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COMBINED WIENER-HOPF AND PERTURBATION ANALYSIS OF THE H-POLARIZED PLANE WAVE DIFFRACTION BY A SEMI-INFINITE PARALLEL-PLATE WAVEGUIDE WITH SINUSOIDAL WALL CORRUGATION

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Abstract—The diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation is analyzed for the H-polarized plane wave incidence using the Wiener-Hopf technique combined with the perturbation method. Introducing the Fourier transform for the unknown scattered field and applying an approximate boundary condition together with a perturbation series expansion of the scattered field, the problem is formulated in terms of the zero-order and first-order simultaneous Wiener-Hopf equations. The Wiener-Hopf equations are solved via the factorization and decomposition procedure leading to the exact solutions. Explicit expressions of the scattered field inside and outside the waveguide are derived analytically by taking the inverse Fourier transform and applying the saddle point method. Far field scattering characteristics of the waveguide are discussed in detail via representative numerical examples.

1. INTRODUCTION

The analysis of the wave scattering by gratings and periodic structures is important in electromagnetic theory and optics. Various analytical and numerical methods have been developed thus far and the diffraction phenomena have been investigated for a number of periodic structures. Most efficient among a number of analysis methods for scattering problems involving periodic structures are, the

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Riemann-Hilbert problem technique [1–3], the analytical regularization methods [3–5], the Yasuura method [6–8], the integral and differential method [9], the point-matching method [10], and the Fourier series expansion method [11, 12].

The Wiener-Hopf technique [13–16] is known as a rigorous, function-theoretic approach for analyzing wave propagation and scattering problems related to canonical geometries, and can be applied efficiently to the analysis of the diffraction by specific periodic structures. There are significant contributions to the analysis of the diffraction by gratings based on the Wiener-Hopf technique [17–21]. In the previous papers [22–25], we have considered several transmission-type gratings and analyzed the plane wave diffraction rigorously by means of the Wiener-Hopf technique. It is to be noted, however, that the analysis presented in most of the above-mentioned Wiener-Hopf papers is restricted to periodic structures with infinite extent and plane boundaries.

Das Gupta [26] showed that some class of diffraction problems involving periodic structures with curved boundaries could be analyzed approximately by the Wiener-Hopf technique. He developed a hybrid method based on the Wiener-Hopf technique and the perturbation method, and solved the diffraction by a half-plane with sinusoidal corrugation approximately for the H-polarized plane wave incidence. The method presented in [26] has been subsequently generalized by Chakrabarti and Dowerah [27] for the analysis of the H-polarized plane wave diffraction by two parallel half-planes with sinusoidal corrugation. We have carefully investigated the results obtained by Chakrabarti and Dowerah [27], and found that their analysis is incorrect since important contributions to the scattered field are not taken into account. In addition, no numerical results are presented in their paper. In [28], we have generalized the method developed by Das Gupta from different aspects, and analyzed the E-polarized plane wave diffraction by a finite sinusoidal grating (a strip with sinusoidal corrugation), where the far field scattering characteristics have been investigated in detail via numerical computation.

In our previous paper [29], we have considered the same geometry as in [27], and analyzed the E-polarized plane wave diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation using the Wiener-Hopf technique together with the perturbation method. Our approach established in [29] is different from the method developed by Chakrabarti and Dowerah [27], and is rigorous in the sense that all kinds of scattering effects are explicitly taken into account. We have also carried out numerical computation and investigated in detail on how the sinusoidal corrugation of the waveguide walls affects the far field scattering characteristics. In this paper, we shall consider the same waveguide geometry as in [29], and analyze the H-polarized plane wave diffraction with the aid of the method similar to that established in our previous paper [29] for the E-polarized plane wave incidence. As mentioned above, this problem was solved in the past by Chakrabarti and Dowerah [27] following the hybrid approach based on the Wiener-Hopf technique and the perturbation method, but their analysis is incomplete and incorrect from a mathematical point of view. We shall derive in this paper various new expressions of the scattered field inside and outside the waveguide analytically via a rigorous approach and provide a number of numerical examples on the far field intensity for detailed investigation on the scattering characteristics.

Section 2 presents the statement of the problem. Assuming that the corrugation amplitude of the waveguide walls is small compared with the wavelength, the original problem is approximately replaced by the problem of the H-polarized plane wave diffraction by a flat, semi-infinite parallel-plate waveguide with an impedance-type boundary condition. We also expand the unknown scattered field using a perturbation series and separate the diffraction problem under consideration into the zero-order and the first-order boundary value problems. In Section 3, the Fourier transform for the scattered field is introduced, and the problem is formulated in terms of the zero- and first-order simultaneous Wiener-Hopf equations by applying the boundary conditions in the Fourier transform domain. It should be emphasized that the problem formulation procedure presented in Sections 2 and 3 is newly developed in this paper and is different from our previous analysis for the E-polarized plane wave incidence [29]. In [29], we have first formulated the problem in terms of the simultaneous Wiener-Hopf equations by using the approximate boundary condition and then separated them into the zero- and first-order equations. The method developed in this paper provides a more systematic way leading to the zero- and first-order Wiener-Hopf equations, in comparison to our previous paper [29]. In Section 4, we shall apply the factorization and decomposition procedure [13–16] to obtain the exact solutions of the zero- and first-order Wiener-Hopf equations. It is verified that the zero-order equation provides the exact solution to the H-polarized plane wave diffraction by a flat, semi-infinite parallel-plate waveguide, whereas the solution of the first-order equation corresponds to the correction term due to the corrugation of the waveguide walls. The subsequent Section 5 discusses the derivation of the zero- and first-order scattered fields inside and outside the waveguide. The field inside the waveguide is expressed in terms of orthogonal mode expansions by application of the residue theorem, whereas the field outside the waveguide is evaluated by using the saddle point method together with the pole-singularity extraction procedure leading to the far field asymptotic expressions uniformly valid in incidence and observation angles. The scattered field expressions obtained in Section 5 are entirely new results and are not found in the paper by Chakrabarti and Dowerah [27]. Again it is to be noted that the results obtained by Chakrabarti and Dowerah are incorrect as important contributions to the scattered field are not taken into account in their analysis. In Section 6, we shall present numerical examples on the far field intensity for various physical parameters, and discuss the scattering characteristics of the waveguide in detail. In particular, the effect of the sinusoidal corrugation of the waveguide walls will be clarified, and some comparisons with our previous analysis for the E-polarized case [29] will be shown.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. STATEMENT OF THE PROBLEM

We consider the diffraction of an H-polarized plane wave by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation as shown in Fig. 1, where the waveguide walls are assumed to be infinitely thin, perfectly conducting, and uniform in the y-direction, being defined by

$$x = \pm b + h \sin mz \quad (m > 0, \quad h > 0) \tag{1}$$

for $z < 0$. In view of the waveguide geometry and the characteristics of the incident field, this is a two-dimensional problem.

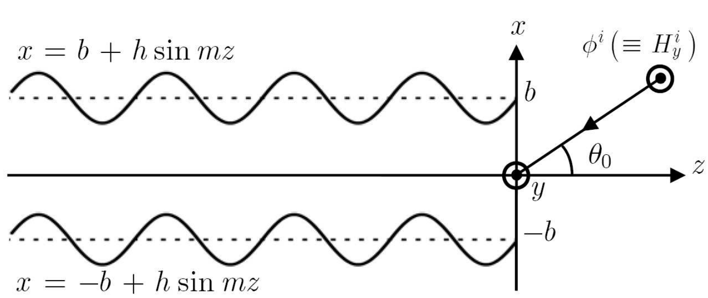

Figure 1. Geometry of the problem.

Let us define the total magnetic field $\phi^t(x,z)[\equiv H_y^t(x,z)]$ by

$$\phi^t(x,z) = \phi^i(x,z) + \phi(x,z), \tag{2}$$

where $\phi^i(x,z)$ is the incident field of H polarization given by

$$\phi^i(x,z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)}, \quad 0 < \theta_0 < \pi/2 \tag{3}$$

with $k\left[\equiv \omega(\varepsilon_0\mu_0)^{1/2}\right]$ being the free-space wavenumber. The scattered field $\phi(x,z)$ satisfies the two-dimensional Helmholtz equation

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi(x,z) = 0. \tag{4}$$

Nonzero components of the scattered electromagnetic fields are derived from the following relation:

$$(H_y, E_x, E_z) = \left(\phi, \frac{1}{i\omega\varepsilon_0}\frac{\partial\phi}{\partial z}, \frac{i}{\omega\varepsilon_0}\frac{\partial\phi}{\partial x}\right). \tag{5}$$

According to the boundary condition, tangential components of the total electric field $E_{\tan}^t$ satisfies

$$E_{\tan}^t = \frac{\partial\phi^t(\pm b + h\sin mz, z)}{\partial n} = 0, \quad z < 0, \tag{6}$$

where $\partial/\partial n$ denotes the normal derivative on the waveguide surface. We assume that the corrugation amplitude $2h$ of the waveguide walls is small compared with the wavelength and expand (6) in terms of the Taylor series. Then by ignoring the $O\left(h^2\right)$ terms from the Taylor expansion, we derive that

$$\frac{\partial\phi^t(\pm b,z)}{\partial x} + h\left[\sin mz\frac{\partial^2\phi^t(\pm b,z)}{\partial x^2} - m\cos mz\frac{\partial\phi^t(\pm b,z)}{\partial z}\right] + O\left(h^2\right) = 0, \quad z < 0. \tag{7}$$

Equation (7) is the approximate boundary condition used throughout the remaining part of this paper.

We express the unknown scattered field $\phi(x,z)$ in terms of a perturbation series expansion in h as

$$\phi(x,z) = \phi^{(0)}(x,z) + h\phi^{(1)}(x,z) + O\left(h^2\right), \tag{8}$$

where $\phi^{(0)}(x,z)$ and $\phi^{(1)}(x,z)$ are the zero-order and the first-order terms contained in the scattered field, respectively. Substituting (8) into (4) and taking into account (2), (3), and (7), the original problem is separated into the following two boundary value problems.

(1) Zero-order Problem

The zero-order scattered field $\phi^{(0)}(x,z)$ satisfies

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi^{(0)}(x,z) = 0, \tag{9}$$

where the boundary conditions are given by

$$\phi^{(0)}(\pm b + 0, z) = \phi^{(0)}(\pm b - 0, z)\left[\equiv \phi^{(0)}(\pm b, z)\right], \tag{10}$$

$$\frac{\partial\phi^{(0)}(\pm b + 0, z)}{\partial x} = \frac{\partial\phi^{(0)}(\pm b - 0, z)}{\partial x}\left[\equiv \frac{\partial\phi^{(0)}(\pm b, z)}{\partial x}\right] \tag{11}$$

for $z > 0$, and

$$\phi^{(0)}(\pm b \pm 0, z) - \phi^{(0)}(\pm b \mp 0, z) = j^{(0)}(\pm b, z), \tag{12}$$

$$\frac{\partial\phi^{(0)}(\pm b, z)}{\partial x} = ik\sin\theta_0 e^{-ik(\pm b\sin\theta_0 + z\cos\theta_0)}, \tag{13}$$

for $z < 0$.

(2) First-order Problem

The first-order scattered field $\phi^{(1)}(x,z)$ satisfies

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi^{(1)}(x,z) = 0, \tag{14}$$

where the boundary conditions are given by

$$\phi^{(1)}(\pm b + 0, z) = \phi^{(1)}(\pm b - 0, z)\left[\equiv \phi^{(1)}(\pm b, z)\right], \tag{15}$$

$$\frac{\partial\phi^{(1)}(\pm b + 0, z)}{\partial x} = \frac{\partial\phi^{(1)}(\pm b - 0, z)}{\partial x}\left[\equiv \frac{\partial\phi^{(1)}(\pm b, z)}{\partial x}\right] \tag{16}$$

for $z > 0$, and

$$\phi^{(1)}(\pm b \pm 0, z) - \phi^{(1)}(\pm b \mp 0, z) = j^{(1)}(\pm b, z) \tag{17}$$

$$\begin{aligned}&\frac{\partial\phi^{(1)}(\pm b, z)}{\partial x} + \sin mz\frac{\partial^2\phi^{(0)}(\pm b, z)}{\partial x^2} - m\cos mz\frac{\partial\phi^{(0)}(\pm b, z)}{\partial z}\\ &= \frac{ike^{\mp ikb\sin\theta_0}}{2}\left[k\sin\theta_0\sum_{n=1}^{2}(-1)^n e^{-ikz\cos\theta_n} - m\cos\theta_0\sum_{n=1}^{2}e^{-ikz\cos\theta_n}\right],\end{aligned} \tag{18}$$

for $z < 0$ with

$$\cos\theta_{1,2} = \cos\theta_0 \pm m/k. \tag{19}$$

In (12) and (17), $j^{(0)}(\pm b, z)$ and $j^{(1)}(\pm b, z)$ are the zero- and first-order terms of the unknown surface currents induced on the waveguide walls, respectively. As seen from the above discussion, the zero-order problem is a classical diffraction problem involving a flat, semi-infinite parallel-plate waveguide [13–15]. On the other hand, the first-order problem is very important since it contains the effect of the sinusoidal corrugation of the waveguide walls.

3. ZERO- AND FIRST-ORDER WIENER-HOPF EQUATIONS

For convenience of analysis, we assume that the medium is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. The solution for real k is obtained by letting $k_2 \to 0$ at the end of analysis. In view of the radiation condition, it follows from (8) that, for any fixed x, the zero- and first-order scattered fields $\phi^{(n)}(x, z)$ for $n = 0, 1$ show the asymptotic behavior

$$\begin{aligned} \phi^{(n)}(x, z) &= O(e^{k_2 z \cos\theta_0}), \quad z \to -\infty, \\ &= O(e^{-k_2 z}), \quad z \to \infty. \end{aligned} \tag{20}$$

Let us introduce the Fourier transform of the scattered field $\phi^{(n)}(x, z)$ as in

$$\Phi^{(n)}(x, \alpha) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \phi^{(n)}(x, z) e^{i\alpha z} dz, \tag{21}$$

where $\alpha = \mathrm{Re}\alpha + i\mathrm{Im}\alpha\,(\equiv \sigma + i\tau)$. Using (20), we find from (21) that $\Phi^{(n)}(x, \alpha)$ for $n = 0, 1$ are regular in the strip $-k_2 < \tau < k_2 \cos\theta_0$ of the complex α-plane. We further introduce the Fourier integrals as

$$\Phi_{\pm}^{(n)}(x, \alpha) = \pm(2\pi)^{-1/2} \int_{0}^{\pm\infty} \phi^{(n)}(x, z) e^{i\alpha z} dz, \tag{22}$$

$$J_{-}^{(n)}(\pm b, \alpha) = (2\pi)^{-1/2} \int_{-\infty}^{0} j^{(n)}(\pm b, z) e^{i\alpha z} dz. \tag{23}$$

Then it is seen from (20) that $\Phi_{+}^{(n)}(x, \alpha)$ is regular in $\tau > -k_2$ whereas $\Phi_{-}^{(n)}(x, \alpha)$ and $J_{-}^{(n)}(\pm b, \alpha)$ are regular in $\tau < k_2 \cos\theta_0$. It follows from (21) and (22) that

$$\Phi^{(n)}(x, \alpha) = \Phi_{+}^{(n)}(x, \alpha) + \Phi_{-}^{(n)}(x, \alpha). \tag{24}$$

We also see with the aid of (20) that $\Phi^{(n)}(x, \alpha)$ is bounded as $|x| \to \infty$. Taking the Fourier transform of (9) and (14) and making use of (20), we derive that

$$\left[d^2/dx^2 - \gamma^2(\alpha) \right] \Phi^{(n)}(x, \alpha) = 0 \tag{25}$$

for $n = 0, 1$, where $\gamma(\alpha) = (\alpha^2 - k^2)^{1/2}$ with $\mathrm{Re}\gamma(\alpha) > 0$. Equation (25) is the transformed wave equation and holds for any α in the strip $-k_2 < \tau < k_2 \cos\theta_0$.

Taking into account the radiation condition, the solution of (25) is expressed as

$$\begin{aligned} \Phi^{(n)}(x, \alpha) &= A^{(n)}(\alpha) e^{-\gamma(\alpha)x}, \quad x > b, \\ &= C^{(n)}(\alpha) e^{-\gamma(\alpha)x} + D^{(n)}(\alpha) e^{\gamma(\alpha)x}, \quad |x| < b, \\ &= B^{(n)}(\alpha) e^{\gamma(\alpha)x}, \quad x < -b, \end{aligned} \tag{26}$$

where $A^{(n)}(\alpha), B^{(n)}(\alpha), C^{(n)}(\alpha)$, and $D^{(n)}(\alpha)$ for $n = 0, 1$ are unknown functions. From (26), we find that

$$\begin{aligned} &\Phi^{(n)}(b + 0, \alpha) - \Phi^{(n)}(b - 0, \alpha) \\ &= A^{(n)}(\alpha) e^{-\gamma(\alpha)b} - C^{(n)}(\alpha) e^{-\gamma(\alpha)b} - D^{(n)}(\alpha) e^{\gamma(\alpha)b}, \end{aligned} \tag{27}$$

$$\begin{aligned} &\Phi^{(n)}(-b + 0, \alpha) - \Phi^{(n)}(-b - 0, \alpha) \\ &= C^{(n)}(\alpha) e^{\gamma(\alpha)b} + D^{(n)}(\alpha) e^{-\gamma(\alpha)b} - B^{(n)}(\alpha) e^{-\gamma(\alpha)b}, \end{aligned} \tag{28}$$

$$\begin{aligned} &\Phi^{(n)\prime}(b + 0, \alpha) - \Phi^{(n)\prime}(b - 0, \alpha) \\ &= -\gamma(\alpha) \left[A^{(n)}(\alpha) e^{-\gamma(\alpha)b} - C^{(n)}(\alpha) e^{-\gamma(\alpha)b} + D^{(n)}(\alpha) e^{\gamma(\alpha)b} \right], \end{aligned} \tag{29}$$

$$\begin{aligned} &\Phi^{(n)\prime}(-b + 0, \alpha) - \Phi^{(n)\prime}(-b - 0, \alpha) \\ &= -\gamma(\alpha) \left[B^{(n)}(\alpha) e^{-\gamma(\alpha)b} + C^{(n)}(\alpha) e^{\gamma(\alpha)b} - D^{(n)}(\alpha) e^{-\gamma(\alpha)b} \right], \end{aligned} \tag{30}$$

where the prime denotes differentiation with respect to x. In the following, we shall derive the zero- and first-order Wiener-Hopf equations.

(1) Zero-order Problem

The zero-order problem corresponds to the H-polarized plane wave diffraction by a flat, semi-infinite parallel-plate waveguide, and the method based on the Wiener-Hopf technique is discussed elsewhere. In the following, we shall formulate the zero-order problem in the manner somewhat different from the literature [13–15]. Solving (27)–(30) with $n = 0$ for $A^{(0)}(\alpha)$, $B^{(0)}(\alpha)$, $C^{(0)}(\alpha)$, and $D^{(0)}(\alpha)$ and using the boundary conditions for tangential magnetic fields as given by (10) and (12), we obtain that

$$A^{(0)}(\alpha) = (1/2)\left[e^{\gamma(\alpha)b}J_-^{(0)}(b,\alpha) + e^{-\gamma(\alpha)b}J_-^{(0)}(-b,\alpha)\right], \quad (31)$$

$$B^0(\alpha) = -(1/2)\left[e^{-\gamma(\alpha)b}J_-^{(0)}(b,\alpha) + e^{\gamma(\alpha)b}J_-^{(0)}(-b,\alpha)\right], \quad (32)$$

$$C^{(0)}(\alpha) = (1/2)e^{-\gamma(\alpha)b}J_-^{(0)}(-b,\alpha), \quad (33)$$

$$D^{(0)}(\alpha) = (1/2)e^{-\gamma(\alpha)b}J_-^{(0)}(b,\alpha), \quad (34)$$

where $J_-^{(0)}(\pm b,\alpha)$ are defined by (23). In view of (22), $J_-^{(0)}(\pm b,\alpha)$ are alternatively written as

$$J_-^{(0)}(\pm b,\alpha) = \Phi_-^{(0)}(\pm b \pm 0,\alpha) - \Phi_-^{(0)}(\pm b \mp 0,\alpha). \quad (35)$$

Substituting (31)–(34) into (26) and using the boundary conditions for tangential electric fields given by (11) and (13), we derive that

$$K(\alpha)U_-^{(0)}(\alpha) + U_+^{(0)}(\alpha) + G_1^{(0)}(\alpha) = 0, \quad (36)$$

$$L(\alpha)V_-^{(0)}(\alpha) + V_+^{(0)}(\alpha) + G_2^{(0)}(\alpha) = 0 \quad (37)$$

for $-k_2 < \tau < k_2\cos\theta_0$, where

$$U_-^{(0)}(\alpha) = J_-^{(0)}(b,\alpha) + J_-^{(0)}(-b,\alpha), \quad (38)$$

$$V_-^{(0)}(\alpha) = J_-^{(0)}(b,\alpha) - J_-^{(0)}(-b,\alpha), \quad (39)$$

$$U_+^{(0)}(\alpha) = \Phi_+^{(0)\prime}(b,\alpha) + \Phi_+^{(0)\prime}(-b,\alpha), \quad (40)$$

$$V_+^{(0)}(\alpha) = \Phi_+^{(0)\prime}(b,\alpha) - \Phi_+^{(0)\prime}(-b,\alpha), \quad (41)$$

$$G_1^{(0)}(\alpha) = \frac{2k\sin\theta_0\cos(kb\sin\theta_0)}{(2\pi)^{1/2}(\alpha - k\cos\theta_0)}, \quad (42)$$

$$G_2^{(0)}(\alpha) = -\frac{2ik\sin\theta_0\sin(kb\sin\theta_0)}{(2\pi)^{1/2}(\alpha - k\cos\theta_0)}, \quad (43)$$

$$K(\alpha) = \gamma(\alpha)e^{-\gamma(\alpha)b}\cosh[\gamma(\alpha)b], \quad (44)$$

$$L(\alpha) = \gamma(\alpha)e^{-\gamma(\alpha)b}\sinh[\gamma(\alpha)b]. \quad (45)$$

Equations (36) and (37) are the zero-order Wiener-Hopf equations, where $K(\alpha)$ and $L(\alpha)$ are the kernel functions.

(2) First-order Problem

We solve (27)–(30) with $n = 1$ for $A^{(1)}(\alpha)$, $B^{(1)}(\alpha)$, $C^{(1)}(\alpha)$, and $D^{(1)}(\alpha)$ and use the boundary conditions for tangential magnetic fields given by (10), (12), (15), and (17). This yields, after some arrangements,

$$\begin{aligned}A^{(1)}(\alpha) \;=\; & e^{\gamma(\alpha)b}\left\{\frac{i}{4\gamma(\alpha)}\left[\delta(\alpha)J_-^{(0)}(b,\alpha+m)\right.\right.\\ & \left.\left.-\delta(\alpha-m)J_-^{(0)}(b,\alpha-m)\right]+\frac{J_-^{(1)}(b,\alpha)}{2}\right\}\\ & +e^{-\gamma(\alpha)b}\left\{\frac{i}{4\gamma(\alpha)}\left[\delta(\alpha)J_-^{(0)}(-b,\alpha+m)\right.\right.\\ & \left.\left.-\delta(\alpha-m)J_-^{(0)}(-b,\alpha-m)\right]+\frac{J_-^{(1)}(-b,\alpha)}{2}\right\},\end{aligned} \tag{46}$$

$$\begin{aligned}B^{(1)}(\alpha) \;=\; & e^{-\gamma(\alpha)b}\left\{\frac{i}{4\gamma(\alpha)}\left[\delta(\alpha)J_-^{(0)}(b,\alpha+m)\right.\right.\\ & \left.\left.-\delta(\alpha-m)J_-^{(0)}(b,\alpha-m)\right]-\frac{J_-^{(1)}(b,\alpha)}{2}\right\}\\ & +e^{\gamma(\alpha)b}\left\{\frac{i}{4\gamma(\alpha)}\left[\delta(\alpha)J_-^{(0)}(-b,\alpha+m)\right.\right.\\ & \left.\left.-\delta(\alpha-m)J_-^{(0)}(-b,\alpha-m)\right]-\frac{J_-^{(1)}(-b,\alpha)}{2}\right\},\end{aligned} \tag{47}$$

$$\begin{aligned}C^{(1)}(\alpha) \;=\; & e^{-\gamma(\alpha)b}\left\{\frac{i}{4\gamma(\alpha)}\left[\delta(\alpha)J_-^{(0)}(-b,\alpha+m)\right.\right.\\ & \left.\left.-\delta(\alpha-m)J_-^{(0)}(-b,\alpha-m)\right]+\frac{J_-^{(1)}(-b,\alpha)}{2}\right\},\end{aligned} \tag{48}$$

$$\begin{aligned}D^{(1)}(\alpha) \;=\; & e^{-\gamma(\alpha)b}\left\{\frac{i}{4\gamma(\alpha)}\left[\delta(\alpha)J_-^{(0)}(b,\alpha+m)\right.\right.\\ & \left.\left.-\delta(\alpha-m)J_-^{(0)}(b,\alpha-m)\right]-\frac{J_-^{(1)}(b,\alpha)}{2}\right\},\end{aligned} \tag{49}$$

where $J_-^{(1)}(\pm b,\alpha)$ are defined by (23), and

$$\delta(\alpha) = k^2 - \alpha^2 - m\alpha. \tag{50}$$

The functions $J_-^{(1)}(\pm b,\alpha)$ in (46)–(49) are also expressed as

$$J_-^{(1)}(\pm b,\alpha) = \Phi_-^{(1)}(\pm b \pm 0,\alpha) - \Phi_-^{(1)}(\pm b \mp 0,\alpha) \tag{51}$$

by taking into account (23). Using the boundary conditions for tangential electric fields given by (11), (13), (16), and (18) and carrying out some manipulations, we arrive at

$$\begin{aligned} U_+^{(1)}(\alpha)+G_1^{(1)}(\alpha) = & -K(\alpha)U_-^{(1)}(\alpha) \\ & -\frac{i}{4}\left\{\delta(\alpha)V_-^{(0)}(\alpha+m)\left[e^{-2\gamma(\alpha+m)b}-e^{-2\gamma(\alpha)b}\right]\right. \\ & \left.-\delta(\alpha-m)V_-^{(0)}(\alpha-m)\left[e^{-2\gamma(\alpha-m)b}-e^{-2\gamma(\alpha)b}\right]\right\}, \quad (52) \end{aligned}$$

$$\begin{aligned} V_+^{(1)}(\alpha)+G_2^{(1)}(\alpha) = & -L(\alpha)V_-^{(1)}(\alpha) \\ & +\frac{i}{4}\left\{\delta(\alpha)U_-^{(0)}(\alpha+m)\left[e^{-2\gamma(\alpha+m)b}-e^{-2\gamma(\alpha)b}\right]\right. \\ & \left.-\delta(\alpha-m)U_-^{(0)}(\alpha-m)\left[e^{-2\gamma(\alpha-m)b}-e^{-2\gamma(\alpha)b}\right]\right\}, \quad (53) \end{aligned}$$

where

$$U_+^{(1)}(\alpha) = \Phi_+^{(1)'}(b,\alpha)+\Phi_+^{(1)'}(-b,\alpha), \quad (54)$$

$$V_+^{(1)}(\alpha) = \Phi_+^{(1)'}(b,\alpha)-\Phi_+^{(1)'}(-b,\alpha), \quad (55)$$

$$U_-^{(1)}(\alpha) = J_-^{(1)}(b,\alpha)+J_-^{(1)}(-b,\alpha), \quad (56)$$

$$V_-^{(1(}(\alpha) = J_-^{(1)}(b,\alpha)-J_-^{(1)}(-b,\alpha), \quad (57)$$

$$G_1^{(1)}(\alpha) = \frac{2mk\left(\alpha\cos\theta_0-k\right)\cos\left(kb\sin\theta_0\right)}{(2\pi)^{1/2}\left[m^2-(\alpha-k\cos\theta_0)^2\right]}, \quad (58)$$

$$G_2^{(1)}(\alpha) = -\frac{2imk\left(\alpha\cos\theta_0-k\right)\sin\left(kb\sin\theta_0\right)}{(2\pi)^{1/2}\left[m^2-(\alpha-k\cos\theta_0)^2\right]}. \quad (59)$$

Equations (52) and (53) are the first-order Wiener-Hopf equations satisfied by the unknown functions $U_+^{(1)}(\alpha)$, $V_+^{(1)}(\alpha)$, $U_-^{(1)}(\alpha)$, and $V_-^1(\alpha)$, which hold for any α in the strip $-k_2 < \tau < k_2\cos\theta_0$. In the above, $K(\alpha)$ and $L(\alpha)$ are the kernel functions defined by (44) and (45), respectively.

4. EXACT SOLUTIONS OF THE WIENER-HOPF EQUATIONS

In this section, we shall solve the zero- and first-order Wiener-Hopf equations to derive exact solutions. First we note that the kernel functions $K(\alpha)$ and $L(\alpha)$ defined by (44) and (45) are factorized as [13–

15]

$$K(\alpha) = K_+(\alpha)K_-(\alpha) = K_+(\alpha)K_+(-\alpha), \tag{60}$$
$$L(\alpha) = L_+(\alpha)L_-(\alpha) = L_+(\alpha)L_+(-\alpha), \tag{61}$$

where $K_\pm(\alpha)$ and $L_\pm(\alpha)$ are the split functions given by

$$K_+(\alpha) = (\cos kb)^{1/2} e^{i3\pi/4} (k+\alpha)^{1/2} \exp\left[\frac{i\gamma(\alpha)b}{\pi} \ln \frac{\alpha - \gamma(\alpha)}{k}\right] \cdot \exp\left[\frac{i\alpha b}{\pi}\left(1 - C + \ln\frac{\pi}{2kb} + i\frac{\pi}{2}\right)\right] \prod_{n=1,\text{odd}}^{\infty} \left(1 + \frac{\alpha}{i\gamma_n}\right) e^{2i\alpha b/n\pi}, \tag{62}$$

$$L_+(\alpha) = (k \sin kb)^{1/2} e^{i\pi/2} \exp\left[\frac{i\gamma(\alpha)b}{\pi} \ln \frac{\alpha - \gamma(\alpha)}{k}\right] \left(1 + \frac{\alpha}{i\gamma_0}\right) \cdot \exp\left[\frac{i\alpha b}{\pi}\left(1 - C + \ln\frac{2\pi}{kb} + i\frac{\pi}{2}\right)\right] \prod_{n=2,\text{even}}^{\infty} \left(1 + \frac{\alpha}{i\gamma_n}\right) e^{2i\alpha b/n\pi} \tag{63}$$

with $C\,(= 0.57721566\ldots)$ being Euler's constant, and

$$\gamma_0 = -ik; \quad \gamma_n = \left[(n\pi/2b)^2 - k^2\right]^{1/2} \text{ for } n \geq 1. \tag{64}$$

It is seem from (62) and (63) that $K_\pm(\alpha)$ and $L_\pm(\alpha)$ are regular and nonzero in $\tau \gtrless \mp k_2$. We can also verify that

$$K_\pm(\alpha) \sim -(\mp i\alpha/2)^{1/2}, \quad L_\pm(\alpha) \sim -(\mp i\alpha/2)^{1/2} \tag{65}$$

as $\alpha \to \infty$ with $\tau \gtrless \mp k_2$.

(1) Zero-order Problem

Equations (36) and (37) are the simultaneous Wiener-Hopf equations arising in the classical diffraction problem for a flat, semi-infinite parallel-plate waveguide [13-1-5], and can be solved exactly with the result that

$$U_-^{(0)}(\alpha) = -\frac{G_1^{(0)}(\alpha)}{K_+(k\cos\theta_0)K_-(\alpha)}, \tag{66}$$

$$V_-^{(0)}(\alpha) = -\frac{G_2^{(0)}(\alpha)}{L_+(k\cos\theta_0)L_-(\alpha)}. \tag{67}$$

Equations (66) and (67) are the exact solution to the zero-order Wiener-Hopf equations (36) and (37), respectively.

(2) First-order Problem

We shall now solve the first-order Wiener-Hopf equations as given by (52) and (53). Multiplying both sides of (52) and (53) by $1/K_+(\alpha)$ and $1/L_+(\alpha)$, respectively and applying the decomposition procedure, we obtain that

$$\begin{aligned}
&\frac{U_+^{(1)}(\alpha)}{K_+(\alpha)}+\frac{k\cos(kb\sin\theta_0)}{(2\pi)^{1/2}(\alpha-k\cos\theta_1)}\left[\frac{\alpha\cos\theta_0-k}{K_+(\alpha)}-\frac{k\cos\theta_1\cos\theta_0-k}{K_+(k\cos\theta_1)}\right]\\
&-\frac{k\cos(kb\sin\theta_0)}{(2\pi)^{1/2}(\alpha-k\cos\theta_2)}\left[\frac{\alpha\cos\theta_0-k}{K_+(\alpha)}-\frac{k\cos\theta_2\cos\theta_0-k}{K_+(k\cos\theta_2)}\right]\\
&-\frac{1}{8\pi}\int_C\frac{\delta(u)V_-^{(0)}(u+m)\left[e^{-2\gamma(u+m)b}-e^{-2\gamma(u)b}\right]}{K_+(\alpha)(u-\alpha)}du\\
&-\frac{1}{8\pi}\int_C\frac{\delta(u-m)V_-^{(0)}(u-m)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}\right]}{K_+(\alpha)(u-\alpha)}du\\
=&\;-K_-(\alpha)U_-^{(1)}(\alpha)-\frac{k\cos(kb\sin\theta_0)(k\cos\theta_1\cos\theta_0-k)}{(2\pi)^{1/2}K_+(k\cos\theta_1)(\alpha-k\cos\theta_1)}\\
&+\frac{k\cos(kb\sin\theta_0)(k\cos\theta_2\cos\theta_0-k)}{(2\pi)^{1/2}K_+(k\cos\theta_2)(\alpha-k\cos\theta_2)}-H_1(\alpha)-H_2(\alpha),
\end{aligned}\tag{68}$$

$$\begin{aligned}
&\frac{V_+^{(1)}(\alpha)}{L_+(\alpha)}-\frac{ik\sin(kb\sin\theta_0)}{(2\pi)^{1/2}(\alpha-k\cos\theta_1)}\left[\frac{\alpha\cos\theta_0-k}{L_+(\alpha)}-\frac{k\cos\theta_1\cos\theta_0-k}{L_+(k\cos\theta_1)}\right]\\
&+\frac{ik\sin(kb\sin\theta_0)}{(2\pi)^{1/2}(\alpha-k\cos\theta_2)}\left[\frac{\alpha\cos\theta_0-k}{L_+(\alpha)}-\frac{k\cos\theta_2\cos\theta_0-k}{L_+(k\cos\theta_2)}\right]\\
&+\frac{1}{8\pi}\int_C\frac{U_-^{(0)}(u+m)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u+m)b}\right]}{L_+(\alpha)(u-\alpha)}du\\
&+\frac{1}{8\pi}\int_C\frac{U_-^{(0)}(u-m)\left[e^{-2\gamma(u-m)b}-e^{-2\gamma(u)b}\right]}{L_+(\alpha)(u-\alpha)}du\\
=&\;-L_-(\alpha)V_-^{(1)}(\alpha)+\frac{ik\sin(kb\sin\theta_0)(k\cos\theta_1\cos\theta_0-k)}{(2\pi)^{1/2}L_+(k\cos\theta_1)(\alpha-k\cos\theta_1)}\\
&-\frac{ik\sin(kb\sin\theta_0)(k\cos\theta_2\cos\theta_0-k)}{(2\pi)^{1/2}L_+(k\cos\theta_2)(\alpha-k\cos\theta_2)}+I_1(\alpha)+I_2(\alpha),
\end{aligned}\tag{69}$$

where

$$H_1(\alpha)=\frac{1}{8\pi}\int_D\frac{\delta(u)V_-^{(0)}(u+m)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u+m)b}\right]}{K_+(u)(u-\alpha)}du,\tag{70}$$

$$H_2(\alpha) = \frac{1}{8\pi}\int_D \frac{\delta(u-m)V_-^{(0)}(u-m)\left[e^{-2\gamma(u-m)b}-e^{-2\gamma(u)b}\right]}{K_+(u)(u-\alpha)}du, \quad (71)$$

$$I_1(\alpha) = \frac{1}{8\pi}\int_D \frac{\delta(u)U_-^{(0)}(u+m)\left[e^{-2\gamma(u+m)b}-e^{-2\gamma(u)b}\right]}{L_+(u)(u-\alpha)}du, \quad (72)$$

$$I_2(\alpha) = \frac{1}{8\pi}\int_D \frac{\delta(u-m)U_-^{(0)}(u-m)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}\right]}{L_+(u)(u-\alpha)}du. \quad (73)$$

In (68)–(73), C and D are the infinite integration paths running parallel to the real axis in the u-plane, as shown in Fig. 2, where c and d are some constants such that $-k_2 < c < \tau < d < k_2 \cos\theta_0$ with $\tau = \mathrm{Im}\alpha$.

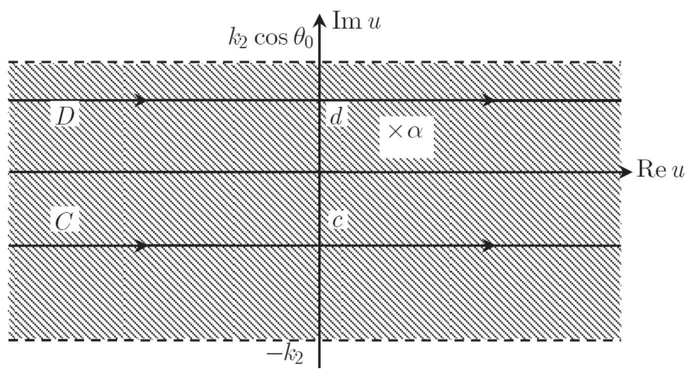

Figure 2. Integration paths C and $D\,(-k_2 < c < \tau < d < k_2\cos\theta_0)$.

It is seen that the left-hand and right-hand sides of (68) are regular in the upper ($\tau > -k_2$) and lower ($\tau < k_2\cos\theta_0$) half-planes, respectively, and both sides have a common strip of regularity $-k_2 < \tau < k_2\cos\theta_0$. Hence by analytic continuation, we can show that both sides of (68) must be equal to an entire function, which is found to be identically zero by taking into account the edge condition and Liouville's theorem. The same argument can also be applied to (69) and its both sides are identically equal to zero. Therefore, it follows that

$$U_-^{(1)}(\alpha) = -\frac{1}{K_-(\alpha)}\left[H_1(\alpha)+H_2(\alpha)\right] + \frac{A_1}{K_-(\alpha)\,(\alpha-k\cos\theta_1)} + \frac{A_2}{K_-(\alpha)\,(\alpha-k\cos\theta_2)}, \quad (74)$$

$$V_-^{(1)}(\alpha) = \frac{1}{L_-(\alpha)}\left[I_1(\alpha)+I_2(\alpha)\right] + \frac{B_1}{L_-(\alpha)(\alpha-k\cos\theta_1)} - \frac{B_2}{L_-(\alpha)(\alpha-k\cos\theta_2)}, \quad (75)$$

where

$$A_{1,2} = \mp\frac{k\cos(kb\sin\theta_0)\left(k\cos\theta_{1,2}\cos\theta_0-k\right)}{(2\pi)^{1/2}K_+(k\cos\theta_{1,2})}, \quad (76)$$

$$B_{1,2} = \pm\frac{ik\sin(kb\sin\theta_0)\left(k\cos\theta_{1,2}\cos\theta_0-k\right)}{(2\pi)^{1/2}L_+(k\cos\theta_{1,2})}. \quad (77)$$

Equations (74) and (75) are the exact solutions to the first order Wiener-Hopf equations (52) and (53), respectively, where the infinite integrals $H_{1,2}(\alpha)$ and $I_{1,2}(\alpha)$ defined by (70)–(73) are involved. These integrals can be evaluated in closed form by using the residue theorem. Substituting (66) and (67) into (70)–(73) and evaluating the resultant integrals, we obtain that

$$\begin{aligned}
H_{1,2}(\alpha) = &\mp\frac{B_0}{4}\left(\frac{1}{k\sin kb}\right)^{1/2}\exp\left[-\frac{ikb}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right] \\
&\cdot\prod_{n=2,\text{even}}^{\infty}\left\{\frac{i\gamma_n\exp(-2iub/n\pi)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}\right]}{(u+i\gamma_n)\,(u\pm k\cos\theta_{1,2})K_\mp(u)(u\pm\alpha)}\right\}_{u=\mp i\gamma_0+m} \\
&\mp\frac{B_0}{4}\left(\frac{1}{k\sin kb}\right)^{1/2}\sum_{q=2,\text{even}}^{\infty}\frac{i\gamma_0\gamma_q}{\gamma_0+\gamma_q}\exp\left[\frac{ib\gamma(i\gamma_q)}{\pi}\ln\frac{-i\gamma_q-\gamma(i\gamma_q)}{k}\right] \\
&\cdot\exp\left[-\frac{2\gamma_q b}{q\pi}-\frac{\gamma_q b}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right] \\
&\cdot\left\{\frac{\delta(u\mp m)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}\right]}{(u\pm k\cos\theta_{1,2})K_\mp(u)(u\pm\alpha)}\right\}_{u=\mp i\gamma_q+m} \\
&\cdot\prod_{\substack{n=2,\,\text{even},\\ n\neq q}}^{\infty}\frac{\gamma_n\exp(-2\gamma_q b/n\pi)}{\gamma_n-\gamma_q}, \quad (78)
\end{aligned}$$

$$\begin{aligned}
I_{1,2}(\alpha) &= \mp\frac{iC_0}{4\exp(i3\pi/4)(\cos kb)^{1/2}} \\
&\quad\cdot\sum_{p=1,\mathrm{odd}}^{\infty} i\gamma_p\,(k-i\gamma_p)^{1/2}\prod_{\substack{n=1,\mathrm{odd}\\ n\neq p}}^{\infty}\frac{\gamma_n\exp(-2\gamma_p b/n\pi)}{\gamma_n-\gamma_p} \\
&\quad\cdot\exp\left[-\frac{2\gamma_p b}{p\pi}-\frac{\gamma_p b}{\pi}\left(1-C+\ln\frac{\pi}{2kb}+i\frac{\pi}{2}\right)\right] \\
&\quad\cdot\exp\left[\frac{ib\gamma(\alpha)}{\pi}\ln\frac{-i\gamma_p-\gamma(\alpha)}{k}\right] \\
&\quad\cdot\left\{\frac{\delta(u\mp m)\left[e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}\right]}{(u\pm k\cos\theta_{1,2})L_{\mp}(u)(u\pm\alpha)}\right\}_{u=\mp i\gamma_p+m}, \qquad (79)
\end{aligned}$$

where

$$B_0 = \frac{2ik\sin\theta_0\sin(kb\sin\theta_0)}{(2\pi)^{1/2}\,L_+(k\cos\theta_0)}, \qquad (80)$$

$$C_0 = -\frac{2k\sin\theta_0\cos(kb\sin\theta_0)}{(2\pi)^{1/2}\,K_+(k\cos\theta_0)}. \qquad (81)$$

Substituting (78) and (79) into (74) and (75), respectively, explicit expressions of the exact solution of the first-order Wiener-Hopf equations will be derived.

5. SCATTERED FIELD

In this section, we shall derive analytical expressions of the scattered field by using the results obtained in Section 4. The scattered field $\phi^{(n)}(x,z)$ with $n=0,1$ in the real space can be derived by taking the inverse Fourier transform according to the formula

$$\phi^{(n)}(x,z) = (2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\Phi^{(n)}(x,\alpha)e^{-i\alpha z}d\alpha, \qquad (82)$$

where c is a constant satisfying $-k_2 < c < k_2\cos\theta_0$. In the following, we shall derive explicit expressions of the zero- and first-order scattered fields inside and outside the waveguide.

5.1. Field inside the Waveguide

(1) Zero-order Problem

Substituting the zero-order field representation for $|x| < b$ in (26) into (82) and taking into account (33), (34), (38), and (39), an integral representation of the zero-order scattered field is derived as

$$\phi^{(0)}(x,z) = -\frac{1}{2}(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\Big[U_-^{(0)}(\alpha)\sinh\gamma(\alpha)x + V_-^{(0)}(\alpha)\cosh\gamma(\alpha)x\Big]e^{-\gamma(\alpha)b-i\alpha z}d\alpha. \quad (83)$$

Substituting (66) and (67) into (83) and evaluating the resultant integral for $z < 0$, we derive the zero-order scattered field inside the waveguide as in

$$\phi^{(0)}(x,z) = -\phi^i(x,z) + \sum_{n=0}^{\infty} T_n^{(0)}\cos\frac{n\pi}{2b}(x+b)e^{\gamma_n z}, \quad (84)$$

where

$$T_0^{(0)} = \frac{\sin(kb\sin\theta_0)}{kb\sin\theta_0 L_-(k)\,L_-(k\cos\theta_0)}, \quad (85)$$

$$T_n^{(0)} = -\left(\frac{\pi}{2}\right)^{1/2}\frac{n\pi}{b\gamma_n}U_-^{(0)}(i\gamma_n)K(i\gamma_n) \text{ for } n = 1,3,5,\ldots,$$
$$= \left(\frac{\pi}{2}\right)^{1/2}\frac{n\pi}{b\gamma_n}V_-^{(0)}(i\gamma_n)L(i\gamma_n) \text{ for } n = 2,4,6,\ldots. \quad (86)$$

It is seen that (84) is the scattered field expression for the diffraction problem involving a flat, semi-infinite parallel-plate waveguide [13–15].

(2) First-order Problem

Substituting the first-order field representation for $|x| < b$ in (26) into (82) and taking into account (38), (39), (48), (49), (56), and (57), we obtain the following integral representation of the first-order scattered field:

$$\phi^{(1)}(x,z) = (2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\Big(\Big\{[4i\gamma(\alpha)]^{-1}\Big[\delta(\alpha-m)U_-^{(0)}(\alpha-m) - \delta(\alpha)U_-^{(0)}(\alpha+m)\Big] - V_-^{(1)}(\alpha)\Big\}\cosh[\gamma(\alpha)x] + \Big\{[4i\gamma(\alpha)]^{-1}\Big[\delta(\alpha-m)V_-^{(0)}(\alpha-m) - \delta(\alpha)V_-^{(0)}(\alpha+m)\Big] - U_-^{(1)}(\alpha)\Big\}\sinh[\gamma(\alpha)x]\Big)e^{-\gamma(\alpha)b-i\alpha z}d\alpha. \quad (87)$$

Substituting (66), (67), (74), and (75) into (87) and evaluating the resultant integral with the aid of (78) and (79) for $z < 0$, we derive, after some manipulations, that

$$\phi^{(1)}(x,z) = T_0^{(1)} e^{-ikz} + \sum_{n=1}^{\infty} T_n^{(1)} \sin\frac{n\pi}{2b}(x+b)e^{\gamma_n z}, \tag{88}$$

where

$$\begin{aligned} T_0^{(1)} &= \frac{1}{2bK_-(k)}\left[-\frac{\sin(kb\sin\theta_0)}{\sin\theta_1 L_+(k\cos\theta_1)} + \frac{\sin(kb\sin\theta_0)}{\sin\theta_2 L_+(k\cos\theta_2)}\right] \\ &\quad + \frac{k^{1/2}\sin\theta_0\cos(kb\sin\theta_0)}{2(1+\cos\theta_0)L_+(k\cos\theta_0)}, \end{aligned} \tag{89}$$

$$\begin{aligned} T_n^{(1)} &= \frac{iC_0}{2}\left[\frac{\sin(kb\sin\theta_1)}{K_-(k\cos\theta_1)}K_n^1(i\gamma_n - m) - \frac{\sin(kb\sin\theta_2)}{K_-(k\cos\theta_2)}K_n^1(i\gamma_n + m)\right. \\ &\quad \left. + \frac{n\pi}{2^{1/2}be^{i\pi/4}}K_n^1(i\gamma_n)\right] \quad \text{for} \quad n = 1,3,5,\ldots, \\ &= \frac{B_0}{2}\left[\frac{\cos(kb\sin\theta_1)}{L_-(k\cos\theta_1)}L_n^1(i\gamma_n - m) + \frac{\cos(kb\sin\theta_2)}{L_-(k\cos\theta_2)}L_n^1(i\gamma_n + m)\right. \\ &\quad \left. - \frac{n\pi}{2^{1/2}b}L_n^1(i\gamma_n)\right] \quad \text{for} \quad n = 2,4,6,\ldots \end{aligned} \tag{90}$$

with

$$\begin{aligned} K_n^1(i\gamma_n \pm m) = &\frac{\exp[\pm mi - \gamma_n - ikb\sin(i\gamma_n \pm m)]}{\exp\{(\gamma_n b/\pi)[1 - C + \ln(\pi/2kb) + i\pi/2]\}} \\ &\cdot\frac{\sin[kb\sin(i\gamma_n \pm m)]}{\exp([ib\gamma(i\gamma_n \pm m)/\pi]\ln\{[-i\gamma_n - \gamma(i\gamma_n \pm m)]/k\})} \\ &\cdot\frac{\delta[\pm(i\gamma_n - m)](k - i\gamma_n \pm m)}{\prod\limits_{\substack{p=1,\,\text{odd}\\ p\neq n}}^{\infty}[(\gamma_p - \gamma_n)/\gamma_p]\exp(2\gamma_p b/p\pi)}, \end{aligned} \tag{91}$$

$$\begin{aligned} L_n^1(i\gamma_n \pm m) = &\left(\frac{1}{k\sin kb}\right)^{1/2}\frac{\exp[\pm mi - \gamma_n - ikb\sin(i\gamma_n \pm m)]}{\exp\{(\gamma_n b/\pi)[1 - C + \ln(2\pi/kb) + i\pi/2]\}} \\ &\cdot\frac{\cos[kb\sin(i\gamma_n \pm m)]}{\exp([ib\gamma(i\gamma_n \pm m)/\pi]\ln\{[-i\gamma_n - \gamma(i\gamma_n \pm m)]/k\})} \\ &\cdot\frac{\delta[\pm(i\gamma_n - m)](k - i\gamma_n \pm m)}{\prod\limits_{\substack{q=2,\,\text{even}\\ q\neq n}}^{\infty}[(\gamma_q - \gamma_n)/\gamma_q]\exp(2\gamma_q b/q\pi)}, \end{aligned} \tag{92}$$

$$K_n^1(i\gamma_n) = \frac{\exp\left[-\gamma_n - ikb\sin(i\gamma_n)\right]}{\exp\left\{(\gamma_n b/\pi)\left[1 - C + \ln(\pi/2kb) + i\pi/2\right]\right\}} \cdot \frac{\sin\left[kb\sin(i\gamma_n \pm m)\right]}{\exp\left(\left[ib\gamma(i\gamma_n)/\pi\right]\ln\left\{\left[-i\gamma_n - \gamma(i\gamma_n)\right]/k\right\}\right)} \cdot \frac{\delta(i\gamma_n)(k - i\gamma_n)}{\prod\limits_{\substack{p=1,\,\mathrm{odd}\\ p\neq n}}^{\infty}\left[(\gamma_p - \gamma_n)/\gamma_p\right]\exp(2\gamma_p b/p\pi)}, \tag{93}$$

$$L_n^1(i\gamma_n) = \left(\frac{1}{k\sin kb}\right)^{1/2} \frac{\exp\left[-\gamma_n - ikb\sin(i\gamma_n)\right]}{\exp\left\{(\gamma_n b/\pi)\left[1 - C + \ln(2\pi/kb) + i\pi/2\right]\right\}} \cdot \frac{\cos\left[kb\sin(i\gamma_n)\right]}{\exp\left(\left[ib\gamma(i\gamma_n)/\pi\right]\ln\left\{\left[-i\gamma_n - \gamma(i\gamma_n)\right]/k\right\}\right)} \cdot \frac{\delta(i\gamma_n)(k - i\gamma_n)}{\prod\limits_{\substack{q=2,\,\mathrm{even}\\ q\neq n}}^{\infty}\left[(\gamma_q - \gamma_n)/\gamma_q\right]\exp(2\gamma_q b/q\pi)}. \tag{94}$$

Equation (88) is the explicit expression of the first-order scattered field inside the waveguide.

5.2. Field outside the Waveguide

We now consider the field outside the waveguide and derive the scattered far field. The region outside the waveguide actually includes $z > 0$ with $|x| < b$, but contributions from this region are negligibly small at large distances from the origin. Therefore, only the scattered far field for $|x| > b$ will be discussed in the following.

(1) Zero-order Problem

Substituting the field representation for $|x| > b$ with $n = 0$ in (26) into (82) and taking into account (31), (32), (38), and (39), we derive an integral representation of the zero-order scattered field for $x \gtrless \pm b$ as in

$$\phi^{(0)}(x,z) = \pm\frac{1}{2}(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\left[U_-^{(0)}(\alpha)\cosh\gamma(\alpha)b - V_-^{(0)}(\alpha)\sinh\gamma(\alpha)b\right]e^{\mp\gamma(\alpha)x - i\alpha z}d\alpha. \tag{95}$$

Let us introduce the cylindrical coordinates $(\rho_\pm, \theta_\pm)$ centered at the waveguide edges $(x, z) = (\pm b, 0)$ as follows:

$$x - b = \rho_+ \sin\theta_+,\; z = \rho_+ \cos\theta_+ \quad \text{for} \quad 0 < \theta_+ < \pi, \tag{96}$$

$$x + b = \rho_- \sin\theta_-,\; z = \rho_- \cos\theta_- \quad \text{for} \quad -\pi < \theta_- < 0. \tag{97}$$

Equation (95) can be evaluated asymptotically using the saddle point method together with the pole-singularity extraction procedure. Omitting the details, the zero-order scattered far field is obtained as

$$\phi^{(0)}(\rho_\pm, \theta_\pm) \sim \phi_{r\pm}^{(0)}(\rho_\pm, \theta_\pm) + \phi_{d\pm}^{(0)}(\rho_\pm, \theta_\pm) \tag{98}$$

for $x \gtrless \pm b$ as $k\rho_\pm \to \infty$, where

$$\begin{aligned}\phi_{r\pm}^{(0)}(\rho_\pm, \theta_\pm) = R_\pm^{(0)}(\theta_0) \Bigg\{ & e^{-ik\rho_\pm \cos(\theta_\pm - \theta_0)} F\left[(2k\rho_\pm)^{1/2} \cos\frac{\theta_\pm - \theta_0}{2}\right] \\ & + e^{-ik\rho_\pm \cos(\theta_\pm + \theta_0)} F\left[(2k\rho_\pm)^{1/2} \cos\frac{\theta_\pm + \theta_0}{2}\right]\Bigg\},\end{aligned} \tag{99}$$

$$\phi_{d\pm}^{(0)}(\rho_\pm, \theta_\pm) = D_\pm^{(0)}(\theta_\pm, \theta_0) \frac{e^{i(k\rho_\pm - 3\pi/4)}}{(k\rho_\pm)^{1/2}} \tag{100}$$

with

$$R_\pm^{(0)}(\theta_0) = -e^{\mp ikb\sin\theta_0}, \tag{101}$$

$$\begin{aligned}D_\pm^{(0)}(\theta_\pm, \theta_0) = & \frac{2ik\sin(\theta_0/2)\sin(\theta_\pm/2)}{\cos\theta_\pm + \cos\theta_0} \\ & \cdot \Big[U_-^{(0)}(-k\cos\theta_\pm)\cos(kb\sin\theta_\pm) \\ & - V_-^{(0)}(-k\cos\theta_\pm)\sin(kb\sin\theta_\pm)\Big].\end{aligned} \tag{102}$$

In (99), $F(\cdot)$ is the Fresnel integral defined by

$$F(x) = \frac{e^{-i\pi/4}}{\pi^{1/2}} \int_x^\infty e^{it^2} dt. \tag{103}$$

Equation (98) gives a uniform asymptotic expression of the zero-order scattered far field. Here $\phi_{r\pm}^{(0)}(\rho_\pm, \theta_\pm)$ contain the geometrical optics fields (incident and reflected waves) and the singly diffracted fields emanating from the waveguide edges at $x = \pm b$, whereas $\phi_{d\pm}^{(0)}(\rho_\pm, \theta_\pm)$ account for the multiply diffracted fields corresponding to the higher order interaction between the two waveguide edges.

(2) First-order Problem

Substituting (26) for $|x| > b$ with $n = 1$ into (82) and making use of (38), (39), (46), (47), (56), and (57), we obtain the integral representation of the first-order scattered field as

$$\begin{aligned}\phi^{(1)}(x,z) \;=\; &(2\pi)^{-1/2}\int_{-\infty+ic}^{\infty+ic}\bigg(\Big\{[4i\gamma(\alpha)]^{-1}\Big[\delta(\alpha)U_-^{(0)}(\alpha+m)\\ &-\delta(\alpha-m)U_-^{(0)}(\alpha)\Big]\pm U_-^{(1)}(\alpha)\Big\}\cosh[\gamma(\alpha)x]\\ &+\Big\{\pm[4i\gamma(\alpha)]^{-1}\Big[\delta(\alpha)V_-^{(0)}(\alpha+m)\\ &-\delta(\alpha-m)V_-^{(0)}(\alpha-m)\Big]+V_-^{(1)}(\alpha)\Big\}\sinh[\gamma(\alpha)x]\bigg)\\ &\cdot e^{\mp\gamma(\alpha)x-i\alpha z}d\alpha \qquad (104)\end{aligned}$$

for $x \gtrless \pm b$. We apply the saddle point method together with the pole-singularity extraction procedure by using the cylindrical coordinates defined by (96) and (97). This leads to a far field asymptotic expression of the first-order scattered field with the result that

$$\phi^{(1)}(\rho_\pm,\theta_\pm) \sim \phi_{r\pm}^{(1)}(\rho_\pm,\theta_\pm) + \phi_{d\pm}^{(1)}(\rho_\pm,\theta_\pm) \qquad (105)$$

for $x \gtrless \pm b$ as $k\rho_\pm \to \infty$, where

$$\begin{aligned}\phi_{r\pm}^{(1)}(\rho_\pm,\theta_\pm) = &\sum_{j=1}^{2} R_{j\pm}^{(1)}(\theta_0,\theta_j)\bigg\{e^{-ik\rho_\pm\cos(\theta_\pm-\theta_j)}F\left[(2k\rho_\pm)^{1/2}\cos\frac{\theta_\pm-\theta_j}{2}\right]\\ &+e^{-ik\rho_\pm\cos(\theta_\pm+\theta_j)}F\left[(2k\rho_\pm)^{1/2}\cos\frac{\theta_\pm+\theta_j}{2}\right]\bigg\}, \qquad (106)\end{aligned}$$

$$\phi_{d\pm}^{(1)}(\rho_\pm,\theta_\pm) = \sum_{j=0}^{2} D_{j\pm}^{(1)}(\theta_\pm,\theta_0,\theta_j)\,\frac{e^{i(k\rho_\pm-3\pi/4)}}{(k\rho_\pm)^{1/2}} \qquad (107)$$

with

$$\begin{aligned}R_{j+}^{(1)}(\theta_0,\theta_j) \;=\; &\frac{kb\sin\theta_0}{2}e^{ikb\sin\theta_0}\left(1+e^{-2ikb\sin\theta_j}\right)\\ &+\frac{1}{8}\left(e^{-2ikb\sin\theta_0}-e^{-2ikb\sin\theta_j}\right)\\ &\cdot\left[\frac{C_0}{K_-(k\cos\theta_0)}-\frac{B_0}{L_-(k\cos\theta_0)}\right], \qquad (108)\end{aligned}$$

$$R_{j-}^{(1)}(\theta_0,\theta_j) \;=\; 0, \qquad (109)$$

$$\begin{aligned} D_{0\pm}^{(1)}\left(\theta_{\pm},\theta_0,\theta_0\right) \;=\; & \frac{iC_0}{8}\sum_{n=1,\mathrm{odd}}^{\infty}\left\{\frac{\delta\left(i\gamma_n\right)L_-\left(k\cos\theta_{\pm}\right)E_{1n}}{p_n^2\left(\theta_{\pm}\right)\left(m-k\cos\theta_{\pm}-i\gamma_n\right)}\right.\\ & \left.+\frac{\delta\left(i\gamma_n-m\right)L_-\left(k\cos\theta_{\pm}\right)E_{2n}}{p_n^1\left(\theta_{\pm}\right)\left(-m-k\cos\theta_{\pm}-i\gamma_n\right)}\right\}\\ & +\frac{iB_0}{8}\sum_{n=0,\mathrm{even}}^{\infty}\left\{\frac{\delta\left(i\gamma_n\right)K_-\left(k\cos\theta_{\pm}\right)E_{1n}}{q_n^2\left(\theta_{\pm}\right)\left(m-k\cos\theta_{\pm}-i\gamma_n\right)}\right.\\ & \left.+\frac{\delta\left(i\gamma_n-m\right)K_-\left(k\cos\theta_{\pm}\right)E_{2n}}{q_n^1\left(\theta_{\pm}\right)\left(-m-k\cos\theta_{\pm}-i\gamma_n\right)}\right\}, \end{aligned} \tag{110}$$

$$\begin{aligned} D_{j\pm}^{(1)}\left(\theta_{\pm},\theta_0,\theta_j\right) \;=\; & \frac{(-1)^{j-1}ik^{1/2}\sin\theta_{\pm}\left(1-\cos\theta_0\cos\theta_j\right)}{2^{3/2}\pi^{1/2}\left(\cos\theta_{\pm}+\cos\theta_j\right)}\\ & \cdot\left\{U_-^{(0)}\left[(-1)^{j-1}m-k\cos\theta_{\pm}\right]\cos\left(kb\sin\theta_{\pm}\right)\right.\\ & \left.+V_-^{(0)}\left[(-1)^{j-1}m-k\cos\theta_{\pm}\right]\sin\left(kb\sin\theta_{\pm}\right)\right\}\\ & +\frac{(-1)^{j-1}i\sin\theta_{\pm}\left(1-\cos\theta_0\cos\theta_j\right)}{2^{3/2}\pi^{1/2}k\left(\cos\theta_{\pm}+\cos\theta_j\right)}\\ & \cdot\left[\frac{A_j\cos\left(kb\sin\theta_{\pm}\right)}{K_+\left(k\cos\theta_{\pm}\right)}+\frac{B_j\sin\left(kb\sin\theta_{\pm}\right)}{L_+\left(k\cos\theta_{\pm}\right)}\right], \end{aligned} \tag{111}$$

$$p_n^j\left(\Theta\right)=\frac{L_-\left(k\cos\Theta\right)}{\left(i\gamma_n-k\cos\theta_0\right)L_+\left[i\gamma_n+(-1)^{j-1}m\right]\mathrm{Res}\left[K_-\left(i\gamma_n\right)\right]}, \tag{112}$$

$$q_n^j\left(\Theta\right)=\frac{K_-\left(k\cos\Theta\right)}{\left(i\gamma_n-k\cos\theta_0\right)K_+\left[i\gamma_n+(-1)^{j-1}m\right]\mathrm{Res}\left[L_-\left(i\gamma_n\right)\right]}, \tag{113}$$

$$\begin{aligned} \mathrm{Res}\left[K_-\left(i\gamma_n\right)\right]=\; & \frac{e^{i3\pi/4}(\cos kb)^{1/2}\left(k+\alpha\right)^{1/2}e^{2\gamma_n b/n\pi}}{\exp\left(\left[ib\gamma(i\gamma_n)/\pi\right]\ln\{[-i\gamma_n-\gamma(i\gamma_n)]/k\}\right)}\\ & \cdot\frac{\exp\{(\gamma_n b/\pi)[1-C+\ln(\pi/2kb)+i\pi/2]\}}{\prod\limits_{\substack{p=1,\,\mathrm{odd}\\ p\neq n}}^{\infty}\dfrac{\gamma_p\exp(-2\gamma_p b/p\pi)}{\gamma_p-\gamma_n}}\\ & \text{for}\quad n=1,3,5,\ldots, \end{aligned} \tag{114}$$

$$\begin{aligned}
\operatorname{Res}\left[L_-\left(i\gamma_n\right)\right] &= (k\sin kb)^{-1/2}\,e^{-i\pi/2}\prod_{q=2,\mathrm{even}}^{\infty}\left[\frac{\gamma_0+\gamma_q}{\gamma_0\exp(2\gamma_0 b/q\pi)}\right]\\
&\quad\cdot\exp\left[-\frac{ikb}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right]\ \text{for } n=0,\\
&= \frac{(k\sin kb)^{-1/2}\,e^{-i\pi/2}\left[\gamma_0/\left(\gamma_n+\gamma_0\right)\right]}{\exp\{(\gamma_n b/\pi)[1-C+\ln(2\pi/kb)+i\pi/2]\}}\\
&\quad\cdot\frac{\exp\left(\left[ib\gamma\left(i\gamma_n\right)/\pi\right]\ln\left\{\left[-i\gamma_n-\gamma\left(i\gamma_n\right)\right]/k\right\}\right)}{\prod\limits_{\substack{q=2,\,\mathrm{even}\\ q\neq n}}^{\infty}\dfrac{\gamma_q\exp(-2\gamma_q b/q\pi)}{\gamma_q-\gamma_n}}\\
&\quad\text{for}\ \ n=2,4,6,\ldots, \qquad (115)\\
E_{1n} &= e^{-2\gamma(i\gamma_n)b}-e^{-2\gamma(i\gamma_n-m)b}, \qquad (116)\\
E_{2n} &= e^{-2\gamma(i\gamma_n+m)b}-e^{-2\gamma(i\gamma_n-m)b}. \qquad (117)
\end{aligned}$$

It is to be noted that (105) gives the uniform asymptotic expression of the first-order scattered far field, and holds for arbitrary incidence and observation angles.

We shall now make physical interpretation of the results obtained above. First we should note that the first-order scattered field is the correction term to the zero-order scattered field (corresponding to the flat waveguide) due to the existence of the sinusoidal corrugation of the waveguide walls and vanishes when $h \to 0$. In (105), $\phi_{r\pm}^{(1)}(\rho_\pm, \theta_\pm)$ denote the geometrical optics fields (reflected waves) and the singly diffracted fields from the two half-planes located solely at $x = \pm b$. On the other hand, the second term $\phi_{d\pm}^{(1)}(\rho_\pm, \theta_\pm)$ in (105) corresponds to the multiply diffracted fields between the edges of the two corrugated half-planes. Using the asymptotic expansion of the Fresnel integral, we can show that $\phi_{r\pm}^{(1)}(\rho_\pm, \theta_\pm)$ contain the reflected waves propagating along the specific directions at $\pi - \theta_1$ and $\pi - \theta_2$. It is important to note that $\pi - \theta_1$ and $\pi - \theta_2$ are, respectively, propagation directions of the (-1) and $(+1)$ order diffracted waves involved in the Floquet space harmonic modes arising in periodic structures of infinite extent.

6. NUMERICAL RESULTS AND DISCUSSION

In this section, we shall show numerical results of the scattered far field for various physical parameters and investigate the scattering characteristics of the waveguide in detail. For convenience, we introduce the cylindrical coordinate

$$x = \rho \sin\theta, \quad z = \rho\cos\theta \quad \text{for } -\pi < \theta < \pi \tag{118}$$

and define the scattered far field intensity as

$$|\phi(\rho,\theta)|\,[\text{dB}] = 20\log_{10}\left[\frac{\lim\limits_{\rho\to\infty}\left|(k\rho)^{1/2}\phi(\rho,\theta)\right|}{\max\limits_{|\theta|<\pi}\lim\limits_{\rho\to\infty}\left|(k\rho)^{1/2}\phi(\rho,\theta)\right|}\right], \tag{119}$$

where

$$\begin{aligned}\phi(\rho,\theta) &= \phi^{(0)}(\rho_+,\theta_+) + h\phi^{(1)}(\rho_+,\theta_+) \quad \text{for} \quad 0<\theta<\pi,\\ &= \phi^{(0)}(\rho_-,\theta_-) + h\phi^{(1)}(\rho_-,\theta_-) \quad \text{for} \quad -\pi<\theta<0.\end{aligned} \tag{120}$$

In (120), $\phi^{(0)}(\rho_\pm,\theta_\pm)$ and $\phi^{(1)}(\rho_\pm,\theta_\pm)$ denote the zero- and first-order scattered far fields, respectively, which are given by (98) and (105). In view of (96), (97), and (118), we can show that, in the far field region, $\rho_\pm$ and $\theta_\pm$ can be approximately replaced by ρ and θ, respectively for the amplitude terms involved in (120). We have found from numerical experimentation that, when the corrugation depth $2h$ and the corrugation period $2\pi/m$ satisfy $kh \leq 1.0$ and $mh/kh \leq 0.3$, the approximate boundary condition given by (7) can be used to simulate a perfectly conducting sinusoidal surface with reasonable accuracy.

Figures 3–6 show numerical examples of the scattered far field intensity as a function of observation angle θ for various values of kh, mh and kb. As has been mentioned in Section 1, the diffraction problem involving the same waveguide geometry for the E-polarized plane wave incidence has been analyzed in our previous paper [29] by following a method similar to that developed in this paper. In order to enable comparison between different polarizations, we have chosen here the same parameters as in the case of the E polarization. The incidence angle θ_0 and the normalized waveguide aperture width kb are taken as $\theta_0 = 60°$ and $kb = 15.7, 31.4$, respectively. In order to investigate the effect of sinusoidal corrugation of the waveguide walls in detail, we have chosen the normalized corrugation depth kh and the periodicity parameter mh/kh as $kh = 0.1, 0.5, 1.0$ and $mh/kh = 0.2, 0.3$. The results for a flat, semi-infinite parallel-plate waveguide have also been added for comparison.

We see from all the figures that, as in the E-polarized case [29], the scattered far field intensity has maximum peaks at $\theta = -120°$, which can be expected as this direction corresponds to the incident shadow boundary. The other common feature for both polarizations is that the

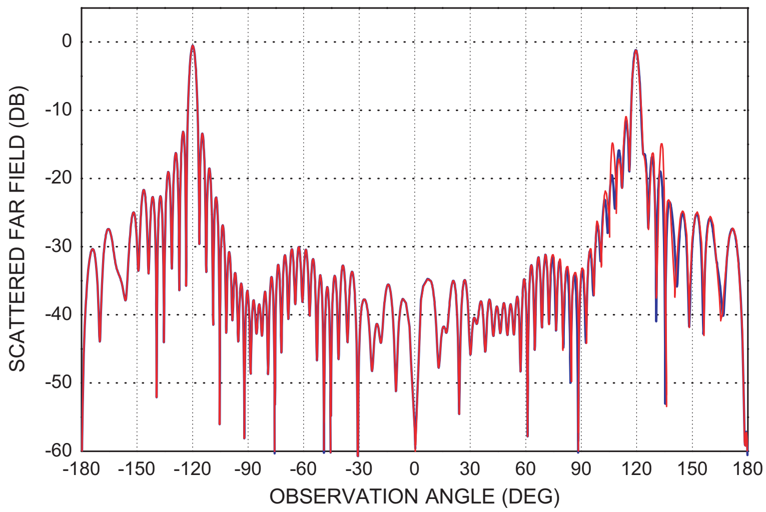

Figure 3a. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 15.7$, $kh = 0.1$. ———: corrugated waveguide. ———: flat waveguide.

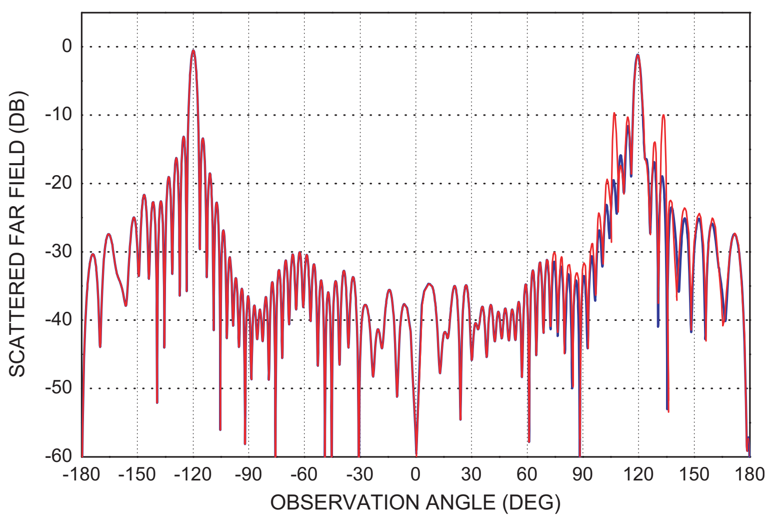

Figure 3b. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 15.7$, $kh = 0.5$. ———: corrugated waveguide. ———: flat waveguide.

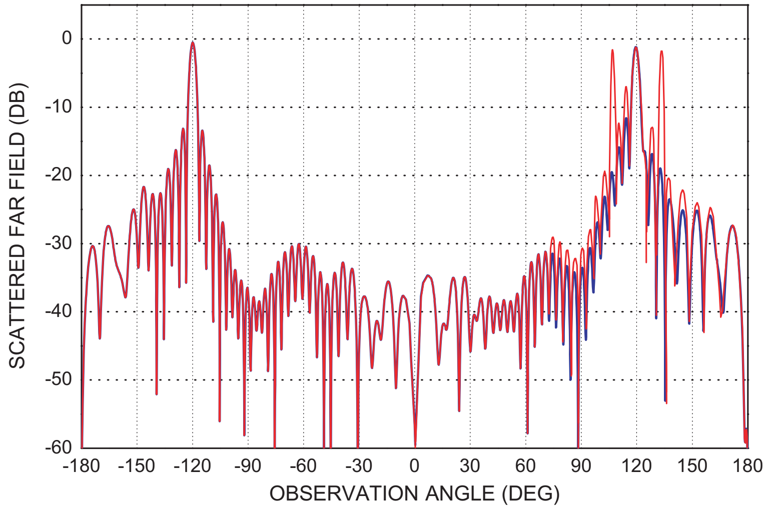

Figure 3c. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 15.7$, $kh = 1.0$. ——: corrugated waveguide. ——: flat waveguide.

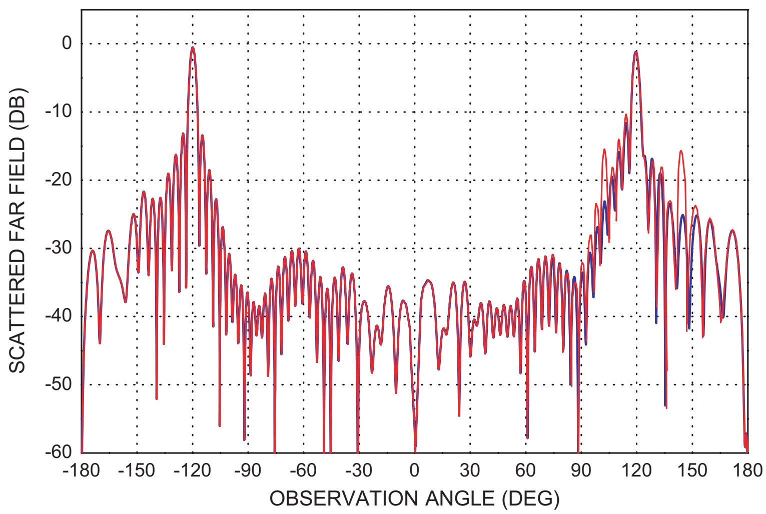

Figure 4a. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 15.7$, $kh = 0.1$. ——: corrugated waveguide. ——: flat waveguide.

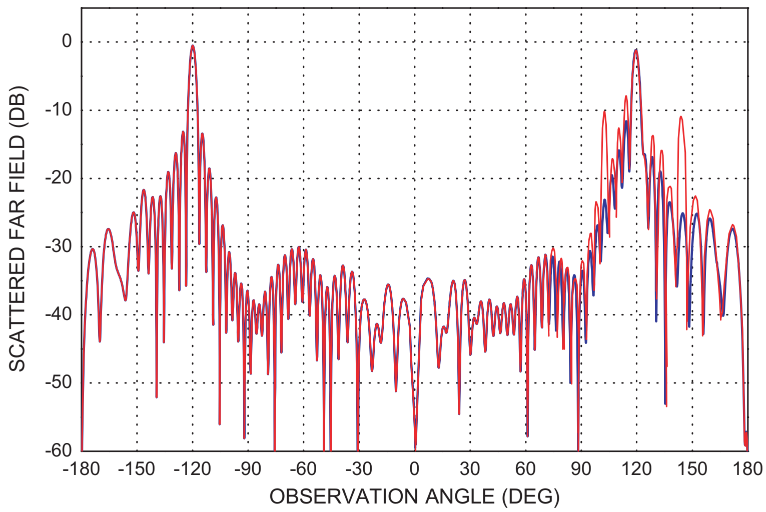

Figure 4b. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 15.7$, $kh = 0.5$. ———: corrugated waveguide. ———: flat waveguide.

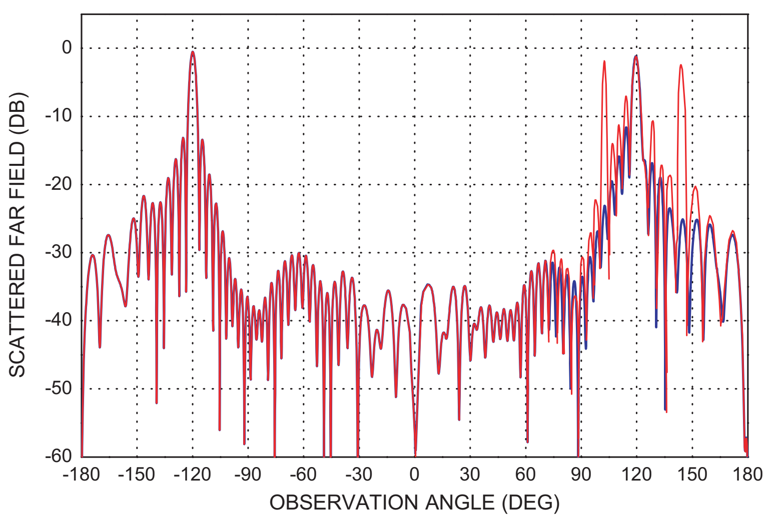

Figure 4c. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 15.7$, $kh = 1.0$. ———: corrugated waveguide. ———: flat waveguide.

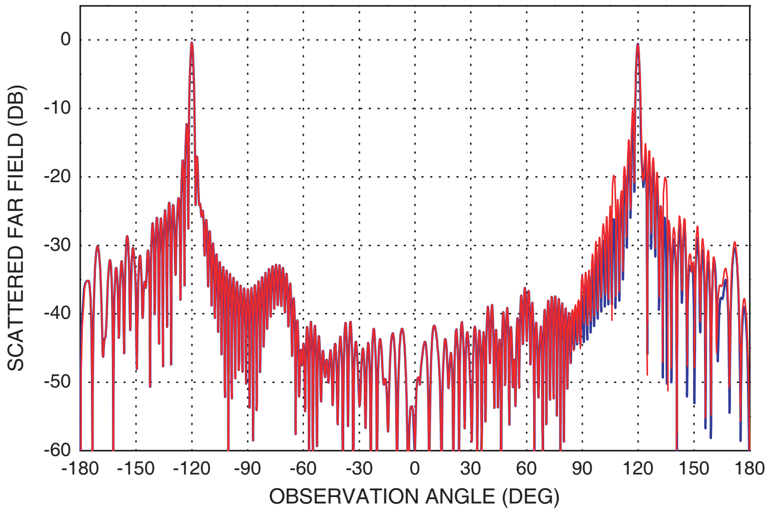

Figure 5a. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 31.4$, $kh = 0.1$. ——: corrugated waveguide. ——: flat waveguide.

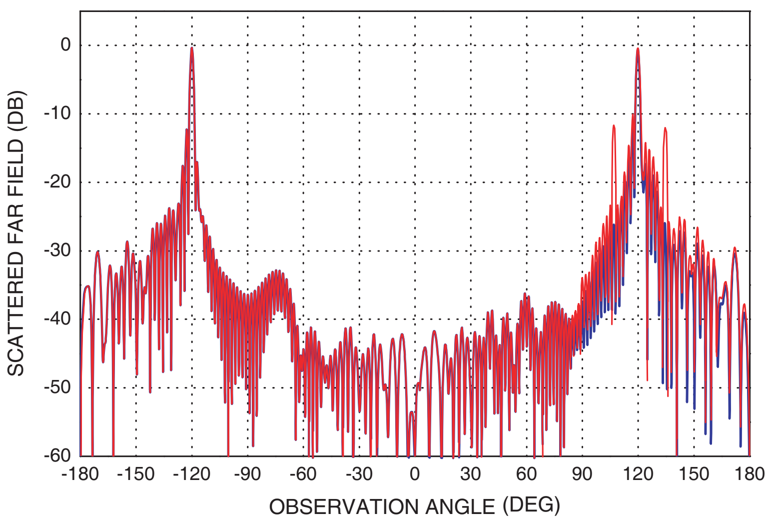

Figure 5b. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 31.4$, $kh = 0.5$. ——: corrugated waveguide. ——: flat waveguide.

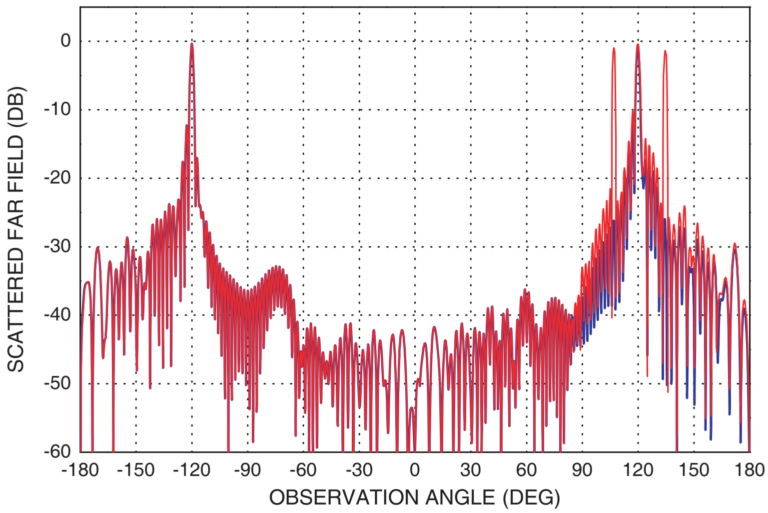

Figure 5c. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 31.4$, $kh = 1.0$. ———: corrugated waveguide. ———: flat waveguide.

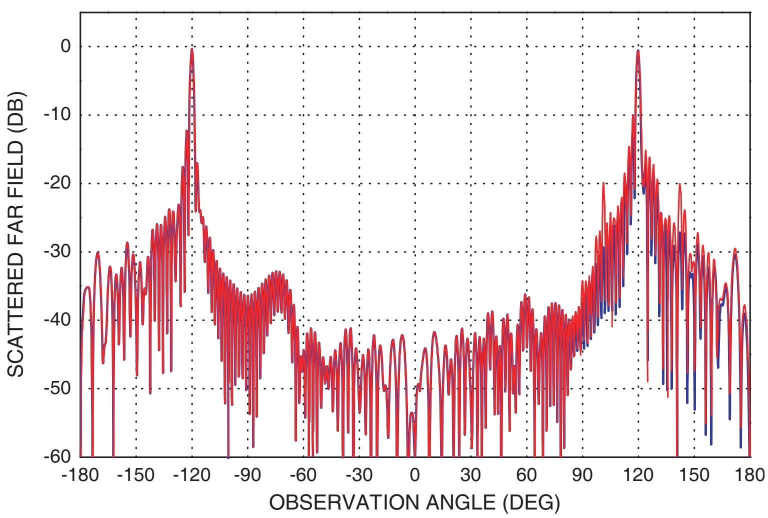

Figure 6a. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 31.4$, $kh = 0.1$. ———: corrugated waveguide. ———: flat waveguide.

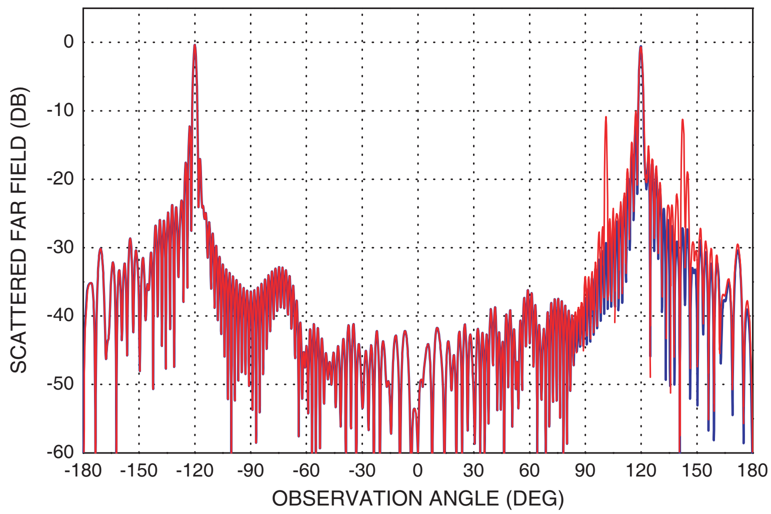

Figure 6b. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 31.4$, $kh = 0.5$. ———: corrugated waveguide. ———: flat waveguide.

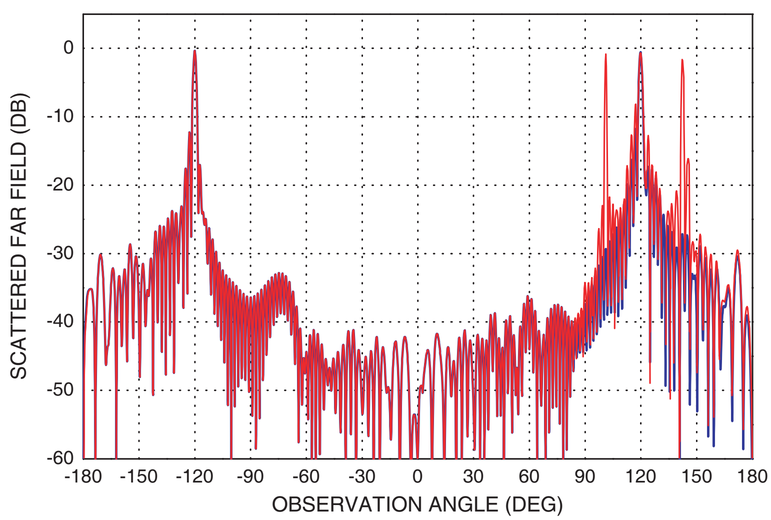

Figure 6c. Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 31.4$, $kh = 1.0$. ———: corrugated waveguide. ———: flat waveguide.

effect of the sinusoidal corrugation of the waveguide walls is noticeable for the range $90^\circ < \theta < 180^\circ$ and the scattered far field intensity has sharp peaks at two particular observation angles around the specularly reflected direction at $\theta = 120^\circ$. For the case of an infinite sinusoidal surface, the field is expressed in terms of Floquet's space harmonic modes due to the periodicity of the structure. Referring to (19), it is also seen that the first three dominant modes in the Floquet field expression are the zero, (-1), and $(+1)$ order diffracted waves and these modes propagate along the directions at $\pi - \theta_0$, $\pi - \theta_1$, and $\pi - \theta_2$, respectively. The angles corresponding to the zero, (-1), and $(+1)$ order Floquet modes are, respectively, $120^\circ, 107.5^\circ$, and 134.4° for the parameters chosen in Figs. 3 and 5, and $120^\circ, 101.5^\circ$, and 143.1° for those in Figs. 4 and 6. At these specific angles, somewhat large reflection is expected due to the existence of the semi-infinite sinusoidal (periodic) surface. In fact, we can see sharp peaks at the three particular observation angles in all the figures. Hence it is confirmed that the three peaks at $\pi-\theta_0$, $\pi-\theta_1$, and $\pi-\theta_2$ in numerical examples are all due to the effect of periodicity of the sinusoidal surface of the waveguide. It can also be observed from the figures for fixed kb and mh/kh that the peaks occurring in the $\pi-\theta_1$ and $\pi-\theta_2$ directions become sharper with an increase of kh as the waves in the propagation directions of the particular Floquet modes are strongly excited for larger kh. We also find by comparing the results for $kb = 15.7$ in Figs. 3 and 4 with those for $kb = 31.4$ in Figs. 5 and 6 that the effect of sinusoidal corrugation of the waveguide walls is noticeable for larger kb.

We shall now make some comparisons between two different polarizations. As mentioned, we have already analyzed the diffraction problem involving the same waveguide geometry for the E-polarized plane wave incidence [29]. Comparing the results in Figs. 3–6 of this paper with the corresponding results in [29], there are some differences in the scattering characteristics depending on the incident polarization. In particular, stronger oscillation is observed in the H-polarized case in comparison to the results for the E polarization. In addition, we see nulls in the scattering characteristics at $\theta = 0^\circ$, $\pm 180^\circ$ for the H-polarized case.

7. CONCLUDING REMARKS

In this paper, we have solved the H-polarized plane wave diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation approximately using the Wiener-Hopf technique combined with the perturbation method. Assuming the corrugation depth to be small compared with the wavelength and expanding the scattered field in the form of a perturbation series, the problem has been formulated in terms of the zero- and first-order simultaneous Wiener-Hopf equations. The Wiener-Hopf equations have been solved via application of the factorization and decomposition procedure leading to the exact solutions. The scattered field inside and outside the waveguide has been derived analytically. In particular, the field outside the waveguide has been evaluated asymptotically using the saddle point method together with the pole-singularity extraction procedure leading to the far field expressions, which are uniformly valid in incidence and observation angles. We have carried out numerical computation of the scattered far field for various physical parameters, and investigated the effect of the sinusoidal corrugation of the waveguide walls in detail. As a result, it has been shown that, in the reflection region, scattered waves are strongly excited along the specific directions corresponding to the three dominant Floquet modes arising in periodic structures of infinite extent. We have also made some comparisons with the results obtained in our previous paper [29] for the case of the E-polarized plane wave incidence.

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