

Novel MRA-Based Sparse MIMO and SIMO Antenna Array for Automotive Radar Applications

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Abstract—Automotive radars make use of angle information obtained from antenna arrays to distinguish objects that lie in the same range-doppler cell (relative to the ego vehicle). This paper proposes novel ways of using presently known minimum redundancy arrays (MRAs) in single-input multiple-output (SIMO) and multiple-input multiple-output (MIMO) automotive radars. Firstly, an MRA-based sparse MIMO array is proposed as a novel modification to the nested MIMO array. The proposed sparse MIMO array uses MRAs as the transmitting and receiving modules, unlike the nested MIMO array, which uses two-level nested arrays (TLNAs) at the transmitting and receiving blocks. Upper bounds for the sum co-array aperture and the overall attainable degrees of freedom (DOF) offered by the MIMO radar have been derived in terms of the number of sensors. Secondly, the suitability of large Low-Redundancy Linear Arrays (LRLAs) in SIMO automotive radars is also studied. A long-range automotive radar driving scenario was assumed for DOA estimation and simulations were carried out in MATLAB using the co-array Multiple Signal Classification (co-array MUSIC) algorithm. Simulation results confirm that the proposed MRA-based MIMO array provides better angular resolutions than the nested MIMO array for the same number of sensors and that LRLAs can serve as a handy replacement for ULAs in SIMO radars owing to their acceptable performance. As MIMO and SIMO radars designed from currently known MRAs were sufficient to satisfy the angular resolution requirements of modern automotive radars, a need to synthesize new MRAs did not arise.

1. INTRODUCTION

Autonomous or self-driving vehicles make use of radar signal processing methods to detect obstacles along their path [1]. Automotive radars are classified as long-range radars (LRRs), medium-range radars (MRRs), and short-range radars (SRRs) based on the distance that they can cover. Typical ranges for LRR, MRR, and SRR are 10–250 m, 1–80 m, and 0.15–30 m, respectively [2].

Front-facing radars make use of LRRs and MRRs to detect obstacles that appear in front of the automobile. The radar determines the range and the relative velocities of the obstacles (relative to its velocity) and categorizes them into different range-doppler bins [3]. This radar technique cannot distinguish two objects that fall in the same range-doppler bin. In such situations, angle information is used to distinguish the objects [4]. Usually, angle estimation in automotive radars is performed using antenna arrays.

Automotive radars are classified as Single Input Multiple Output (SIMO) and Multiple Input Multiple Output (MIMO) radars. SIMO radars are simple. There is a single antenna at the transmitter and a linear antenna array (LAA) at the receiver [5]. In general, uniform linear arrays (ULAs) are used as receivers in SIMO radars.

Sparse arrays contain voids that are deliberately created to fulfil certain mathematical constraints and can be created from ULAs by careful elimination of select sensors such that the sensor elements

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thus retained are capable of meeting the desired constraints [6]. A sparse array, therefore, needs fewer sensors than a ULA to provide the same aperture. Alternatively, for a given number of sensors, sparse arrays provide wider apertures than ULAs. Sparse arrays are generally analyzed in the co-array domain. A difference co-array (DCA) is formed from the physical sparse array by considering all the spatial lags (differences) that can be generated using the available sensors [7]. A missing spatial lag forms a hole in the DCA. The DCA should be hole-free as the presence of holes introduces ambiguity in the estimation of spatial correlation and spatial angles [7, 8]. The usefulness of arrays with holes in the DCA is limited by the span of the central continuous portion of the DCA [9, 10]. Though there are methods [11, 12] that can extend the continuous portion of the DCA, they are computationally intense. The number of continuous entries in the DCA gives the uniform degrees of freedom (DOFs) of the sparse array [11].

Co-located MIMO arrays make use of co-located, parallel transmitting and receiving arrays. These transmitting and receiving arrays work in tandem and produce the effect of a large virtual array. The transmitting array consists of M antennas and the receiving array contains N antennas. Hence, the total number of physical antennas in the MIMO array is $K = M + N$. Each transmitting antenna emits an independent waveform. At each receiving antenna, there are M matched filters which are used to extract the reflected signals. Since there are N receiving elements, the total number of extracted signals is MN . It has been shown that the matched filter output is equivalent to the signals received by an array of MN elements. This results in a large virtual array of MN elements [13], whereas physically there are only $M + N$ elements. The virtual array is also known as the sum co-array as it is obtained by adding all possible element positions in the transmitting and receiving arrays. For example, the transmitting and receiving arrays with sensors at $\{0, 1, 2\}$ and $\{0, 3, 6\}$ normalized to the basic unit of inter-element spacing (half wavelength), respectively, result in a sum co-array which has sensors at $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. This sum co-array is a ULA. However, it could be made sparse by careful positioning of antenna elements in the transmitting and receiving blocks. Early attempts to create sparse MIMO arrays have been reported in [14]. From this point on, we refer the virtual array of the MIMO radar as its sum co-array.

To carry out parameter estimation using MIMO radar, the sum co-array is considered as the starting point. Since the sum co-array is designed to be a linear sparse array, its DCA, i.e., the difference co-array of the sum co-array (DCSC) should be hole-free.

Designing sparse MIMO radar arrays remained as a herculean task for many years [15–17], mainly owing to the exhaustive search space and the non-availability of closed-form expressions for element positions. Early designs made use of exhaustive search mechanisms to determine the optimum positioning of sensors. Of late, authors in [18] proposed the design of co-prime MIMO radar which provides closed-form expressions for sensor positions. A generalized co-prime MIMO array structure with increased transmitter spacing was proposed in [19]. This MIMO radar uses prototype co-prime arrays at the transmitter and receiver. However, the problem with co-prime MIMO radars is that their DCSC is not hole-free and hence causes ambiguity in parameter estimation or limits the usefulness of the DCSC to the central continuous portion [18, 19].

Recently, an excellent way of designing a MIMO array using nested arrays was proposed [20]. It provides closed-form expressions for sensor positions in the transmitting and receiving arrays alongside generating a hole-free DCSC. The design is scalable for any number of sensors and the computational complexity of finding the optimal sensor locations does not increase with the number of sensors. The transmitting and receiving modules in this procedure can be designed using any sparse array that has a hole-free co-array. Hence, only the two-level nested array (TLNA) [21], the super-nested array [22], the Yang's improved nested array (YINA) [23] and the minimum redundancy array (MRA) [24] could be made use of. Other sparse arrays such as the co-prime array, the triple primed array [25], the Huang's nested array [9], the grade nested array [26] and the minimum hole array (MHA) are not suitable for this procedure as they contain holes in the co-array.

In the present work, we propose a new extension to the method given in [20], by making use of MRAs as the basic building blocks at the transmitter and the receiver instead of TLNAs. We explain how existing off-the-shelf MRA configurations could be reasonably and cleverly used to design MIMO radars that provide massive apertures using just a handful of sensors. The design just needs a table look-up for the MRA configuration when the number of sensors is given. No exhaustive search procedures are required as there is no need to find new MRA configurations. Hence, the fact that MRAs do not

have closed-form expressions for sensor positions does not have any bearing in the present context.

The choice of MRAs is logical as they need fewer sensors than nested, super-nested, and improved nested arrays to provide the same aperture [7, 23]. Additionally, MRAs are more immune to the effects of mutual coupling compared to TLNAs [22, 27]. YINA is superior to the TLNA in terms of the aperture offered for a given number of sensors. However, given the dense ULA portion in level 1, it is easy to predict that even the YINA is not much different from the TLNA in terms of the vulnerability to mutual coupling. Though super-nested arrays match the brilliance of MRAs in many aspects, they cannot offer apertures as large as the MRAs do [22]. Therefore, MRAs remain as an obvious choice for designing the MIMO array. MRAs with twelve or more sensors are known as low-redundancy linear arrays (LRLAs). LRLAs cannot achieve minimum redundancies and are, therefore, designed to provide the least possible redundancy for a given number of sensors [6].

The specific contributions of this paper are as follows:

- A novel modification to the design of the nested MIMO array given in [20] is presented here by using MRAs as the basic building blocks at the transmitter and the receiver instead of TLNAs. As a result, the sum co-array aperture increases. Additionally, the transmitting and receiving blocks become more immune to the effects of mutual coupling compared to those in the nested MIMO array.
- Expressions for the upper bounds on the achievable sum co-array aperture and the overall degrees of freedom (DOFs) are derived when the total number of sensors in the MIMO array is known.
- For SIMO radars, it is shown that LRLAs can be used as a handy alternative to ULAs for high-resolution angle estimation. The use of LRLAs provides significant savings in the system hardware as they need less than 20% of the sensors needed by a ULA to provide the same aperture.

The paper does not deal with any new algorithms for DOA estimation nor does it propose to synthesize new MRA configurations. The main focus here is to explain how currently known MRAs could be efficiently used to design sparse MIMO radar arrays. Apart from the methods listed above, sparse MIMO arrays have also been designed by thinning regular arrays through exhaustive optimization techniques to obtain a desired beam pattern or side lobe performance [28–30]. A cyclic algorithm was used in [28] to determine the antenna positions in the transmitting and receiving arrays such that the array attains a desired response. Genetic algorithm was used in [30] to thin a ULA in the receiver of the MIMO array to obtain a sparse array of similar side lobe performance. These methods need complicated computational searches to determine the optimum sensor locations in the transmitting and receiving arrays such that the array achieves a desired radiation pattern. However, in the present work, we did not focus on such techniques. Instead, our objective was to design a sparse MIMO array that offers the largest hole-free DCSC for a given number of sensors using known MRAs.

The remainder of this paper is organized as follows. Section 2 presents the array model and the methodology to design the MRA-based MIMO array. Section 3 presents the procedure to construct the MIMO array design book. Section 4 describes the simulation methodology and numerical simulation results for DOA estimation using the proposed MIMO array. Section 5 deals with the use of LRLAs for DOA estimation in SIMO radars. Section 6 provides a comparison between SIMO and MIMO arrays. Section 7 provides a few future directions and concludes the paper.

Note: The terms sensors, antennas, elements, and antenna elements have been used interchangeably in this paper. The following sparse array terminology is followed in the rest of the paper: — the sum co-array refers to the cross summation obtained from the sensor positions in transmitting and receiving arrays. The term DCSC is used to refer the unique entries obtained from the difference set of the sum co-array. Likewise, the term DCA refers to the unique entries obtained from the difference set of the physical array.

2. ARRAY MODEL AND DESIGN PRELIMINARIES

This section discusses the MIMO array model and the procedure to design it using MRAs as the base. The approach is similar to the one explained in [20] except the fact that MRAs are used as the basic modules at the transmitting and the receiving arrays instead of TLNAs.

2.1. The Proposed MRA-Based Sparse MIMO Array

It is well known that the MIMO radar array consists of co-located transmitting and receiving arrays. In the proposed method, the transmitting as well as receiving arrays are MRAs with M and N antenna elements, respectively. $x_{T,m}$ denotes the position of the m th antenna in the transmitter in units of the inter-element spacing. The default inter-element spacing is assumed to be half of the wavelength. Similarly, $x_{R,n}$ denotes the position of the n th antenna in the receiver. The resulting sum co-array is given by

$$\{x_v\} = \{x_{T,m} + x_{R,n} | m = 1, 2, \dots, M; n = 1, 2, \dots, N\}, \quad (1)$$

where $v = 1, 2, \dots, MN$.

The proposed MIMO array with MRA base is obtained as follows. The equations described below are inspired from the equations used in [20] to design the nested MIMO array.

- The transmitting array has M elements and is denoted by

$$\{x_{T,m}\} = \{t_m | m = 1, 2, \dots, M\}, \quad (2)$$

where $\{t_m\}_{m=1}^M$ indicates the transmitting MRA which can be obtained through table look-up for a known value of M . If L_t is the aperture provided by the transmitting array, then $D = 2L_t + 1$ denotes the span of its DCA. It is already known that the MRA provides a hole-free DCA.

- The receiving array has N elements where the basic unit of inter-element spacing is D times that of the inter-element spacing in the transmitter. It is denoted by

$$\{x_{R,n}\} = \{Dr_n | n = 1, 2, \dots, N\}, \quad (3)$$

where $\{r_n\}_{n=1}^N$ indicates the N -element MRA which serves as the base. This base MRA offers an aperture of L_r and has to be obtained through table look-up. The actual receiving array is then obtained using Eq. (3) and provides an aperture of DL_r .

- The resulting sum co-array is obtained by combining Eqs. (1), (2), and (3)

$$\{x_v\} = \{t_m + Dr_n | m = 1, 2, \dots, M; n = 1, 2, \dots, N\}. \quad (4)$$

It provides an aperture of $L_v = DL_r + L_t$.

- The DCA of the sum co-array, i.e., the difference co-array of the sum co-array (DCSC) is given by

$$\{x_{dcsc}\} = \{x_v - x_{v'} | v, v' = 1, 2, \dots, MN\} \quad (5)$$

The element positioning in Eqs. (2) and (3) ensures that the DCSC is hole-free. The span of the DCSC or the number of unique entries in it gives the overall achievable degrees of freedom (DOF).

The following example shown in Table 1 with $M = 4$ and $N = 5$ helps to get an understanding of the proposed MIMO array model. The MRA configuration for a given number of antenna elements

Table 1. Example of the proposed MIMO radar array using MRAs.

	Array element positions	Remarks
Transmitting Array	$\{x_{T,m}\} = \{0, 1, 4, 6\}$	$L_t = 6; D = 13$.
Basic Receiving Module	$\{r_n\} = \{0, 2, 5, 8, 9\}$	$L_r = 9$
Actual Receiving array	$\{x_{R,n}\} = \{D.r_n\} = \{0, 26, 65, 104, 117\}$	$DL_r = 117$
Sum co-array	$\{x_v\} = \{0, 1, 4, 6, 26, 27, 30, 32, 65, 66, 69, 71, 104, 105, 108, 110, 117, 118, 121, 123\}$	$L_v = 123$
Difference co-array of the sum co-array (DCSC)	$\{x_{dcsc}\} = \{0, \pm 1, \pm 2, \pm 3, \dots, \pm 123\}$	$DOF = 247$

is obtained through table look-up. For the moment, let us consider that the MRAs for four and five elements are given.

As a comparison, the DOF of the nested MIMO array [20] for the same number of antennas is 187 whereas the DOF of the proposed MIMO array with MRA base is 247. This difference grows bigger as the number of sensors increase. However, our motive is not to prove the superiority of the proposed design but to drive home the point that currently known MRAs can gracefully embrace the design of MIMO radars to provide large sum co-arrays and increased DOFs.

2.2. Designing the MIMO Array Using MRA Blocks

The following steps must be followed to design the proposed MIMO array with MRA base.

- Given a total of K antenna elements in the MIMO array, the optimum values of M and N to obtain the maximum possible DOFs are same as those formulated in [20]

$$\begin{aligned}
 K \text{ is even} &\Rightarrow M = N = \frac{K}{2} \\
 K \text{ is odd} &\Rightarrow M = \frac{K-1}{2}; N = \frac{K+1}{2}.
 \end{aligned}
 \tag{6}$$

- The M -element transmitting MRA and the N -element receiving MRA must be obtained using table look-up from Table 2. Such tabulated MRA configurations are widely available in the literature.
- The sum co-array and its DCSC automatically follow.

Table 2. Tabulated MRA configurations and the basic aperture provided by YINA and TLNA for four to ten sensors.

Number of sensors	Sensor positions for MRA configuration	Aperture of MRA	Aperture of YINA for the same number of sensors	Aperture of TLNA for the same number of sensors
4	[0, 1, 4, 6]	6	6	5
5	[0, 2, 5, 8, 9]	9	9	8
6	[0, 2, 8, 9, 12, 13]	13	13	11
7	[0, 2, 5, 7, 13, 16, 17]	17	17	15
8	[0, 2, 5, 7, 13, 19, 22, 23]	23	22	19
9	[0, 1, 4, 10, 16, 22, 24, 27, 29]	29	27	24
10	[0, 1, 3, 6, 13, 20, 27, 31, 35, 36]	36	33	29

2.3. Upper Bounds for the Sum Co-Array Aperture and the DOFs Offered by the Proposed MIMO Array

The expressions for upper bounds on the achievable aperture of the sum co-array and the overall DOFs, in terms of the number of available sensors, can be derived as follows:

2.3.1. Case 1: K Is Even

From Eq. (6), it is seen that $M = N = K/2$. Because the transmitting and receiving arrays are MRAs, their apertures are related to the number of array elements by a formula given in [24]

$$L_t = L_r = \frac{M(M-1)}{2R},
 \tag{7}$$

where R is the redundancy of the particular configuration. This leads to

$$L_t = L_r = \frac{\frac{K}{2} \times \left(\frac{K}{2} - 1\right)}{2R} = \frac{K(K-2)}{8R}. \quad (8)$$

Now we can find the aperture offered by the sum co-array as follows

$$L_v = DL_r + L_t = L_t(D+1) = L_t(2L_t+2) = 2L_t(L_t+1)$$

Using Eq. (8),

$$\begin{aligned} L_v &= \left(\frac{K^2 - 2K}{4R}\right) \left(\left\{\frac{K^2 - 2K}{8R}\right\} + 1\right) \\ &= \frac{(K^2 - 2K)(K^2 - 2K + 8R)}{32R^2}. \end{aligned} \quad (9)$$

Similarly, the overall DOF offered by the MIMO array with MRA base can be obtained using Eq. (9) as

$$\begin{aligned} DOF &= 2L_v + 1 \\ &= \left[\left\{ \frac{(K^2 - 2K)(K^2 - 2K + 8R)}{16R^2} \right\} + 1 \right] \\ &= \left(\frac{K^2 - 2K + 4R}{4R} \right)^2 \end{aligned} \quad (10)$$

It is known in MRA theory that a maximum aperture is possible when the redundancy is minimum. We consider the ideal redundancy value of $R = 1$. This gives the upper bound on the attainable sum co-array aperture and DOF using Eqs. (9) and (10).

$$\begin{aligned} L_{v,\max} &= \frac{(K^2 - 2K)(K^2 - 2K + 8)}{32} \\ DOF_{\max} &= \left(\frac{K^2 - 2K + 4}{4} \right)^2 \end{aligned} \quad (11)$$

2.3.2. Case 2: K Is Odd

In a similar manner, one can get the expressions for an odd number of sensors. Let R_1 and R_2 be the redundancies of the transmitting and receiving MRAs, respectively. We have,

$$L_t = \frac{M(M-1)}{2R_1}; \quad L_r = \frac{N(N-1)}{2R_2} \quad (12)$$

Substituting M and N from Eq. (6), we get

$$\begin{aligned} L_t &= \frac{K^2 - 4K + 3}{8R_1}; \quad L_r = \frac{K^2 - 1}{8R_2} \\ D &= \frac{K^2 - 4K + 3 + 4R_1}{4R_1} \end{aligned} \quad (13)$$

The aperture of the sum co-array has to be derived from the above values. The maximum aperture is obtained when the redundancies R_1 and R_2 are equal to one. Therefore, the upper bounds for L_v and DOF for the case of odd number of sensors are

$$\begin{aligned} L_{v,\max,odd} &= \frac{K^4 - 4K^3 + 10K^2 - 12K - 4}{32}; \\ DOF &= 2L_v + 1; \\ DOF_{\max,odd} &= \frac{K^4 - 4K^3 + 10K^2 - 12K + 12}{16}. \end{aligned} \quad (14)$$

It is therefore clear from Eqs. (11) and (14) that the proposed MRA-based MIMO provides $O(K^4)$ DOFs using just K sensors which is typically expected of present-day sparse MIMO radar arrays. More specifically, the MIMO array provides $O(M^2N^2)$ DOFs for $M + N$ sensors.

2.4. Signal/Data Model for DOA Estimation

Radar arrays depict an active sensing scenario wherein the transmitting array illuminates the scene of interest and the receiving array records the reflections obtained from the illuminated objects. The received data at the receivers after matched filtering can be perceived as the observations recorded by the sum co-array. As the sum co-array here is a sparse linear array, it has to be translated to the difference co-array domain such that DOA estimation could be performed on its resulting DCSC [31]. In this paper, we make use of the widely known co-array MUltiple SIgnal Classification (co-array MUSIC) algorithm for DOA estimation [32].

From a signal processing perspective, the signal received by the co-located MIMO array (with matched filters at its receivers) is same as the signal received by an imaginary SIMO array which has the sum co-array as its receiver. The procedure for DOA estimation in sparse arrays using the co-array MUSIC algorithm is explained below in Figure 1. For SIMO arrays, \mathbf{x} denotes the signal received at the SIMO receiver. Therefore, DOA estimation is performed using the DCA of the receiver. For MIMO arrays, \mathbf{x} denotes the combined signal received at the output of all the matched filters in the receiver. It could be thought of as the observations recorded by the sum co-array had it existed. DOA estimation is performed using the DCSC of the MIMO array, i.e., the DCA of the sum co-array.

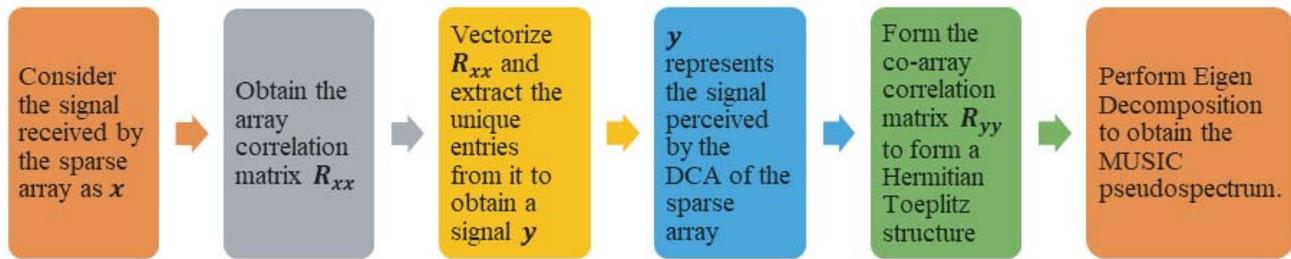


Figure 1. Procedure for DOA estimation in sparse arrays using co-array MUSIC.

3. CREATING THE MIMO ARRAY DESIGN BOOK

First, we construct a look-up table for the MIMO radar array by varying the total number of antenna elements K from 8 to 18. As per Eq. (6), this corresponds to M and N values from 4 to 9. The transmitting and receiving MRAs are obtained through table look-up from Table 2. The sum co-array and its DCSC can be obtained using Eqs. (4) and (5).

Table 3 presents the design book for the proposed MRA-based MIMO radar array. As an example, consider the entries in Table 3 corresponding to $K = 10$. The transmitting and receiving arrays have five sensors each as per Eq. (6). It is evident from Table 2 that a 5-element MRA can provide an aperture of 9. Therefore, $L_t = L_r = 9$; $D = 19$. The receiving module can be constructed using Table 2 and Eq. (3). The resulting sum co-array will have an aperture of $L_v = DL_r + L_t = 180$.

Table 3 also shows the sum co-array apertures that could have been provided by the nested MIMO array and the YINA-based MIMO array for the same number of sensors. As described earlier, the nested MIMO array makes use of prototype TLNAs as transmitting and receiving modules whereas the YINA-based MIMO array uses improved nested arrays at the transmitter and at the receiver. The basic apertures provided by the prototype TLNA and YINA are listed in Table 2.

It can be observed from Table 3 that the proposed MRA-based MIMO array provides an increment of 25% to 45% in the sum co-array aperture compared to the nested MIMO array for the same number of antennas. Additionally, the use of MRAs reduces the effect of mutual coupling in the transmitting

Table 3. Overall aperture of the sum co-array for a given number of sensors.

			MRA-based MIMO array [Proposed]				YINA-based MIMO array				Nested MIMO array			
K — Total sensors	M	N	L_t	D	L_r	L_v	L_{ty}	D_y	L_{ry}	$L_{v,y}$	L_{tn}	D_n	L_{rn}	$L_{v,n}$ $= DL_{rn}$ $+L_{tn}$
8	4	4	6	13	6	84	6	13	6	84	5	11	5	60
9	4	5	6	13	9	123	6	13	9	123	5	11	8	93
10	5	5	9	19	9	180	9	19	9	180	8	17	8	144
11	5	6	9	19	13	256	9	19	13	256	8	17	11	195
12	6	6	13	27	13	364	13	27	13	364	11	23	11	264
13	6	7	13	27	17	472	13	27	17	472	11	23	15	356
14	7	7	17	35	17	612	17	35	17	612	15	31	15	480
15	7	8	17	35	23	822	17	35	22	787	15	31	19	604
16	8	8	23	47	23	1104	22	45	22	1012	19	39	19	760
17	8	9	23	47	29	1386	22	45	27	1237	19	39	24	955
18	9	9	29	59	29	1740	27	55	27	1512	24	49	24	1200

and receiving blocks compared to that of the nested MIMO array given the fact that MRAs are more immune to the effects of mutual coupling than TLNAs [22, 27].

Mutual coupling is a phenomenon where the outputs of closely spaced sensors interfere with each other. The presence of mutual coupling can affect the accuracy of DOA estimation algorithms. Though there are techniques that can overcome the effects of mutual coupling (by decoupling the received data), they are usually computationally expensive and prone to model mismatch [22]. Hence, it is preferable to use a sparse array which is more immune to mutual coupling. The susceptibility of a sparse array to mutual coupling depends on the number of sensor pairs it has with a unit spacing. Two sensors are said to have a unit spacing if they are adjacent to each other and are separated by half wavelength. Arrays that have a large number of adjacent sensors are more susceptible to the effects of mutual coupling than those with fewer unit spacings. As TLNAs have a dense ULA portion in the level 1, they have many sensor pairs with unit spacing and are hence more prone to the effects of mutual coupling compared to MRAs and coprime arrays [27].

A good thing about nested MIMO arrays is that they provide hole-free DCSCs and thereby offer a higher number of DOFs and better angular resolutions than coprime MIMO radar arrays for the same number of sensors [20]. The angular resolution and DOFs offered by a coprime MIMO radar are limited by the central continuous portion of its DCSC. The first nested MIMO array capable of providing $O(K^4)$ DOFs using K sensors was designed using prototype TLNAs at the transmitting and receiving modules and had an enlarged spacing in the transmitter [33]. However, it was pointed out that coprime MIMO radars perform better than such nested MIMO arrays when it comes to mutual coupling [19]. The MRA-MIMO array proposed here combines the best features of both these MIMO array types. Firstly, it has lower mutual coupling than nested MIMO radars owing to the use of MRAs in the transmitting and receiving blocks instead of TLNAs. Secondly, it can provide higher DOFs and better angular resolutions for a given number of sensors compared to nested MIMO array as seen from Table 3. Thirdly, unlike coprime MIMO radars, its DCSC is hole-free and hence the full span of the DCSC is available for DOA estimation. Hence, the choice of MRAs for designing the transmitting and receiving blocks in the proposed MIMO array is justified.

4. SIMULATION METHODOLOGY AND DOA ESTIMATION RESULTS USING THE PROPOSED MRA-MIMO ARRAY

This section describes the simulation methodology and numerical simulation results wherein the angular resolution requirements of a long-range automotive radar driving scenario are considered for DOA estimation.

4.1. Simulation Methodology

Though the MIMO array design book has been created by choosing the total number of available antennas as the starting point, the actual usage of the MIMO array for automotive applications starts from the specification of the desired angular resolution (which gives an indication of the required aperture). The design book has to be then traced in the backward direction to determine the minimum number of sensors needed to achieve the desired resolution.

Assuming that an angular resolution of θ_{res}° is required, the minimum aperture length needed to support this resolution can be obtained using an equation in [34] by solving for L

$$\begin{aligned}\theta_{res} &= \frac{2}{L} \text{ radians}, \\ \theta_{res}^\circ &= \frac{114.6^\circ}{L}, \\ \therefore L &\geq \frac{114.6^\circ}{\theta_{res}^\circ}.\end{aligned}\tag{15}$$

Table 3 has to be checked for the nearest value of L_v , such that $L_v \geq L$. Once this is done, backward tracing can be performed to extract L_t, L_r, M, N and the MRA configurations.

The Root Mean Square Error (RMSE) between the true and the estimated angles gives a measure of DOA estimation accuracy. Considering C independent trials, the RMSE can be expressed as

$$RMSE = \sqrt{\frac{1}{CP} \sum_{p=1}^P \sum_{c=1}^C (\hat{\theta}_{p,c} - \theta_p)^2},\tag{16}$$

where $\hat{\theta}_{p,c}$ denotes the estimate of the p th source in the c th trial.

4.2. Numerical Simulation Results

Simulations were performed using MATLAB 2016a. The widely known co-array MUSIC was used for DOA estimation. The reasons behind the selection of MRAs as the basic building blocks in MIMO radar array are explained hereunder. We illustrate a numerical example to demonstrate the effectiveness of the proposed MRA-based MIMO array over the nested MIMO array design given in [20].

Considering a long-range radar (LRR) driving scenario where a resolution of 1 m at a distance of 125 m is desired. This means that the radar should be able to distinguish two objects which are spaced 1 m apart from each other, 125 m ahead of it as shown in Figure 2. This corresponds to an angular resolution of $1/125$ radians or 0.458° .

As per Eq. (15), an aperture of $L = 251$ would be needed to achieve this resolution. Table 3 shows that the nearest possible aperture offered by the MRA-MIMO sum co-array is $L_v = 256$. Therefore, tracing back the design ladder, a total of 11 sensors would be needed for the MRA-MIMO array. The design needs a 5-element MRA at the transmitter and a 6-element MRA at the receiver.

The transmitting array, receiving array, and the resulting sum co-array are shown in the 2nd row of Table 4. For better illustrations and comparisons, Table 4 additionally lists the MIMO arrays that would have been obtained if the YINA and the TLNA were to be used to design the basic transmitting and receiving blocks using the same number of sensors. For example, the transmitter in the YINA-based MIMO array would be a 5-element YINA. Table 2 shows that such a YINA can offer an aperture of 9. Similarly, the receiver would be a 6-element YINA capable of providing an aperture of 13. The respective transmitting and receiving arrays can then be easily obtained since YINA has closed-form

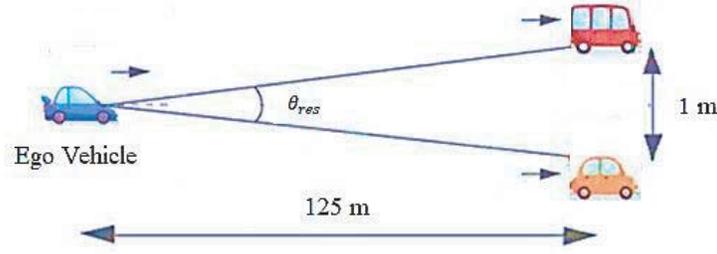


Figure 2. Long-range radar driving scenario.

Table 4. Sum co-arrays of MIMO array with MRAs, YINAs and TLNAs as the basic transmitting-receiving blocks.

Basic blocks	Transmitting array	Receiving array	Sum co-array
MRA	$[0, 2, 5, 8, 9]$	$19 \times [0, 2, 8, 9, 12, 13]$ $\Rightarrow [0, 38, 152, 171, 228, 247]$	$[0, 2, 5, 8, 9, 38, 40, 43, 46, 47, 152, 154, 157, 160, 161, 171, 173, 176, 179, 180, 228, 230, 233, 236, 237, 247, 249, 252, 255, 256]$
YINA	$[0, 1, 4, 7, 9]$	$19 \times [0, 1, 2, 6, 10, 13]$ $\Rightarrow [0, 19, 38, 114, 190, 247]$	$[0, 1, 4, 7, 9, 19, 20, 23, 26, 28, 38, 39, 42, 45, 47, 114, 115, 118, 121, 123, 190, 191, 194, 197, 199, 247, 248, 251, 254, 256]$
TLNA	$[0, 1, 2, 5, 8]$	$17 \times [0, 1, 2, 3, 7, 11]$ $\Rightarrow [0, 17, 34, 51, 119, 187]$	$[0, 1, 2, 5, 8, 17, 18, 19, 22, 25, 34, 35, 36, 39, 42, 51, 52, 53, 56, 59, 119, 120, 121, 124, 127, 187, 188, 189, 192, 195]$

expressions for sensor positions when the number of sensors is given. The 3rd row in Table 4 shows the details of the YINA-based MIMO array. In a similar way, the 4th row describes the sensor positions of the nested MIMO array.

The following simulation parameters were considered. Thirty-one equally-spaced, uncorrelated, non-coherent and stationary sources were assumed to lie in the far-field of the array. An angular separation of $\theta_s^\circ = 0.45^\circ$ was considered between adjacent sources, such that the sources would be uniformly spaced from -6.75° to 6.75° . The Signal-to-Noise Ratio (SNR) was fixed at 10 dB and 500 snapshots were considered. DOA estimation was performed on the DCSC of the MIMO radar using the co-array MUSIC algorithm.

Figure 3 shows the DOA estimation results obtained using co-array MUSIC, ensembled over 200 independent trials. It is seen that the MRA-MIMO and YINA-MIMO arrays have the same performance and can detect all the sources whereas the nested MIMO array fails to do so, owing to its smaller aperture. It has to be noted that all the three MIMO arrays have the same number of total sensors, i.e., $K = 11$.

The RMSE values for the three MIMO arrays were 0.0169, 0.0168, and 7.3114, respectively. It can be concluded that the proposed MRA-based MIMO array provides better resolution than the nested MIMO array. In the present example, the MRA-MIMO provides an increment of 31% ($= \frac{256-195}{195} \times 100\%$) in the sum co-array aperture compared to the nested MIMO array for the same number of sensors.

It is clear that the YINA-based MIMO and MRA-based MIMO arrays perform better than the nested MIMO array as they possess wider apertures for the same number of sensors. Among them, the MRA-based MIMO array offers extra-wide apertures for $K \geq 15$ as seen from Table 3. As an example, for $K = 15$, the sum co-array apertures provided by the MRA MIMO and YINA MIMO arrays are 822 and 787, respectively. Since it is well-known that arrays with larger apertures perform better than those with smaller ones, MRAs are preferred over YINA for constructing the proposed sparse MIMO array.

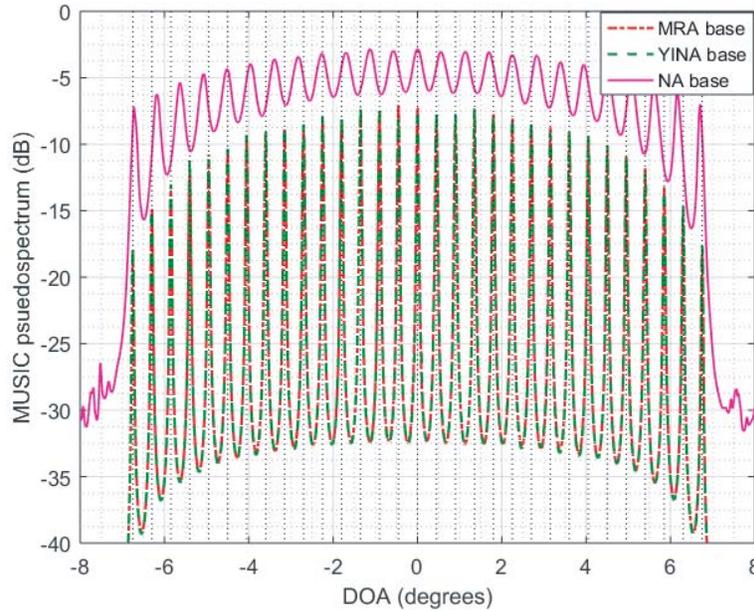


Figure 3. Pseudospectra obtained using the DCSCs of the sparse MIMO arrays given in Table 4. The vertical dotted lines denote the actual DOA angles.

5. DOA ESTIMATION IN SIMO RADARS USING LRLA

In this section, we show how known low-redundancy linear arrays (LRLAs) could be employed for high-resolution DOA estimation in SIMO radars.

Most studies available in the literature compare the performance of sparse arrays and ULAs for a given number of sensors. Obviously, sparse arrays perform well owing to the larger apertures they possess for the same number of sensors. However, the real testimony to the worth of sparse arrays is how closely do they mimic a ULA's performance when the design needs a fixed aperture. Therefore, sparse arrays must be tested against ULAs that offer the same aperture. Automotive radars form one such area where the antenna array is designed for a fixed aperture in view of the space constraints on the vehicle.

SIMO radars are not generally preferred for high-resolution DOA estimation in autonomous vehicles as they would need ULAs with hundreds of antennas to provide the extra-wide apertures needed to achieve the desired resolutions. However, the use of linear sparse arrays as a replacement to ULAs could ease the system complexity if they can provide an acceptable level of estimation accuracy. Here too, MRAs remain a preferred choice among sparse arrays as they provide the largest filled difference co-array and the best resolution for a given number of sensors [7, 35]. As mentioned before, MRAs with twelve or more sensors are named as LRLAs.

5.1. Numerical Simulation Example

Consider the same scenario as described in the simulation example of the previous section. A resolution of 1 m at 125 m, i.e., an angular resolution of 0.458° is desired which corresponds to an aperture of $L = 251$. Sources are uniformly distributed from -6.75° to 6.75° .

First, a table look-up has to be performed to extract an LRLA that can provide the desired aperture. The nearest known LRLA configuration has 28 sensors and can provide an aperture of 259 [36]. The antenna positions for the chosen LRLA are shown in Table 5. To provide the same aperture ($L = 259$), a ULA needs 260 elements.

MRA and LRLA notations given in [6, 36] are based on the $\{a.b.c.d\}$ format or simply $\{a, b, c, d\}$.

Table 5. LRLA configuration for an aperture of 259.

Array	Antenna positions	Aperture
LRLA	[0, 1, 2, 3, 12, 21, 30, 39, 40, 57, 74, 91, 108, 125, 142, 159, 176, 193, 210, 227, 235, 243, 251, 253, 256, 257, 258, 259]	259
ULA	[0, 1, 2, 3, , 259]	259

This format has $n - 1$ entries for a n -element array. For example, an 8-element MRA may be denoted as $\{.1.3.6.6.2.3.2.\}$ or $\{1, 3, 6, 6, 2, 3, 2\}$ or $\{1, 3, 6^2, 2, 3, 2\}$. The power indicates the number of times the given spacing should be repeated. The sensor positions can be deduced from the above representation as $\{0, a, a + b, a + b + c, a + b + c + d\}$. Therefore, the 8-element MRA has sensors at $\{0, 1, 4, 10, 16, 18, 21, 23\}$. The sensor positions shown in Table 5 have been obtained using the LRLA notation $\{1, 1, 1, 9, 9, 9, 9, 1, 17^{11}, 8, 8, 8, 2, 3, 1, 1, 1\}$ in [36].

For simulations, two SIMO radars were considered. One had the LRLA as its receiver and the other had the ULA. DOA estimation was performed using the DCAs of the receiving arrays in both the cases. The ensembled pseudospectra for 200 independent trials are shown in Figure 4. It can be seen that the LRLA provides an acceptable level of performance and can detect all the sources properly.

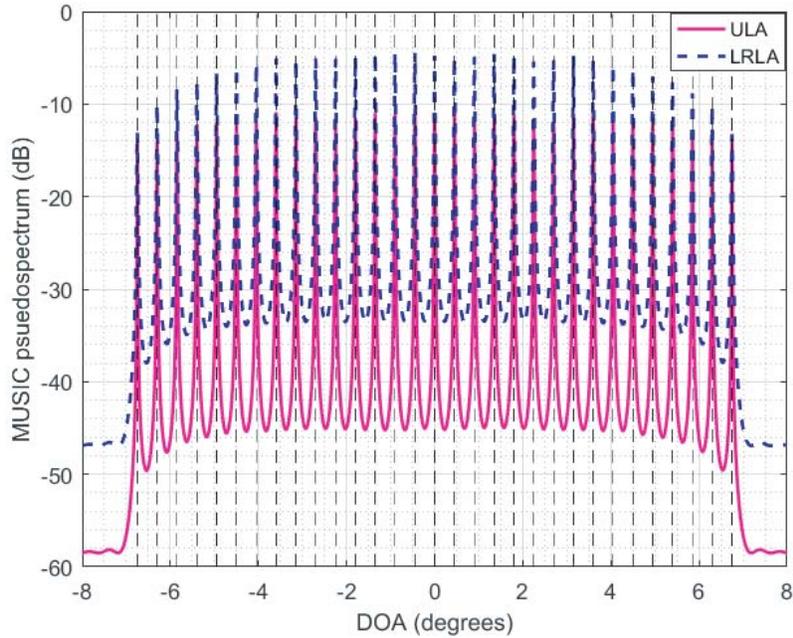


Figure 4. Pseudospectra obtained using the DCAs of the ULA and the LRLA given in Table 5.

5.2. Extreme Resolution Obtainable from a Known LRLA

The largest known LRLA has 37 sensors and can offer an aperture of 465 [36]. As per Eq. (15), this means a resolution of 0.25° or $1/232$ radians. This corresponds to a resolution of 1 m at 232 m from the vehicle. If resolutions finer than 0.25° are needed, the designer must either opt for super-nested arrays (using 43 or more elements, i.e., $N_1 + N_2 \geq 43$; where N_1, N_2 are as defined in [22]) or make use of super-resolution algorithms on existing LRLAs.

6. COMPARING SIMO AND MIMO ARRAYS

This section discusses the pros and cons of each type of radar in the context of the methods presented in the previous sections.

6.1. DOA Estimation Using SIMO and MIMO Radars

The sum co-array of the MRA MIMO array in the second row of Table 4 and the LRLA in Table 5 were used for DOA estimation under the simulation settings mentioned in Section 4. DOA estimation was performed using the DCSC of the MIMO array and the DCA of the SIMO receiver. The results are shown in Figure 5. It is seen that the performance of SIMO is slightly better than that of MIMO, although the difference is not significant. It should, however, be noted that the SIMO array provides an aperture of 259 using 28 sensors whereas the MIMO sum co-array provides an aperture of 256 using 11 sensors.

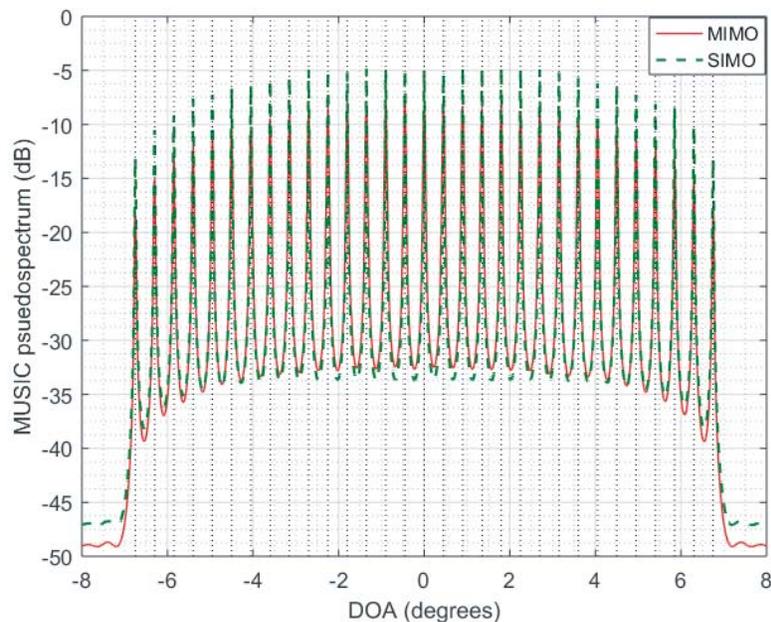


Figure 5. Pseudospectra obtained by using the DCSC of the proposed MIMO array and the DCA of the LRLA in SIMO array.

Firstly, the system designer must choose between SIMO and MIMO arrays. SIMO radars using LRLAs at the receiver are quite simple. Though MIMO radars need fewer sensors than SIMO radars to realize the same aperture, they depend on matched filters and orthogonal processing. They have huge computational load owing to the coherent processing at the receiver and additionally suffer from crosstalk between the transmitting and receiving arrays.

The designer should obtain a trade-off among the factors such as system complexity (antennas, radio frequency (RF) chains, data converters etc.), computational cost, coupling effects, and power consumption. Another important consideration is that the SIMO radar offers a real aperture (needs more physical space on the vehicle) whereas the MIMO array provides a virtual aperture (needs less physical space on the vehicle). MIMO radar, therefore, provides the technology to synthesize larger virtual apertures than those physically realizable on the vehicle given its space constraints.

6.2. Applications

The arrays proposed here could be applied in many practical applications such as highway driving assistance, predictive automated emergency braking, adaptive cruise control, urban driving, driverless

public transportation, robot-taxi, warehouse monitoring vans, and traffic monitoring at intersections [4]. Additionally, risk-free and low-speed applications such as lightweight electric buggies could be made fully autonomous and deployed inside airports, railway stations, malls, and convention centers.

7. CONCLUSIONS AND FUTURE SCOPE

It is shown how off-the-shelf MRAs and LRLAs could be efficiently used to achieve high-resolution angle estimation in MIMO and SIMO radars, respectively. The fact that MRAs do not have closed-form expressions for sensor positions or that they need exhaustive searching procedures could easily be overlooked as the angular resolution requirements of automotive radars can be conveniently realized using currently known MRAs and LRLAs. It has been observed that the proposed MRA-MIMO array provides at least 25% wider apertures than the nested MIMO array for the same number of sensors and also makes the transmitting and receiving blocks more immune to the effects of mutual coupling. Additionally, since LRLAs provide an acceptable performance, they can serve as an alternative to ULAs in SIMO radars.

This work offers ample scope for future extensions as described below: —

- Algorithms that perform well in the presence of coherent arrivals, low signal-to-noise ratio (SNR) conditions and fewer snapshots could be used in place of co-array MUSIC for practical DOA estimation in automotive radars.
- Given their robustness against single-element failures, two-fold redundancy arrays (TFRAs) [37] could be used as the basic building blocks for designing newer sparse MIMO arrays.

APPENDIX A. PROOF THAT THE PROPOSED MIMO ARRAY GENERATES A HOLE-FREE DCSC

Consider the DCSC of the proposed MIMO array as given in Eq. (5). By substituting Eq. (4) in Eq. (5), the following equation can be obtained

$$\begin{aligned} \{x_{dcsc}\} &= \{t_m + Dr_n - (t_{m'} + DR_{n'}) | m, m' = 1, 2, \dots, M; n, n' = 1, 2, \dots, N\}, \\ \{x_{dcsc}\} &= \{t_m - t_{m'} + D(r_n - r_{n'}) | m, m' = 1, 2, \dots, M; n, n' = 1, 2, \dots, N\}. \end{aligned} \quad (\text{A1})$$

The DCA of the transmitting array is given by the unique and sorted entries of the difference set $\{x_{dca,tx}\} = \{t_m - t_{m'} | m, m' = 1, 2, \dots, M\}$. As the transmitting array is an MRA of aperture L_t , its DCA is hole-free and contains all the spatial lags between $-L_t$ to L_t and can be represented in a simplified form as follows

$$\{x_{dca,tx}\} = \{t_m - t_{m'} | m, m' = 1, 2, \dots, M\} = \{p | -L_t \leq p \leq L_t\}, \quad (\text{A2})$$

where p denotes consecutive integers between $-L_t$ to L_t representing the hole-free DCA. Similar to all the other sensor positions considered in this paper, p is also normalized to half wavelength and indicates the locations of the imaginary sensors in the DCA of the transmitter.

In a similar manner, the DCA of the receiving array can also be obtained in a two-step process. Firstly, as mentioned in Table 1, the basic receiving module is an MRA of aperture L_r (before incorporating the enlarged spacing factor D). Since, this receiving module is an MRA, its DCA is also hole-free between $-L_r$ to L_r and is given by

$$\{x_{dca,rn}\} = \{r_n - r_{n'} | n, n' = 1, 2, \dots, N\} = \{q | -L_r \leq q \leq L_r\}, \quad (\text{A3})$$

where q denotes consecutive integers between $-L_r$ to L_r representing the hole-free DCA of the basic receiving module. Now, the actual receiving array is obtained by multiplying the sensor positions of the base MRA with the enlarged spacing factor D . Therefore, the DCA of the actual receiving array is given by

$$\{x_{dca,rx}\} = \{D(r_n - r_{n'}) | n, n' = 1, 2, \dots, N\} = \{qD | -L_r \leq q \leq L_r\}. \quad (\text{A4})$$

At this point, let D represent an arbitrary integer denoting the enlarged spacing in the receiver. The following steps help in determining the optimum value of D that provides the largest hole-free DCSC in the MIMO array. By substituting Eqs. (A2) and (A4) in Eq. (A1), we obtain

$$\{x_{dcsc}\} = \{p + qD | -L_t \leq p \leq L_t; -L_r \leq q \leq L_r\} \quad (\text{A5})$$

Table A1. Entries of the DCSC of Eq. (A5) for various values of p and q .

		$p = -L_t$	$p = -L_t + 1$...	$p = 0$	$p = 1$...	$p = L_t$
1	$q = -L_r$	$-L_t - DL_r$	$-L_t + 1 - DL_r$...	$-DL_r$	$1 - DL_r$...	$L_t - DL_r$
2	$q = -L_r + 1$	$-L_t - DL_r + D$	$-L_t + 1 - DL_r + D$...	$-DL_r + D$	$1 - DL_r + D$...	$L_t - DL_r + D$
3	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
4	$q = 0$	$-L_t$	$-L_t + 1$...	0	1	...	L_t
5	$q = 1$	$-L_t + D$	$-L_t + 1 + D$...	D	$1 + D$...	$L_t + D$
6	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
7	$q = L_r$	$-L_t + DL_r$	$-L_t + 1 + DL_r$...	DL_r	$1 + DL_r$...	$L_t + DL_r$

The DCSC of the MIMO array will be hole-free if the term $p + qD$ in Eq. (A5) yields consecutive integers. For further analysis, we evaluate the term $p + qD$ for various values of p and q as shown in the following table. Although the values of $\{x_{dcsc}\}$ in Eq. (A5) are arranged row-wise, we use a row and column approach in Table A1 for simplicity in evaluating the term $p + qD$.

Now for all these numbers to be consecutive integers, i.e., for the DCSC to be continuous and hole-free, the difference between the first number of a given row (n th row) and the last number of its previous row ($n - 1$ th row) should be one. For example, consider the first element in the 2nd row and the last element in the 1st row. They will be consecutive integers if the difference between them is one. This gives the optimum value of the spacing factor D .

$$-L_t - DL_r + D - (L_t - DL_r) = 1; \quad \therefore D = 2L_t + 1. \quad (\text{A6})$$

Upon substituting the value of D in Eq. (A5), the term $p + qD$ represents a set of consecutive integers and indicates that the DCSC is hole-free. For example, after substituting the D value, the last number in row 1 becomes $L_t - L_r - 2L_tL_r$ and the first number of row 2 becomes $L_t - L_r - 2L_tL_r + 1$, thereby indicating consecutive integers.

If $D < 2L_t + 1$, there would be repetitions in the DCSC and hence its effective span decreases. This lowers the DOFs offered by the MIMO array. However, the DCSC would still be hole-free. On the other hand, if $D > 2L_t + 1$, there would be missing integers in the term $p + qD$ and hence holes would occur in the DCSC. A part of the above proof for the case of passive sensing using the generalized nested sub array (GNSA) under similar considerations has been discussed in [38].

Therefore, in view of the preceding discussion, it is proved that the optimum value of the enlarged spacing factor D to guarantee the largest hole-free DCSC is $D = 2L_t + 1$.

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