral expansion

$$c_n = \varphi_n \int\limits_V (\mathbf{h}\mathbf{m}_n^*)\, dV, \qquad (10)$$

where

$$\varphi_n = \frac{i\omega_M}{\Phi_{nn}} \cdot \frac{1}{\omega_n - \omega}. \qquad (11)$$

A more accurate solution, which takes into account contributions of non-diagonal matrix elements in (8), can be obtained on the base of a perturbation theory [11]. In a number of practically important cases a solution (10) is rigorous one (e.g., for surface magnetostatic waves and for spin waves). This takes place when there is only one propagating wave at the frequency ω or eigenwave magnetization vectors form a closed set of orthogonal functions. Losses in ferrite can be easily taken into account phenomenologically by introducing complex eigenfrequencies $\omega_n = \omega_n' + i\omega_n''$ in (11).

Obtained solution (4), (10) gives propagating wave beams in a ferrite layer and a local magnetization in the vicinity of an external source and is characterized by a simplicity and a high genericity, since all peculiarities of the electrodynamic system are taken into account in eigenwaves of the magnetization $\mathbf{m}_n$ and in dispersion characteristics of waves $\omega_n(\mathbf{k}_n)$, while the used Landau-Lifshits equation is universal one. The solution holds for any layered waveguiding structure containing, in particular, multi-layered ferrite films, dielectric layers, metal screens. The excited magnetization is obtained in the electrodynamic, magnetostatic or dipole-exchange approximation depending on the approximation used for deriving eigenwave functions of the magnetization. In the case of multi-layered ferrite films eigenwave functions $\mathbf{m}_n$ correspond to collective waves of the magnetization which satisfy boundary conditions on surfaces of all layers with a different saturation magnetization. A high-frequency magnetization of each layer is given by the formula (4), while in solutions (10), (11) all the parameters should be taken for the saturation magnetization of a corresponding layer.

Consider an excitation of an infinite ferrite waveguide by a high-frequency magnetic field $\mathbf{h}$. A waveguide is supposed to be magnetized to a saturation in an arbitrary direction z of an internal stationary field. Let ξ be a longitudinal axis of the waveguide, while ζ and η are transverse cartezian coordinates. As earlier, eigenwaves of the magnetization $\mathbf{m}_n = \mathbf{m}_n^0(\zeta,\eta)\exp(-ik_n\xi)$ and dispersion characteristics of waves $\omega_n(k_n)$ are assumed to be known. Let us find the magnetization excited in the waveguide as the integral modal expansion in longitudinal wave vectors k_n

$$\mathbf{M} = \sum_n \int_{-\infty}^{+\infty} c_n \mathbf{m}_n dk_n, \tag{12}$$

where the sum is taken over the waveguide eigenwaves. Performing transformations analogues to that done for a ferrite layer, we come to the formula (10) for expansion coefficients c_n with the following new normalization coefficient instead of Φ_{nn} in (11)

$$\Phi_n = 2\pi \int_{S_\perp} \left[\mathbf{m}_n^0 \times \mathbf{m}_n^{0*}\right]_z dS_\perp, \tag{13}$$

where the integral is taken over the waveguide's cross section $S_\perp$. Residues of the intergrand (12) in poles, located on the real axis, give waves propagating along ξ-axis, while the principal value of the integral comprises a local magnetization. The magnetic field induced by this part of the magnetization, in the sum with the given field $\mathbf{h}$, represents the near field of a source. As it is seen from (10), the most effective excitation takes place for waves with the structure of the magnetization corresponding to the structure of the magnetic field $\mathbf{h}$.

The solution (12), (10), (13) holds also for the case of a transversally non-uniform internal magnetic field $\mathbf{H}_i(\zeta,\eta)$ and, correspondingly, for an arbitrary cross section of a ferrite waveguide. This can be easily proved by performing above transformations taking into account the dependence of parameters χ, v, Ω_H on the transverse coordinates ζ, η of the waveguide and assuming that in every point of the waveguide the stationary magnetization is equal to the saturation magnetization and is oriented in the direction of the stationary internal

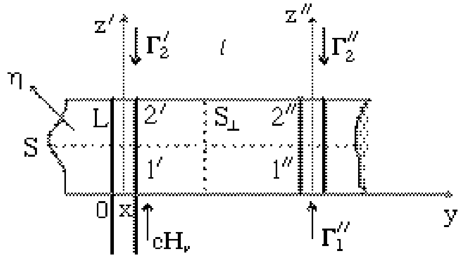

Figure 1. Filter (delay line) using a ferrite film.

magnetic field. An external magnetizing field for this case can be also transversally non-uniform one.

3. SELF-CONSISTENT ELECTRODYNAMIC PROBLEM OF EXCITATION AND RECEPTION OF WAVES IN FERRITE FILMS BY TRANSMISSION LINES OF AN ARBITRARY TYPE

Consider an electrodynamic system shown schematically in the Fig. 1, that is formed by two parallel transmission lines of an arbitrary type with longitudinal axes z' and z'' coupled to each other through a ferrite film which is infinite along the y axis and is located between the cross sections $z' = z'' = 0$ and $z' = z'' = L$. Such structure is usually referred as a magnetostatic wave filter or a delay line in which transmission lines (transducers) realize the function of the transformation of electromagnetic waves in feeding lines into magnetostatic waves propagating in a ferrite film. The port $1'$ is the input port of the device, x is the axis orthogonal to the film surface, $S_\perp$ and S are cross and longitudinal sections of the film, l is the distance between the axes z' and z''. A ferrite film is magnetized to the saturation in an arbitrary direction η of an internal field.

Let in transmission lines forming input and output transducers there can propagate (in the absence of a ferrite film) only fundamental electromagnetic modes of the types correspondingly v and μ with the propagation constants γ and β and the electromagnetic fields

$$\mathbf{E}_{\pm v} = \mathbf{E}_{\pm v0}(x,y)e^{\mp i\gamma z'}, \qquad \mathbf{H}_{\pm v} = \mathbf{H}_{\pm v0}(x,y)e^{\mp i\gamma z'}, \tag{14}$$

$$\mathbf{E}_{\pm \mu} = \mathbf{E}_{\pm \mu0}(x,y)e^{\mp i\beta z''}, \qquad \mathbf{H}_{\pm \mu} = \mathbf{H}_{\pm \mu0}(x,y)e^{\mp i\beta z''}. \tag{15}$$

Input and output transducers are loaded by arbitrary loadings giving in the sections $z' = L,\ z'' = 0,\ z'' = L$ reflection coefficients for transverse components of magnetic fields of transmission lines correspondingly Γ'_2, Γ''_1, Γ''_2 (here and thereafter introduced values are assigned with indexes that correspond to the notations of the device ports in the Fig. 1). The magnetic field of a falling electromagnetic wave of the fundamental v-mode at the port $1'$ in the section $z' = 0$ is $c\mathbf{H}_v$. Eigenwaves of the magnetization $\mathbf{m}_n = \mathbf{m}_n^0(x, z)\exp(-ik_n y)$ and dispersion characteristics of waves in a ferrite film $\omega_n(k_n)$ are assumed to be known.

Let us formulate a self-consistent electrodynamic problem of the excitation of waves of an arbitrary type in a ferrite film by the input transducer. According to (12), (10) the magnetization excited in a ferrite waveguide by the given high-frequency magnetic field $\mathbf{h}_v$ can be written down in the form

$$\mathbf{M}' = \sum_n \int_{-\infty}^{+\infty} c_n \mathbf{m}_n dk_n, \qquad c_n = \varphi_n \int_V (\mathbf{h}_v \mathbf{m}_n^*)\, dV, \tag{16}$$

where

$$\varphi_n = \frac{i\omega_M}{\Phi_n} \cdot \frac{1}{\omega_n - \omega}, \tag{17}$$

$$\Phi_n = 2\pi \int_{S_\perp} \left[\mathbf{m}_n^0 \times \mathbf{m}_n^{0*}\right]_\eta dS_\perp. \tag{18}$$

We shall treat the field $\mathbf{h}_v$ as the magnetic field of the input transducer in the locus of the ferrite film. For taking into account the "back" influence of the excited magnetization on the transducer, this field should be represented through the given high-frequency magnetization of the film, thus deriving a system of equations of a self-consistent electrodynamic problem. A required representation can be obtained on the base of the theory of an excitation of waveguides by given electric and magnetic currents [7]. The field $\mathbf{h}_v$ consists of the field $c\mathbf{H}_v$ of the electromagnetic wave falling at the port $1'$ and of the field excited by the ferrite film (with the reflection from a load at the port $2'$ being taken into account). The latter can be found as the field of a transmission line excited by the equivalent given magnetic current $i\omega\mu_0\mathbf{M}'$ locally excluded from the section under consideration. In the case when ferrite fills a small part of the transmission line cross section (that takes place in the most of cases when ferrite films are used), a one mode approximation for this field can be applied. In these assumptions we can write down

$$\mathbf{h}_v = [c + c_v(z')]\mathbf{H}_v + \left\{c_{-v}(z') + [c + c_v(L)]\Gamma'_2 e^{-2i\gamma L}\right\}\mathbf{H}_{-v}, \tag{19}$$

where

$$c_v(z') = -\frac{i\omega\mu_0}{N_v} \int\limits_0^{z'} dz' \int\limits_S (\mathbf{M}'\mathbf{H}_{-v}) dS,$$
$$c_{-v}(z') = -\frac{i\omega\mu_0}{N_v} \int\limits_{z'}^{L} dz' \int\limits_S (\mathbf{M}'\mathbf{H}_{v}) dS, \tag{20}$$

$$N_v = \int\limits_S \{[\mathbf{E}_{-v} \times \mathbf{H}_v] - [\mathbf{E}_v \times \mathbf{H}_{-v}]\}_{z'} \, dS. \tag{21}$$

The normalization coefficient N_v is four times greater than the power transmitted by the fundamental mode through a cross section of the line [7].

The system of singular integral equations (16), (19), (20) for unknowns $\mathbf{M}'$ and $\mathbf{h}_v$ (the singularity arises in φ_n at $\omega_n = \omega$) is a mathematical formulation of the self-consistent electrodynamic problem of the excitation of waves in a ferrite film by a transmission line of an arbitrary type. It is convenient to take the amplitudes of electromagnetic waves in the transmission line $c_v(z')$ and $c_{-v}(z')$ as new independent variables. In this case the system of integral equations can be comparatively easy resolved, that is caused by an integral character of these variables.

Not limiting the general character of a consideration, we assume the magnetic fields of transmission lines $\mathbf{H}_v$ and $\mathbf{H}_\mu$ to be transverse. In addition, for transmission lines without losses $\mathbf{H}_{v0}$ and $\mathbf{H}_{\mu0}$ in (14), (15) can be taken as real functions. In this case, according [7], one can assume that $\mathbf{H}_{v0} = \mathbf{H}_{-v0}$, $\mathbf{H}_{\mu0} = \mathbf{H}_{-\mu0}$. Then the reflection coefficient at the input port $1'$ at the section $z' = 0$ is the following

$$\Gamma_1' = \left\{ c_{-v}(0) + \Gamma_2' e^{-2i\gamma L} \left[c + c_v(L) \right] \right\} / c, \tag{22}$$

and the input impedance normalized to the characteristic impedance of the transmission line forming a transducer (in the absence of a ferrite film) can be calculated as

$$Z_1' = (1 - \Gamma_1')/(1 + \Gamma_1'). \tag{23}$$

The transmission coefficient for magnetic fields of falling waves from the port $1'$ to the port $2'$ reads

$$D_2' = [c + c_v(L)] e^{-i\gamma L}/c. \tag{24}$$

The value $|D_2'|^2$ is the power transmission coefficient for falling waves of a band-stop filter formed by the transmission line coupled to a ferrite film (port $2'$ is considered in this case as output one).

In the further consideration we assume that eigenwave functions of the magnetization $\mathbf{m}_n$ do not depend of the coordinate z'. In the Subsection 4.3 it is proved that the variation of the high-frequency magnetization over the width of a ferrite film does not change results significantly for a fundamental waveguide mode. Performing the integration over the longitudinal coordinate z' in the integral representations of the amplitudes of electromagnetic waves $c_v(z')$ and $c_{-v}(z')$ in (20), we can obtain the following functional dependence

$$c_v(z') = \left(e^{i\gamma z'} - 1\right) c_{v0}, \qquad c_{-v}(z') = \left(e^{-i\gamma z'} - e^{-i\gamma L}\right) c_{v0}. \tag{25}$$

Substituting the solution in the form (25) into the system of singular integral equations (16), (19), (20), we reduce the latter to the linear algebraic system for the unknown constant c_{v0}. In the result, we can find

$$c_{v0} = \frac{c\left(1 + \Gamma_2' e^{-i\gamma L}\right)\left(e^{-i\gamma L} - 1\right)}{2\left(i\gamma L + e^{-i\gamma L} - 1\right) + \Gamma_2'\left(1 - e^{-i\gamma L}\right)^2 - \gamma^2 F_v^{-1}}, \tag{26}$$

where the following notations are introduced:

$$F_v = \frac{i\omega\mu_0}{N_v} \sum_n \int_{-\infty}^{+\infty} \varphi_n |I_{vn}|^2 dk_n, \tag{27}$$

$$I_{vn} = \int_S \left(\mathbf{H}_{v0}\mathbf{m}_n^*\right) dS. \tag{28}$$

From the solution (25), (26) it follows that in the self-consistent problem the interaction of a transducer with a ferrite film is completely characterized by the complex parameter F_v. Hereafter this parameter will be referred as the parameter of coupling, while the value I_{vn} — as the integral of an excitation. The real part $\mathrm{Re}F_v$, which is given by semi-residues of the integrand in (27), determines the coupling of a transducer with propagating waves of the magnetization, while the imaginary part $\mathrm{Im}F_v$, equal to the principle value of the integral, describes the coupling with the local magnetization in the vicinity of a transducer. The case $F_v \equiv 0$ corresponds to the problem of the excitation of a ferrite film by the given field of a transducer, when the "back" reaction of the magnetization on a transducer is neglected.

Let us find amplitudes of excited waves in a ferrite film. Performing the integration in (16), we take into account that the function $\varphi_n(k_n)$ has on the integration path over k_n the poles $k_{n\omega+}$

and $k_{n\omega-}$, which correspond to the fulfilment of the resonant condition of the excitation of waves propagating in the positive and negative directions of the axis y. Positions of poles on the axis of the integration are determined by the solution of the dispersion equation $\omega_n(k_n) = \omega$ and in a general case are not symmetric relatively the imaginary axis, i.e., $|k_{n\omega+}| \neq |k_{n\omega-}|$. For instance, this takes place for a tangentially magnetized (along z') ferrite film with different boundary conditions on its surfaces (e.g., in the presence of metal screens). When losses in ferrite are introduced, mentioned poles displace in the complex plane from the real axis: $|k_{n\omega-}|$ - upward and $|k_{n\omega+}|$ - downward. This means that, when integrating, the poles in the left complex semi-plane should be rounded from below and the poles in the right semi-plane — from above. Residues in (16) can be easily calculated using the expansion of the denominator of the function φ_n (17) into the Taylor series in the vicinity of poles

$$\omega_n - \omega \approx \frac{\partial \omega_n}{\partial k_n}(k_n - k). \tag{29}$$

In the result, we find the amplitudes of waves excited by the transducer in the ferrite film:

$$\begin{aligned} c_n \;=\; & -2c\frac{\pi\omega_M}{|V_{n\omega}|} \cdot \frac{I_{vn}(k_{n\omega})}{\Phi_n(k_{n\omega})} \\ & \cdot \frac{\left(e^{-i\gamma L} - 1\right)\left(1 + \Gamma_2' e^{-i\gamma L}\right)}{i\gamma - iF_v\gamma^{-1}\left[2(i\gamma L + e^{-i\gamma L} - 1) + \Gamma_2'\left(1 - e^{-i\gamma L}\right)^2\right]}, \end{aligned} \tag{30}$$

where $V_{n\omega} = \left.\frac{\partial \omega_n}{\partial k_n}\right|_{\omega_n=\omega}$ is the group velocity of a wave (subscripts $\pm$ indicating the direction of the propagation of waves are omitted). Note that the amplitude of each mode depends on the coupling of the transducer with all other modes, since the parameter of coupling F_v includes the sum over the whole spectrum of waves.

Assuming $\Gamma_1' = 0$ in (22), it is not difficult to obtain the optimum value of the parameter of coupling at which the transducer is matched at the section $z' = 0$ as

$$F_{v\,\mathrm{opt}} = \frac{\gamma^2}{2\left(1 + i\gamma L - e^{i\gamma L}\right) - \left(e^{i\gamma L} - 1\right)^2 / \Gamma_2'}. \tag{31}$$

For a reactive load at the second end of the transducer, this case corresponds to the full transformation of the input power into the power transmitted by excited waves in the ferrite film. Note that the amplitudes of waves c_n (30), besides F_v, also depend on the integrals of an excitation I_{vn}, therefore in a general case the relation (31) may not coincide with the condition of achieving the maximum value of the amplitude $|c_n|$ for corresponding mode.

Let us analyze the realizability of the optimum coupling in the sense (31). Calculating semi-residuals in (27), we assume the same paths of rounding the poles $k_{n\omega+}$, $k_{n\omega-}$, as in the computation of the amplitudes c_n. In the result, taking into account that the normalization coefficient Φ_n is a pure imaginary quantity, real and imaginary parts of the parameter of coupling can be obtained as

$$\mathrm{Re}F_v = \frac{i\omega\mu_0}{N_v}\pi\omega_M\left(\sum_n \frac{|I_{vn}(k_{n\omega+})|^2}{|V_{n\omega+}|\Phi_{n\omega+}} + \sum_n \frac{|I_{vn}(k_{n\omega-})|^2}{|V_{n\omega-}|\Phi_{n\omega-}}\right), \quad (32)$$

$$\mathrm{Im}F_v = \frac{i\omega\mu_0\omega_M}{N_v}\sum_n \mathrm{V}\cdot\mathrm{p}\cdot\int_{-\infty}^{+\infty}\frac{|I_{vn}(k_n)|^2}{\Phi_n(k_n)}\cdot\frac{1}{\omega_n(k_n)-\omega}dk_n. \quad (33)$$

Both real and imaginary parts of the parameter of coupling depend on the integral of an excitation I_{vn} and therefore are not independent. Consequently, the system of equations (31), defining their optimum values, does not always have solutions. The physical realizability of the optimum coupling depends on the type of waves propagating in the ferrite film and its parameters, and also on the type and the length of the transmission line. In practice, the integral of an excitation I_{vn}, which in a considerable degree determines the values $\mathrm{Re}F_v$ and $\mathrm{Im}F_v$, can be easily changed by approaching or moving away the transducer from the ferrite film. Note that the integrand in (33) changes the sign in the interval of the integration, therefore the principle value of the integral will monitonically change from positive to negative values in the frequency band in which waves in the ferrite film exist. Consequently, the frequency dependent condition of the optimum coupling (31) can be achieved (if only possible) at the only one frequency of this band.

In the case of a small length of the transducer short circuited at the end (in practice for $\gamma L \leq 0.5$), the optimum value of the parameter of coupling appears to be a real quantity equal to

$$F_{v\,\mathrm{opt}} \approx 1/(2L^2). \quad (34)$$

In the same approximation $\gamma L \ll 1$ the relationship (30) can be rewritten in the form

$$c_n \approx 2c\frac{\pi\omega_M}{|V_{n\omega}|}\cdot\frac{I_{vn}(k_{n\omega})}{\Phi_n(k_{n\omega})}\cdot\frac{L(1+\Gamma_2')}{1+F_vL^2(1+\Gamma_2')}, \quad (35)$$

from which it follows that the short open circuited transducer ($\Gamma_2' = -1$) practically does not excite waves in the film, while the effectiveness of the excitation of waves by the shortcircuited transducer ($\Gamma_2' = 1$) always higher than for the transducer matched at the second end ($\Gamma_2' = 0$).

Let us write down equations of the self-consistent electrodynamic problem of the reception of waves by the output transducer. Denote as $\mathbf{M}_{\text{in}}$ the high-frequency magnetization of the ferrite film in the locus of the output transducer, which corresponds to propagating waves excited by the input transducer. Then, using arguments analogous to that put forward for the input transducer, the self-consistent problem of the reception of waves can be mathematically formulated in the following way:

$$\mathbf{M}'' = \sum_n \int_{-\infty}^{+\infty} d_n \mathbf{m}_n dk_n + \mathbf{M}_{\text{in}}, \qquad d_n = \varphi_n \int_V (\mathbf{h}_\mu \mathbf{m}_n^*)\, dV, \tag{36}$$

$$\mathbf{h}_\mu = c_\mu(z'')\mathbf{H}_\mu^{\text{r}} + c_{-\mu}(z'')\mathbf{H}_{-\mu}^{\text{r}}, \tag{37}$$

where

$$\begin{aligned} c_\mu(z'') &= -\frac{i\omega\mu_0}{N_\mu^{\text{r}}} \int_0^{z''} dz'' \int_S \left(\mathbf{M}''\mathbf{H}_{-\mu}^{\text{r}}\right) dS, \\ c_{-\mu}(z'') &= -\frac{i\omega\mu_0}{N_\mu^{\text{r}}} \int_{z''}^{L} dz'' \int_S \left(\mathbf{M}''\mathbf{H}_{\mu}^{\text{r}}\right) dS, \end{aligned} \tag{38}$$

$$\mathbf{H}_\mu^{\text{r}} = \mathbf{H}_\mu + \Gamma_2''\mathbf{H}_{-\mu}, \qquad \mathbf{H}_{-\mu}^{\text{r}} = \mathbf{H}_{-\mu} + \Gamma_1''\mathbf{H}_\mu, \qquad N_\mu^{\text{r}} = (1 - \Gamma_1''\Gamma_2'')N_\mu. \tag{39}$$

In the eqn. (36) the magnetization $\mathbf{M}''$, exciting the output transducer, is the sum of the magnetization excited by the transducer itself and the magnetization $\mathbf{M}_{\text{in}}$ corresponding to waves propagating from the input transducer from the section $y = 0$ to the section $y = l$, with the phase shift and fading being taken into account,

$$\mathbf{M}_{\text{in}} = \sum_n c_n \mathbf{m}_n^0 \exp\left(-ik_{n\omega+}l - k'_{n\omega+}l\right). \tag{40}$$

Here $k'_{n\omega+}$ is the imaginary part of the wavenumber at the frequency ω which takes into account losses in ferrite and is equal to [1]

$$k'_{n\omega} = \alpha\Delta H_k / V_{n\omega}, \tag{41}$$

where ΔH_k is the half-linewidth of the ferromagnetic resonance (in a general case depending on the wavenumber k_n). The field of the output transducer in the ferrite film $\mathbf{h}_\mu$ (37) is written down on the base of the theory of the excitation of resonators [7], since the output transducer has arbitrary loads both at $z'' = 0$ and at $z'' = L$. The reflection coefficient Γ_2'' from the load at the second end of the transducer in the formula (39), unlike (19), is reduced to the coordinate origin $z'' = 0$ according to the following relation

$$\Gamma_2''\Big|_{z''=0} = \Gamma_2''\Big|_{z''=L} e^{-2i\beta L}. \tag{42}$$

Thus, for instance, for a transducer shortcircuited at the second end we have in (39) $\Gamma_2'' = \exp(-2i\beta L)$.

Resolving the system of singular integral equations (36), (37) relatively unknown functions $c_\mu(z'')$ and $c_{-\mu}(z'')$ with the same method, as for the input transducer, we obtain the following solution

$$\begin{aligned} c_\mu(z'') &= \left[\left(e^{i\beta z''} - 1\right) - \Gamma_1''\left(e^{-i\beta z''} - 1\right)\right] c_{\mu 0}, \\ c_{-\mu}(z'') &= \left[\left(e^{-i\beta z''} - e^{-i\beta L}\right) - \Gamma_2''\left(e^{i\beta z''} - e^{i\beta L}\right)\right] c_{\mu 0}, \end{aligned} \tag{43}$$

where

$$c_{\mu 0} = -\frac{i\omega\mu_0}{N_\mu} \cdot \frac{\sum_n c_n I_{\mu n}^* \exp(-ik_{n\omega+}l - k'_{n\omega+}l)}{i\beta(1 - \Gamma_1''\Gamma_2'') + 2F_\mu s}, \tag{44}$$

$$s = l_1 + \Gamma_1'' l_2 + \Gamma_2'' l_2 e^{2i\beta L} - \Gamma_1''\Gamma_2'' l_3, \tag{45}$$

$$l_1 = L + \frac{1}{i\beta}(e^{-i\beta L} - 1), \quad l_2 = \frac{1}{2i\beta}(e^{-i\beta L} - 1)^2, \quad l_3 = L - \frac{1}{i\beta}(e^{i\beta L} - 1). \tag{46}$$

The parameter of coupling F_μ and the integral of an excitation $I_{\mu n}$ are introduced for the output transmission line similarly to (27), (28), with the substitution of the subscript μ instead of v, while the normalization coefficient N_μ for the μ-mode is defined for this line, as in (21).

Assuming a matched load at the port $1''$ ($\Gamma_1'' = 0$), we find the amplitude of the electromagnetic wave $c_1''\mathbf{H}_{-\mu}$ falling at the output of the structure at the section $z'' = 0$ in the form

$$c_1'' = \frac{i\omega\mu_0}{N_\mu} \cdot \frac{(e^{-i\beta L} - 1)(1 + \Gamma_2'' e^{-i\beta L}) \sum_n c_n I_{\mu n}^*(k_{n\omega+}) e^{-(ik_{n\omega+} + k'_{n\omega+})l}}{i\beta - iF_\mu \beta^{-1}\left[2(i\beta L + e^{-i\beta L} - 1) + \Gamma_2''(1 - e^{-i\beta L})^2\right]}. \tag{47}$$

The comparison of this relation with (30) shows, that the functional dependence of the transducer's effectiveness from its length and loading is the same in transmission and reception modes of operation and, in particular, above conclusions concerning the effectiveness of the excitation of waves by short transducers hold also for the reception of waves.

The transmission coefficients for the magnetic field of waves falling at the ports $1''$ and $2''$ at the sections $z''=0$ and $z''=L$ in a general case are

$$D_1'' = \frac{c_1''(0)H_{-\mu 0}}{cH_{v0}} = \frac{c_{-\mu}(0)H_{\mu 0}}{cH_{v0}}, \tag{48}$$

$$D_2'' = \frac{c_2''(L)H_{\mu 0}}{cH_{v0}}e^{-i\beta L} = \frac{c_{\mu}(L)H_{\mu 0}}{cH_{v0}}e^{-i\beta L}. \tag{49}$$

From this relations and (43) it follows, that the transmission coefficient does not depend on what port ($1''$ or $2''$) is considered as output one (for the same loadings at the opposite ports). If different types of transmission lines are used as input and output transducers, the computation of the transmission coefficient for falling waves requires the following normalization

$$K_{1\mathrm{p}} = D_{\mathrm{p}}''\frac{H_{v0}}{H_{\mu 0}}\sqrt{\frac{N_\mu}{N_v}}, \qquad (\mathrm{p}=1,2). \tag{50}$$

The squared absolute value of this coefficient is equal to the transmission power coefficient for falling waves, while its argument gives the phase shift between magnetic field of falling waves in feeding transmission lines. This transmission power coefficient (in dB)

$$b_{1\mathrm{p}} = 20\lg|K_{1\mathrm{p}}|, \qquad (\mathrm{p}=1,2). \tag{51}$$

is equal to the insertion losses of the device with the opposite sign.

Using (50), (48), (43), the transmission coefficient for falling waves from the port $1'$ to the port $1''$ can be obtained in a general case, as follows

$$K_{11} = -q_v q_\mu \sqrt{\frac{N_\mu}{N_v}} \cdot \frac{2\pi\omega_M \dfrac{i\omega\mu_0}{N_\mu}\displaystyle\sum_n \left(\frac{I_{vn}I_{\mu n}^*}{V_n\Phi_n}e^{-ik_n l - k_n' l}\right)_{\omega+}}{\left[i\beta(1-\Gamma_1''\Gamma_2'')+2sF_\mu\right](i\gamma+pF_v)}, \tag{52}$$

where the values marked by the index $\omega+$ are taken at $k_{n\omega+}$ and parameters

$$q_v = \left(1-e^{-i\gamma L}\right)\left(1+\Gamma_2' e^{-i\gamma L}\right), \tag{53}$$

$$q_\mu = \left(1-e^{-i\beta L}\right)\left(1+\Gamma_2'' e^{i\beta L}\right), \tag{54}$$

$$p = (i\gamma)^{-1}\left[2\left(i\gamma L + e^{-i\gamma L} - 1\right) + \Gamma_2'\left(1 - e^{-i\gamma L}\right)^2\right] \tag{55}$$

depend only on the electrical lengths and the loads of transducers. Note that in (52) there are taken into account both the interaction of waves with transducers and the interference of different modes at the output transducer. The transmission coefficient of the device in the reverse direction, from the port $1''$ to the port $1'$, is obtained from (52) by substituting the index μ instead of v and $k_{n\omega-}$ instead $k_{n\omega+}$ for corresponding values. In the case of the identical short transducers ($\gamma = \beta,\ \gamma L \ll 1$) and the matched output port $1''$ ($\Gamma_1'' = 0$) from (52), in particular, we have

$$K_{11} = D_1'' \approx -\frac{2\pi\omega\omega_M\mu_0}{N_\mu}\cdot\frac{L^2(1+\Gamma)^2}{\left[1+F_\mu L^2(1+\Gamma)\right]^2}$$
$$\sum_n \frac{i\left|I_{\mu n}(k_{n\omega+})\right|^2}{V_{n\omega+}\Phi_n(k_{n\omega+})}\exp\left[-(ik_{n\omega+}+k'_{n\omega+})l\right], \tag{56}$$

where $\mu \equiv v$ and $\Gamma \equiv \Gamma_2' \equiv \Gamma_2''$ is the reflection coefficient from the load at the second end, identical for both transducers and reduced to the section $z' = z'' = L$. Since the normalization coefficient Φ_n is a pure imaginary quantity, from (56) it follows that, besides the phase delay of a signal $k_{n\omega+}l$ associated with the propagation of waves from the input transducer to output one, there is an additional phase shift in the device caused by the reactive local field of transducers, that is described by the imaginary part of the parameter of coupling $\mathrm{Im}F_\mu$. In the case of the optimum coupling (34), the parameter of coupling F_μ is a real quantity and for the real value of the reflection coefficient Γ this additional phase shift vanishes. Using (56), one can also come to a practically important conclusion about the existence of the optimum coupling of transducers to the ferrite film, which provides the maximum transmission coefficient of the device. This is conditioned by the fact, that F_μ includes the second power of the absolute value of the integral of an excitation and, consequently, the denominator (56) contains its fourth power, while the numerator is proportional to its second power. The corresponding optimum parameter of coupling in a general case does not coincide with (34), although appears to be close to this value. The analysis of the relations (56) and (32) shows, that this is caused by the change in the correlation between the amplitudes of different excited modes and between the amplitudes of the same modes propagating in the opposite directions with the change of the coupling of transducers to the film. In practice, this means that minimum losses in the device do not always correspond to the minimum wave standing ratio at its input. Note that this phenomenon becomes more apparent with the increase of the thickness of the ferrite film.

In the case, when feeding transmission lines of the device and transmission lines forming the transducers have different characteristic impedances, the resultant transmission coefficient for falling waves should take into account these step discontinuouses and can be obtained as the product

$$\tilde{K}_{1\mathrm{p}} = K_{1\mathrm{st}} K_{1\mathrm{p}} K_{2\mathrm{st}}, \qquad (\mathrm{p} = 1, 2), \tag{57}$$

where $K_{1\mathrm{st}}$ and $K_{2\mathrm{st}}$ are the transmission coefficients for the magnetic field of falling waves of the step connections of transmission lines at the input and at the output. These coefficients are given by the following relations which are analogous to that obtained in [12] for filters with spherical ferrite resonators:

$$K_{1\mathrm{st}} = \sqrt{R}\frac{1 + Z_1'}{R + Z_1'}, \tag{58}$$

$$K_{2\mathrm{st}} = \frac{2\sqrt{R}}{1 + R}, \tag{59}$$

where $R = \tilde{W}/W$, $\tilde{W}$ is the characteristic impedance of feeding transmission lines, W is the characteristic impedance of lines forming transducers (in the absence of the ferrite film), Z_1' is the input impedance at the section $z' = 0$ normalized to W and calculated through the reflection coefficient according (23). Using (58), we can write down

$$K_{1\mathrm{st}} = \frac{2\sqrt{R}}{(R + 1) + \Gamma_1'(R - 1)}, \tag{60}$$

where the reflection coefficient is determined by the formula (22).

4. APPLICATION OF THE THEORY TO MAGNETOSTATIC WAVE FILTERS AND DELAY LINES

In this section we demonstrate the application of the above developed general theory to filters and delay lines using surface and volume magnetostatic waves and numerically evaluate assumed approximations. The investigation of electrodynamic characteristics of devices in a frequency band and their dependence on the parameters of structures is presented. Obtained results can be used for the optimization of mentioned devices according their function.

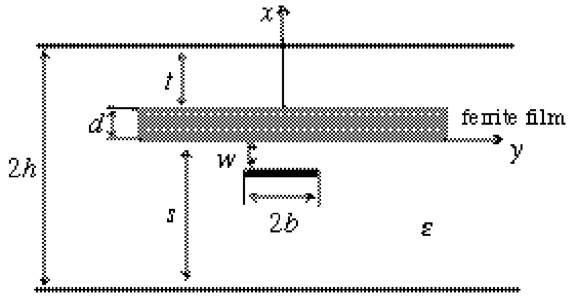

Figure 2. Cross-section of a strip-line MSW transducer.

4.1. Characteristics of MSW Filters and Delay Lines with Strip-Line Transducers

Numerical calculations were performed for a MSW filter (a delay line) with transducers on the base of a symmetrical strip-line with the dielectric filling with the permittivity ε (Fig. 2). Input and output transducers were supposed to be identical and connected to feeding transmission lines having the characteristic impedance $\tilde{W} = 50\,\text{Ohms}$. Results of numerical calculations presented in the figures below correspond to shortcircuited (at the second ends) transducers and to the following set of the parameters of the structure: $h = 0.5\,\text{mm}$, $b = 75\,\mu\text{m}$, $d = 15\,\mu\text{m}$, $w = 15\,\mu\text{m}$, $L = 4\,\text{mm}$, $l = 5\,\text{mm}$, $Z_2' = 0$, $Z_2'' = 0$, $\tilde{W} = 50\,\text{Ohms}$, $M_0 = 140 \cdot 10^3\,\text{A/m}$, $H_i = 89.2 \cdot 10^3\,\text{A/m}$, $\Delta H_0 = 20\,\text{A/m}$, $\varepsilon = 10$. Variable parameters are indicated in figures and captions. Amplitude-frequency responses (frequency dependences of the transmission power coefficients for falling waves) were calculated using formulas (51), (52), (57). Eigenwave functions of the magnetization used in the computation are presented in the appendix. The electromagnetic field of a strip-line was used in the form obtained in [13] .

Consider, at first, characteristics of devices using surface magnetostatic waves. In a general case amplitude-frequency responses (AFR) of filters (delay lines) have a typical form of alternating maximums and minimums (Fig. 3), the number of which in the frequency band of existence of waves depends on the strip width and the thickness of the ferrite film. For the case of the strip located directly on the film surface, frequencies of minimums approximately coincide with the frequencies at which an integer number of wavelengths go into the strip width. Frequency positions of AFR maximums and the slope of curves are mainly determined by the frequency dependences of the

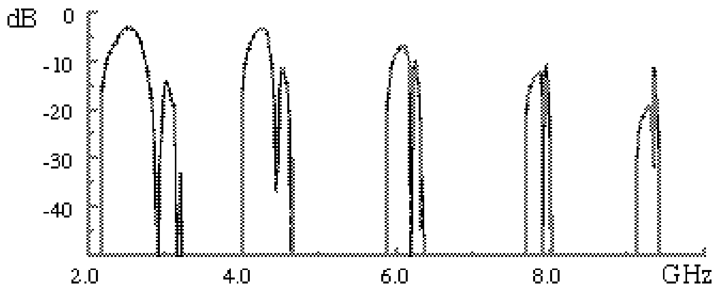

Figure 3. AFR of the SMSW filter (delay line) under the variation of a magnetizing field ($H_i = 24 \cdot 10^3,\ 64 \cdot 10^3,\ 112 \cdot 10^3,\ 160 \cdot 10^3,\ 200 \cdot 10^3$ A/m).

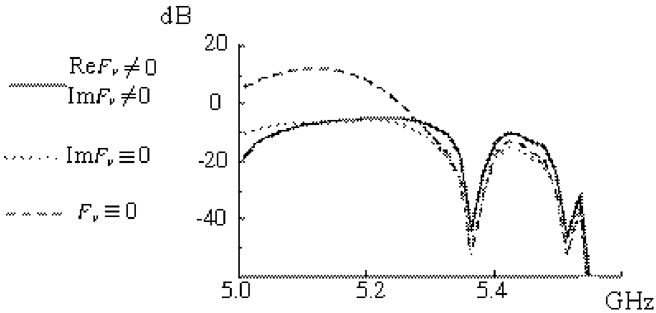

Figure 4. AFR of the filter (delay line) calculated in different approximations.

integral of an excitation and the group velocity of excited waves.

In the Fig. 4 we present the comparison of amplitude-frequency responses (in the frequency band of existence of waves) calculated using the present theory (solid curve) and other approximations. The calculation in the approximation of the given field ($F_v \equiv 0$) results in the transmission coefficient exceeding unity in the region of the main maximum of the AFR, that contrary to the law of conservation of energy. This means that correct computations of characteristics of MSW filters and delay lines can be performed only on the base of a self-consistent electrodynamic problem. The case Im$F_v \equiv 0$, which corresponds to taking into account the "back" influence only of propagating waves on transducers, does not change the transmission coefficient significantly. However, with the increase of a magnetizing field the difference also increases. At rather high frequencies, taking into account the "back" influence of the local magnetization on

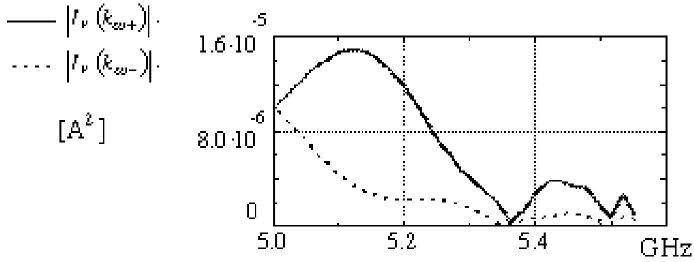

Figure 5. Integrals of excitation for surface waves propagating in the opposite directions.

transducers becomes essential, since this interaction causes the growth of the transmission coefficient in high order maximums, which can prevail over the main maximum (Fig. 3).

The value of the integral of an excitation $|I_v(k_{\omega+})|$ for waves propagating in the direction of the output transducer is significantly larger, than the integral $|I_v(k_{\omega-})|$ for waves propagating in the opposite direction (Fig. 5). This is the consequence of the well known property of nonreciprocity of surface waves, which localize near different surfaces of a ferrite film depending on the direction of propagation and therefore differently interact with transducers. The frequency position of minimums of the integral of an excitation $|I_v(k_{\omega+})|$ coincides with the position of AFR minimums (Fig. 4). Because of the significant difference in values of the integrals of an excitation $|I_v(k_{\omega+})|$, $|I_v(k_{\omega-})|$, the frequency dependence of the real part of the parameter of coupling ReF_v (32) is determined mainly by the value $|I_v(k_{\omega+})|$. Imaginary part of this parameter ImF_v, which characterizes the interaction of transducers with the local magnetization of the ferrite film, changes its sign within the frequency band of existence of waves and amounts to the absolute values close to the real part of the parameter of coupling (Fig. 6). This means that at corresponding frequencies contributions of propagating waves and of the local magnetization are comparable. In the region of the main maximum of the AFR the imaginary part of the parameter of coupling is close to zero, while in the higher order maximums it is negative.

The frequency dependence of the real and imaginary parts of the input impedance of the filter (delay line) Z_{in} at the section $z' = 0$ in the feeding transmission line, normalized to the characteristic impedance of this line (50 Ohms), is presented in the Fig. 7. Frequencies, at which the real part of the input impedance is close to zero correspond to the minimums of the AFR (Fig. 4). At lower frequencies of the band the reactive part of the input impedance has a capacitive character, while

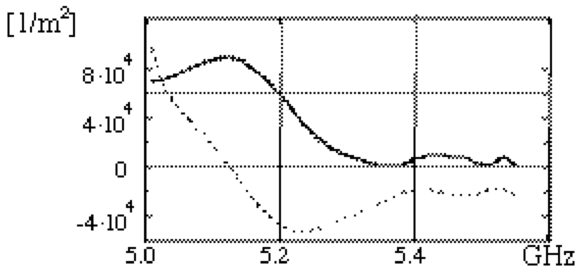

Figure 6. Real (solid curve) and imaginary (dotted curve) parts of the parameter of coupling.

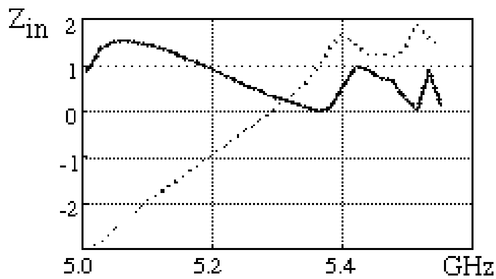

Figure 7. Real (solid curve) and imaginary (dotted curve) parts of the input impedance.

at the upper frequencies it is inductive one. As the analysis shows, the frequency dependence of the input impedance considerably depends on the interaction of transducers with the local magnetization. From Fig. 7 it follows that insertion losses of the filter (delay line) in the main maximum of the AFR for the assumed parameters of the structure are mainly caused by the reflection of the power from the input port.

The phase shift in the filter (delay line) is summed up of the phase shift $k_{n\omega+}l$, associated with propagation of waves from the input transducer to output one, and the additional phase shift $\vartheta(\omega)$ caused by the processes of the excitation and the reception of waves and the excitation of the local magnetization in the vicinity of transducers. In the Fig. 8 the frequency dependence of the mentioned additional phase shift is shown. In the region of the main maximum of the AFR ϑ increases with frequency from 0 to π. Because of the interaction of the transducers with the local magnetization, the phase shift ϑ in the high order maximums of the AFR appears to be close to $\pi/2$ (the solid curve for $\mathrm{Im}F_v \neq 0$), while it is close to zero (the dotted curve for

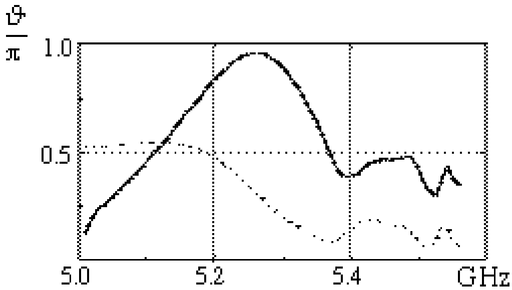

Figure 8. Additional phase shift ϑ for two approximations: taking into account $\mathrm{Im}F_v$ (solid curve) and neglecting $\mathrm{Im}F_v$ (dotted curve).

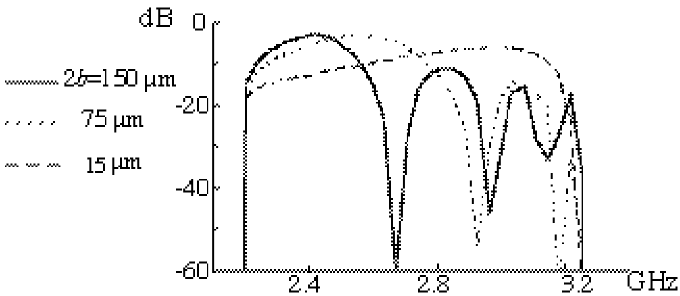

Figure 9. AFR of the filter (delay line) for different widths $2b$ of the strip ($H_i = 24 \cdot 10^3$ A/m).

$\mathrm{Im}F_v \equiv 0$) when this interaction is neglected.

In the Fig. 9 amplitude-frequency responses of the filter (delay line) in the frequency band of existence of waves are shown for different widths of the strip. With the decrease of the width of the strip, the pass-band (at the level 3 dB) increases and high order maximums of the AFR vanish. Minimum insertion losses (for the assumed lengths of transducers $L = 4$ mm) change insignificantly. The AFR of the filter (delay line) in the frequency band of existence of waves are presented in the Fig. 10 for different distances w between transducers and the ferrite film. With the decrease of the coupling of transducers to the film, the pass-band (at the level 3 dB) decreases and additional high order maximums of the AFR appear, that is caused by the growth of the spatial region of uniformity of the magnetic field of the strip-line with moving away form the strip. This effect is analogous to the increase of the width of the strip for a constant distance to the ferrite film. The level of high order maximums decreases with the frequency,

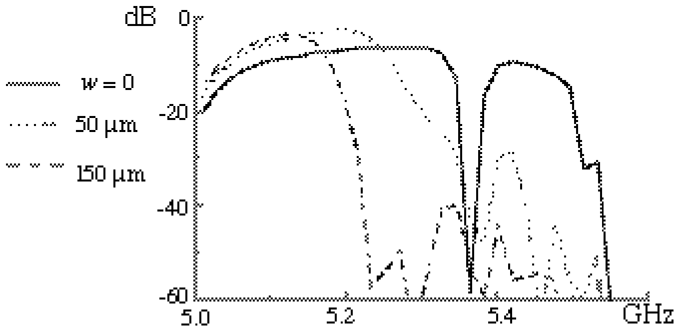

Figure 10. AFR of the SMSW filter (delay line) for different distances w between transducers and a ferrite film.

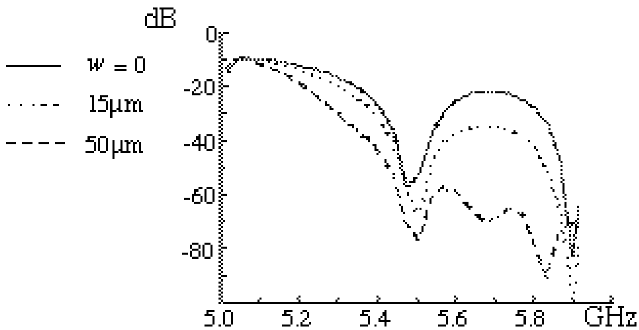

Figure 11. AFR of the FVMSW filter (delay line) for different distances w between transducers and a ferrite film.

at that more significantly for a low coupling of transducers to the film. For every frequency there exists an optimum distance between transducers and the ferrite film, at which insertion losses are minimum.

Above rules of the formation of characteristics of magnetostatic surface wave filters and delay lines are in the main correct also for volume waves, although for this case there are some distinctive peculiarities. As an example, in the Fig. 11 we show the amplitude-frequency responses of forward volume magnetostatic wave filter (delay line) with the variation of a distance between transducers and the ferrite film. Other parameters of the structure are the same, as in the case of SMSW. As it is seen from the Fig. 11, with moving away the ferrite film from transducers, losses in the filter (delay line) increase at all frequencies of the band in which waves exist. The peculiarity of the characteristics in the Fig. 11 is that the optimum value of the

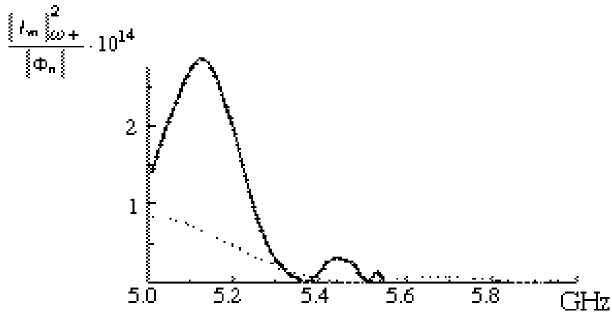

Figure 12. Normalized integrals of excitation for SMSW (solid curve) and FVMSW (dotted curve) for $w = 0$.

parameter of coupling (31) does not achieved even for $w = 0$, because the integral of an excitation for volume waves is much smaller than for surface waves for the same conditions of the excitation of waves. Also, the excitation of higher modes of FVMSW influences on the form of frequency responses. For comparison, in the Fig. 12 there are presented normalized values of the integrals of an excitation for SMSW and for a fundamental mode of FVMSW for identical transducers and for equal low frequencies of the band of existence of waves. Parameters of the structures coincide with given above for SMSW. The necessity of the normalization is caused by the fact that eigenwave functions of the magnetization in the integral of an excitation are different for SMSW and FVMSW. Note that the width of maximums in the frequency dependence of the integral of an excitation and, correspondingly in the AFR, in the case of FWMSW appears to be larger than in the case of SMSW because of the higher slope of a dispersion characteristic.

4.2. Influence of Higher Modes of Transmission Line on Amplitudes of Excited Magnetostatic Waves

The theory of the excitation and the reception of waves in ferrite films by transmission lines developed in the Section 3 is based on the one-mode approximation (19), (37) of the magnetic field of a line. Let us solve the electrodynamic problem of the excitation of waves taking into account higher modes of the transmission line forming a transducer. As earlier, assume that there is only one propagating mode in the transmission line, that is a fundamental mode. For this case, taking v as the index of the summation, the second equation of the self-consistent electrodynamic problem (19) can be written down in the form

$$\mathbf{h} = \left(c\mathbf{H}_v + c\Gamma'_{2v} e^{-2i\gamma_v L}\mathbf{H}_{-v}\right)\Big|_{v=1} + \sum_{v=1}\left[c_v(z')\mathbf{H}_v + c_{-v}(z')\mathbf{H}_{-v} + c_v(L)\Gamma'_{2v} e^{-2i\gamma_v L}\mathbf{H}_{-v}\right], \quad (61)$$

where Γ'_{2v} is the reflection coefficient from a load at the second end of the transducer for transverse components of the magnetic field of the mode v and amplitudes of electromagnetic waves $c_v(z')$ and $c_{-v}(z')$ are determined by eqns. (20). The first equation of the self-consistent problem is given by the formula (16) with the substitution of $\mathbf{h}$ (61) instead of $\mathbf{h}_v$. Resolving the system of singular integral equations similarly to the Section 3, we obtain

$$c_v(z') = \left(e^{i\gamma_v z'} - 1\right) c_{v0}, \qquad c_{-v}(z') = \left(e^{-i\gamma_v z'} - e^{-i\gamma_v L}\right) c_{v0}, \quad (62)$$

where

$$c_{v0} = -\frac{\omega\mu_0}{\gamma_v N_v}\sum_n \int_{-\infty}^{+\infty} c_n I^*_{vn} dk_n, \quad (63)$$

γ_v is the propagation constant of an electromagnetic wave of the type v and the integral of an excitation I_{vn} coincides with (28). Substituting c_n from (16), we reduce the integral equation (63) to the infinite system of linear algebraic equations for unknown coefficients c_{v0}

$$\sum_\mu c_{\mu 0}\left(B_{\mu v} - \delta_{\mu v}\right) = cA_{1v}, \quad (64)$$

where $\delta_{\mu v}$ is the Kronecker symbol and the index of the summation μ takes the same values, as v. Elements of the matrix and the column of absolute terms are as follows

$$B_{\mu v} = \frac{i}{\gamma_v} c_{\mu 0} F_{\mu v}\left[2L + \frac{2}{i\gamma_\mu}\left(e^{-i\gamma_\mu L} - 1\right) + \frac{\Gamma'_{2\mu}}{i\gamma_\mu}\left(e^{-i\gamma_\mu L} - 1\right)^2\right], \quad (65)$$

$$A_{1v} = \frac{c}{\gamma_1\gamma_v}\left(e^{-i\gamma_1 L} - 1\right)\left(1 - \Gamma'_{2\Gamma} e^{-2i\gamma_1 L}\right) F_{1v}, \quad (66)$$

where the parameter of coupling is

$$F_{\mu v} = \frac{i\omega\mu_0}{N_v}\sum_n \int_{-\infty}^{+\infty} \varphi_n I_{\mu n} I^*_{vn} dk_n. \quad (67)$$

With the increase of the number $\mu(v)$ the propagation constant $\gamma_\mu(\gamma_v)$ increases. Since the normalization coefficient N_v is proportional to γ_v, we have $F_{\mu v} \sim 1/\gamma_v$ and the matrix elements $B_{\mu v}$ decrease, with moving away from the diagonal, faster than $1/\gamma_v^2$. This enables to neglect non-diagonal elements and to solve the system of equations (64) approximately. The solution for c_{v0} in such "diagonal" approximation coincides with earlier obtained one (26), with the substitution of γ_v instead of γ and Γ'_{2v} instead of Γ'_2.

Substituting (61) into (16), we find in the result the amplitudes of waves excited in the film as

$$c_n = c_n^{(1)} + c_n^{(\mathrm{hm})}, \tag{68}$$

where $c_n^{(1)}$ is the amplitude of the wave (30) in the one-mode approximation ($v = 1$) and

$$c_n^{(\mathrm{hm})} = \frac{2\pi\omega_M}{\Phi_n |V_{n\omega}|} \sum_{v>1} c_{v0} I_{vn} \left[2L + \frac{2}{i\gamma_v}\left(e^{-i\gamma_v L} - 1\right) + \frac{1}{i\gamma_v}\left(e^{-i\gamma_v L} - 1\right)^2 \Gamma'_{2v}\right] \tag{69}$$

is the correction term caused by higher-order modes of the transmission line.

The numerical calculation of (68) was performed for the case of surface magnetostatic waves in a ferrite film and a transducer on the base of a symmetrical strip-line with the same parameters of the structure, as are given in the previous subsection. When 50 higher-order modes of the strip-line are taken into account, the correction term (69) does not exceed 5% to the one-mode approximation (30) in the whole frequency band. This estimate enables to make a conclusion about the applicability of the one-mode approximation of the field for modeling of magnetostatic wave devices.

4.3. Influence of Width Modes of Ferrite Film on Amplitudes of Excited Magnetostatic Waves

The equations (16), (19), formulating the self-consistent problem, in the most of cases can be resolved analytically for higher-order magnetostatic modes which have a variation of the magnetization over the width of a ferrite film. Dispersion characteristics of magnetostatic waves in ferrite-thin-film waveguides are usually calculated using approximate boundary conditions of "magnetic walls" or of spin-pinning at the side walls of the waveguide [1, 2]. Assume that dispersion characteristics of magnetostatic waves in the ferrite waveguide are known and take the eigenwaves of the magnetization in the form

$$\mathbf{m}_n = \mathbf{m}_n^0(x) \cdot e^{-ik_n y} \cdot \sin(\chi_n z), \qquad n = 1, 2, 3, \ldots \tag{70}$$

where χ_n are transverse wavenumbers which for spin-pinning boundary conditions at $z = 0$ and at $z = L$ (Fig. 1) are equal to [14]

$$\chi_n = n\pi/L. \tag{71}$$

The solution of the system of singular integral equations (16), (19), for the case when the coupling of modes in the waveguide is neglected, can be obtained for eigenwave functions (70) as

$$c_v(z') = \sum_n \left[e^{i\gamma z'} \cdot \left(\frac{i\gamma}{\chi_n} \sin \chi_n z' - \cos \chi_n z' \right) + 1 \right] \cdot c_{vn}^0 \tag{72}$$

$$c_{-v}(z') = \sum_n \left[e^{-i\gamma z'} \cdot \left(\frac{i\gamma}{\chi_n} \sin \chi_n z' + \cos \chi_n z' \right) - (-1)^n e^{-i\gamma L} \right] \cdot c_{vn}^0 \tag{73}$$

where

$$c_{vn}^0 = c \cdot \tag{74}$$

$$\frac{\left[1 - e^{-i\gamma L}(-1)^n\right] + \Gamma_2' e^{-i\gamma L} \left[e^{-i\gamma L} - (-1)^n\right]}{2[e^{-i\gamma L}(-1)^n - 1] - \Gamma_2'[e^{-i\gamma L} - (-1)^n]^2 - i\gamma L\left(1 - \frac{\gamma^2}{\chi_n^2}\right) - F_{vn}^{-1}\chi_n^2\left(1 - \frac{\gamma^2}{\chi_n^2}\right)^2}.$$

Here, unlike (27), the parameter of coupling

$$F_{vn} = \frac{i\omega\mu_0}{N_v} \int_{-\infty}^{+\infty} \varphi_n |I_{vn}|^2 dk_n \tag{75}$$

does not contain the summation over n and in the calculation of the integral of an excitation

$$I_{vn} = \int_S \mathbf{H}_{v0} \cdot \mathbf{m}_n^{0*}(x) e^{ik_n y} dS \tag{76}$$

a functional dependence of eigenwave functions of the magnetization from the transverse coordinate z is excluded.

Calculating the residuals of the integrand in the expansion (16), we obtain the amplitudes of propagating magnetostatic waves as

$$c_n = 2\pi \frac{\omega_M}{|V_{n\omega}|\Phi_n(k_{n\omega})} \int_V (\mathbf{h}_v \cdot \mathbf{m}_n^*)\, dV. \tag{77}$$

Substituting in (77) the field $\mathbf{h}_v$ (19), with the amplitudes of electromagnetic waves (72), (73), we finally come to the following result

$$c_n = 2c\chi_n \frac{\pi\omega_M}{|V_{n\omega}|} \cdot \frac{I_{vn}(k_{n\omega})}{\Phi_n(k_{n\omega})} \cdot \rightarrow \tag{78}$$

$$\cdot \frac{\left[1 - e^{-i\gamma L}(-1)^n\right] + \Gamma_2' e^{-i\gamma L}\left[e^{-i\gamma L} - (-1)^n\right]}{\left(\begin{array}{l} \chi_n^2 - \gamma^2 + 2F_{vn}\left\{i\gamma\frac{L}{2} - \frac{\chi_n^2}{\chi_n^2 - \gamma^2}\left[e^{-i\gamma L}(-1)^n - 1\right]\right\} \\ \qquad\qquad\qquad + F_{vn}\Gamma_2' \cdot \frac{\chi_n^2}{\chi_n^2-\gamma^2}\left[e^{-i\gamma L} - (-1)^n\right]^2 \end{array}\right)}.$$

The structure of this expression for amplitudes of excited waves is identical to the formula (30). The difference is in the terms containing transverse wavenumbers χ_n and the index of summation n. This causes a significant peculiarity of the solution (78), which consists in the appearance of transverse waveguide resonances, when the propagation constant of an electromagnetic wave in the transmission line forming the transducer is equal to the transverse wavenumber of a magnetostatic wave: $\gamma = \chi_n$. Under this condition, amplitudes of waves excited by a shortcircuited transducer turn out to be equal to zero for a real value of the propagation constant γ. If losses in the transmission line are taken into account, i.e. for a complex value of γ, amplitudes c_n are not equal to zero, but are extremely small.

Consider the peculiarities of the excitation of width modes by transducers of a small electrical length, when the condition $\gamma L \ll 1$ is fulfilled. In this approximation and for a fundamental mode ($n = 1,\ \chi_1 = \pi/L,\ \gamma \ll \chi_1$), from (78) we have the following expression

$$c_1 \approx 2c\frac{\pi\omega_M}{|V_{1\omega}|} \cdot \frac{I_{v1}(k_{1\omega})}{\Phi_n(k_{1\omega})} \cdot \frac{\frac{2}{\pi} \cdot \Gamma(1 + \Gamma_2')}{1 + \frac{4}{\pi^2} \cdot F_{v1}L^2(1 + \Gamma_2')}, \tag{79}$$

which differs from (35) only by the multipliers $2/\pi$ in the numerator and $4/\pi^2$ in the denominator. In the result, the amplitude of the fundamental excited mode c_1 turns out to be close to that obtained for the model considered in the Section 3. Consequently, all the rules of the formation of amplitude-frequency responses of MSW filters and delay lines, discussed in the Section 3, can be extended on the case of the excitation of a fundamental mode in thin-ferrite-film waveguides. The formulas (52), (57) for transmission coefficients for falling waves also hold for this case. Note that for all odd modes ($n = 2p + 1,\ p = 0, 1, 2, \ldots$) the dependence of amplitudes c_n from the reflection coefficient Γ_2' of a load at the second end of the transducer turns out to be the same, as for the fundamental mode.

For the second mode ($n = 2,\ \chi_2 = 2\pi/L,\ \gamma \ll \chi_2$) in the same approximation $\gamma L \ll 1$ from (78) one can obtain

$$c_2 \approx 2c\frac{\pi\omega_M}{|V_{2\omega}|} \cdot \frac{I_{v2}(k_{2\omega})}{\Phi_2(k_{2\omega})} \cdot \frac{\dfrac{1}{2\pi} \cdot i\gamma L^2(1-\Gamma_2')}{1 + iF_{v2}\left(\dfrac{L}{2\pi}\right)^2 3\gamma L}, \tag{80}$$

This expression significantly differs from (35). The dependence of c_2 from Γ_2' turns out to be reverse to the case of odd modes and identical for all even modes ($n = 2p,\ p = 1, 2, 3, \ldots$). In particular, as it follows from (80), a shortcircuited transducer does not excite even modes.

5. CONCLUSION

We have suggested a new simple solution of electrodynamic problems of the excitation of ferrite waveguiding structures, having the same form for electromagnetic, magnetostatic and spin waves. It enables to determine the magnetization and, correspondingly, the electromagnetic field excited in ferrite by the high-frequency magnetic field of an external source. The solution holds for an arbitrary direction of a magnetizing field, for an arbitrary thickness of a ferrite layer and for an arbitrary cross section of a ferrite waveguide. A high degree of universality is achieved by using, in the construction of the solution, the Landau-Lifshits equation instead of the equations of electrodynamics. Using this solution, there were formulated and analytically solved electrodynamic problems of the excitation and the reception of all types of waves in ferrite films by transmission lines of an arbitrary type. It was shown that such electrodynamic problems should be solved fundamentally in a self-consistent formulation. Closed form expressions were obtained for transmission coefficients of band-pass and band-stop filters and delay lines using ferrite films. The parameter of coupling which completely characterizes the interaction of a transmission line with a ferrite film was introduced and its optimum value for matching the devices was found. In a general case this value does not coincide with the parameter of coupling providing minimum insertion losses in a filter or a delay line. It was established that a phase shift in a filter (in a delay line) is determined not only by the propagation of waves from the input to the output transmission line, but also by the processes of the excitation and the reception of waves.

A general theory was applied to magnetostatic wave filters and delay lines. Numerical calculations were performed for cases of surface and forward volume magnetostatic waves and strip-line transducers. The frequency dependence of the introduced parameters characterizing coupling of transducers to a ferrite film was explored, as well the dependence of the amplitude-frequency responses of filters and delay lines on the parameters of their structure. It was shown that minimum insertion losses in devices can be achieved in the case of surface magnetostatic waves by putting a ferrite film at the optimum distance from transducers, while for the case of forward volume magnetostatic waves an optimum coupling can not be achieved. It was proved that the developed general theory, which uses a one mode approximation for the electromagnetic field in a transmission line and a fundamental mode in a ferrite film, holds for the case when ferrite fills a small part of the transmission line cross section, that takes place for the most of magnetostatic wave devices.

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APPENDIX A. EIGENWAVES OF MAGNETIZATION FOR MAGNETOSTATIC WAVES

Below are presented analytical expressions for eigenwave functions of the magnetization for surface and forward volume magnetostatic waves propagating along the y axis (Fig. 2)

$$\mathbf{m}_n = \mathbf{m}_n^0(x, z)\exp(-ik_n y) \tag{A1}$$

and their normalization coefficients Φ_n (18). The solutions were obtained using equations of magnetostatics and boundary conditions on the surfaces of a ferrite film and on perfectly conducting metal screens. Arbitrary dimensional multipliers in the eigenwave functions are omitted. In the case of SMSW a magnetizing field is oriented along the z axis and for FVMSW — along the x axis. Dispersion characteristics of waves are given in [2].

A.1. Surface Magnetostatic Waves (SMSW)

$$m_{nx}^0 = k_n \cdot \left[\begin{array}{l} -(\omega_M + (\omega_H + \omega_n)(1 - \tanh k_n s)) \cdot e^{-k_n x} \\ +(\omega_M(\omega_H + \omega_n)(1 + \tanh k_n s)) \cdot e^{k_n x} \end{array} \right], \tag{A2}$$

$$m_{ny}^0 = -ik_n \cdot \left[\begin{array}{l} (\omega_M + (\omega_H + \omega_n)(1 - \tanh k_n s)) \cdot e^{-k_n x} \\ +(\omega_M + (\omega_H + \omega_n)(1 + \tanh k_n s)) \cdot e^{k_n x} \end{array} \right], \tag{A3}$$

$$\Phi_n = 2\pi i k_n L \cdot \left\{ \begin{array}{l} [\omega_M + (\omega_H + \omega_n)(1 - \tanh k_n s)]^2 \cdot \left(e^{-2k_n d} - 1\right) \\ +[(\omega_M + (\omega_H + \omega_n)(1 + \tanh k_n s))]^2 \cdot \left(e^{2k_n d} - 1\right) \end{array} \right\}. \tag{A4}$$

A.2. Forward Volume Magnetostatic Waves (FVMSW)

$$m^0_{nz} = -k_n v \cdot \left[\cos(\eta k_n x) + \frac{\tanh(k_n s)}{\eta} \sin(\eta k_n x)\right], \tag{A5}$$

$$m^0_{ny} = -ik_n \chi \cdot \left[\cos(\eta k_n x) + \frac{\tanh(k_n s)}{\eta} \sin(\eta k_n x)\right], \tag{A6}$$

$$\begin{aligned} \Phi_n = {} & 4\pi i L v k_n^2 \chi \cdot \left[\frac{\sin(2\eta k_n d)}{4k_n \eta}\left(1 - \frac{\tanh(k_n s)^2}{\mu}\right)\right. \\ & \left. + \frac{\tanh(k_n s)^2}{2\mu} d + \frac{d}{2} + \frac{\tanh(k_n s)}{k_n \mu} \sin(\eta k_n d)^2\right], \end{aligned} \tag{A7}$$

where

$$\eta = \sqrt{-\mu} = \sqrt{\frac{(\omega_H^2 + \omega_M \omega_H - \omega_n^2)}{\omega_n^2 - \omega_H^2}}. \tag{A8}$$

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