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DERIVATIVE CONSTRAINED ROBUST LCMV BEAMFORMING ALGORITHM

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Abstract—A class of linear derivative constraints, which provides robustness to the conventional narrowband uniform linear array configuration so as to handle broadband and moving jammer sources problem is presented. The robust modification of linear constrained minimum variance (LCMV) algorithm is given to broaden the null widths in the jammer directions. Numerical results show that the proposed algorithm has a better performance in broadband and moving jammer scenarios in terms of maintaining beamwidth broadening and capability of rejecting interferences.

1. INTRODUCTION

Smart antenna technology has found applications in radar, sonar and communication systems for minimizing degradation in signal-to-noise ratio performance owing to unwanted interference [1–9]. One of the important applications of optimum adaptive beamforming is to suppress jammer signals [10, 19]. Usually, the directional pattern nulls formed by beamforming algorithm are extremely sharp. However, in practical communication systems, the high speed jammer motion or broadband signal may bring the jammers out of the nulls.

Recently, a wide number of papers have been devoted to the problem of robust adaptive beamforming algorithm [20]. The limitation of these algorithms is they only consider broadening the beamwidth in the desired signal direction. Derivative constraints are proposed to improve the performance of beamforming algorithm in a broadband element space antenna array processors [21]. The robustness against fast jammer motion is considered in [22], where the robust Hung-Turner (HT) algorithm is proposed. [23] further develop the approach of [22], and incorporate the robustness property into

SMI, LSMI and EP algorithm. In a subsequent paper, the derivative constraints of jammer directions are incorporated with a maximum likelihood characterization of the so-called jammer subspace [24].

The main idea of this paper is to present a new set of constraints for broadening the width of the nulls. For this purpose, the derivative constraints are added to the LCMV beamformer. The simulation results show that the proposed algorithm increases the width of the nulls and performs better than the conventional LCMV algorithms in moving jammer scenarios.

2. DERIVATIVE CONSTRAINTS

We assume the problem for a uniform linear array of M antennas at $d = \lambda/2$ spacing. Here λ is the wavelength. Let L ($L < M$) narrow band jammers impinge into the array from the direction of $\{\theta_{J1}, \theta_{J2}, \ldots, \theta_{JL}\}$ while the look direction is θ_s. Let the jammers be uncorrelated with each other as well as with the signal.

Therefore, the nth snapshot of $n \times 1$ received data vector can be represented as:

$$x(n) = A_J J(n) + S_d a(\theta_s) + N(n) \tag{1}$$

$J = [J_1(n), J_2(n), \ldots, J_L(n)]^T$ is the jammer signal waveforms, and $A_J = [a(\theta_1), a(\theta_2), \ldots, a(\theta_L)]$ is the $M \times L$ matrix array manifold of them. The vector $N(n)$ denotes Gaussian White Noise. As plane waves, $a(\theta_i) = [1, e^{j\varphi_i}, \ldots, e^{j(M-1)\varphi_i}]^T$, $i = 1, 2, \ldots, L$, and $a(\theta_s) = [1, e^{j\varphi_s}, \ldots, e^{j(M-1)\varphi_s}]^T$.

To broaden the width of the nulls, the pth-order derivative of directional pattern $f(\theta)$ respect to $\varphi = \frac{2\pi d}{\lambda}\sin\theta$ at $\theta_{Ji} (i = 1, 2, \ldots, L)$ is set to be zero as follow:

$$\left.\frac{\partial^p f(\theta)}{\partial \varphi^p}\right|_{\theta=\theta_{Ji}} = 0, \quad i = 1, 2, \ldots, L; \;\; p = 1, 2, \ldots, P \tag{2}$$

where $f(\theta) = w^H a(\theta)$, and let w be the adjustable weights defined by $w = [w_1, w_2, \ldots, w_M]^T$.

And hence,

$$\frac{\partial f(\theta)}{\partial \varphi} = j w_2^* \varphi e^{j\varphi} + \ldots + j w_M^* (M-1)\varphi e^{j(M-1)\varphi} = j w^H B a(\theta) \tag{3}$$

where $B = \mathrm{diag}(0, 1, \ldots, M-1)$.

Similarly we have

$$\frac{\partial^p f(\theta)}{\partial \varphi^p} = j^p w^H B^p a(\theta) \tag{4}$$

So Equation (2) becomes

$$w^H B^p A_J = 0 \tag{5}$$

3. ROBUST MODIFICATION OF LINEAR CONSTRAINED MINIMUM VARIANCE BEAMFORMING

In the previous section, the derivative constrained algorithm is discussed. It set the high order derivative of directional pattern in the directions of nulls to be zero to broad the null widths. With the derivative constraints, one can expect a better performance of the algorithm in the broadband and moving jammer scenarios. Below, how to use these constraints in the LCMV algorithm is studied. Besides the original linear constraints of LCMV algorithm, the derivative constraints are added. Moreover, we add enforcement constraints to deepen the null depth. Finally, the derivative data control parameter is discussed in this section.

The weights of an optimum antenna array processor are often obtained by solving a LCMV problem. The objective function is the mean output power (variance), and the constraint space is a set of linear equations that ensure a constant gain in a specified direction known as the look direction. The LCMV optimization results in a set of weights that attenuate all signals except for the look direction signal.

This is expressed mathematically as:

$$\begin{cases} \min \quad P_{out} = E[w^H R_{xx} w] \\ s.t. \quad S^H w = 1 \end{cases} \tag{6}$$

where R_{xx} denotes the covariance matrix of $x(n)$, i.e., $R_{xx} = E\{x(n)x^H(n)\}$ where $E\{\ \}$ denotes expectation. The constraints vector S is a column vector, which is to assure the output desired signal power to be a constant. The optimal weight vector w_{opt} to (6) is given by

$$w_{opt} = (S^T R_{xx}^{-1} S)^{-1} R_{xx}^{-1} S \tag{7}$$

Adding the derivative constraints into the constraint matrix, Equation (6) becomes:

$$\begin{cases} \min P_{out} & = E[w^H R_{xx} w] \\ S^H w & = 1 \\ w^H B^p A_J & = 0 \end{cases} \tag{8}$$

The algorithm with the derivative constraints of directional pattern steer wider nulls in the directions of the jammer signals, but the depth of the nulls is reduced correspondingly.

To overcome the depth decrease, enforcement constraints of the interference signal directions are added. Then the problem becomes:

$$\begin{cases} \min P_{out} & = E[w^H R_{xx} w] \\ S^H w & = 1 \\ w^H B^p A_J & = 0 \\ w^H A_J & = \varepsilon \end{cases} \tag{9}$$

where ε is a small quantity which is discussed in the following paragraph.

Let us discuss the choice of the parameter ε in the robust LCMV methods. The real positive weight ε controls the relative contribution of the "derivative" data. With ε increasing, the contribution of the derivative data decreases. The optimum choice of ε is depend on the practical situation. If ε is small, the enforcement constraints make the antenna pattern severely decline in the jammer direction. But in this situation, the antenna pattern is too sensitive to the direction error. Conversely, when ε is large, the nulls is wide enough, but the depth decreased significantly. As a result, the jammer power is not sufficiently suppressed. Therefore, we should find a value of ε from the compromise between null depth and width of the adapted pattern. In Fig. 1, the null depth and width performance of the derivative constrained LCMV algorithm is presented in terms of ε. It is observed that the null depth decrease with the increase of ε while the null width does the opposite way.

4. NUMERICAL RESULTS

4.1. Directional Patterns

Before testing wideband and moving jammer scenarios, it is useful to consider how the constraints modify the directional pattern of adaptive array. Experiments on system identifications are carried out

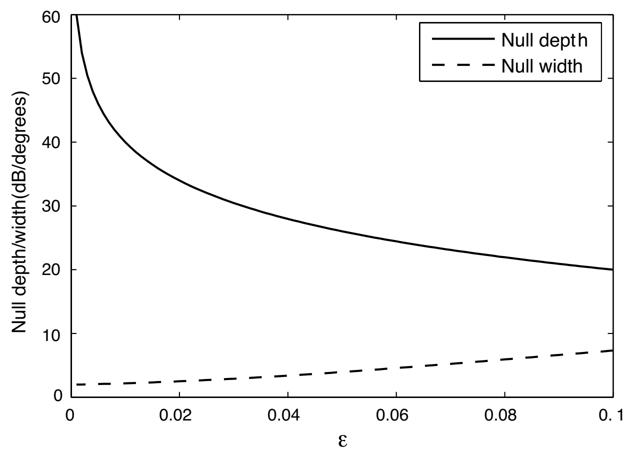

Figure 1. The null depth and the null width versus ε.

to evaluate the performance of the proposed derivative constrained LCMV algorithm, referred as LCMV. It is parameterized by the size of the 8-element linear antenna array.

Figure 2 shows the beam pattern of the conventional LCMV algorithm and the robust LCMV algorithm for a desired signal direction at 80° while two narrowband nonmoving jammers impinging from the directions 30° and 140°. We assumed INR = 40 dB for each jammer and SNR = 0 dB in each sensor, respectively. The value of the ϵ is 0.001. The results of computer simulation verify that the derivative constrained LCMV algorithm achieves broader nulls in the jammer directions. The influence of the ϵ on beam pattern is given in Fig. 3. It obviously shows that when the ϵ increases, the null depth decreases fast, but the raise of the null width is not so distinctly.

4.2. Moving Jammers

In this example, we considered the case of two moving jammers. The jammer angular change with the snapshots, the trajectories of which are $\theta_{J1}(i) = 30° + 5\sin(i/10)$, $\theta_{J2}(i) = 140° - 5\cos(i/15)$. The value of the ϵ is 0.002. The array element number is 16, and other conditions are the same as that in the last example. Fig. 4 shows the output SINR of the LCMV algorithm and the proposed algorithm for 200 snapshots. Simulation results show that the proposed robust algorithms perform better than conventional algorithms in a moving jammer scenarios. By using the derivative constraints, the output SINR is improved about 4 dB.

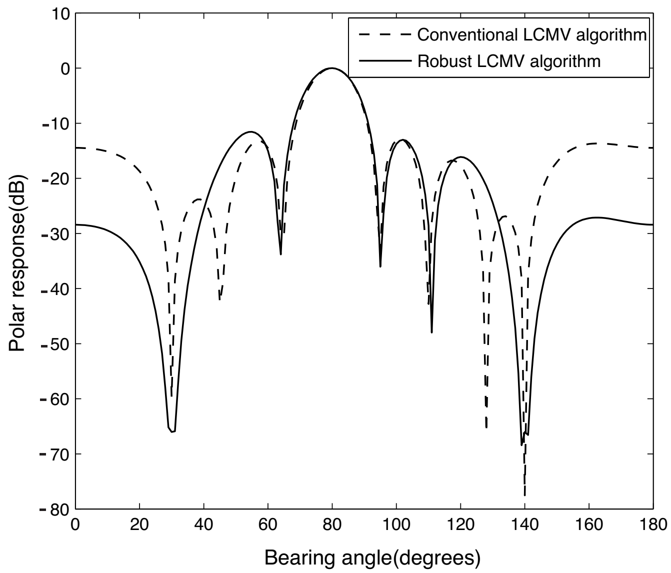

Figure 2. The beam pattern of an 8-element linear array with two nonmoving jammer sources impinging from direction 30° and 140°.

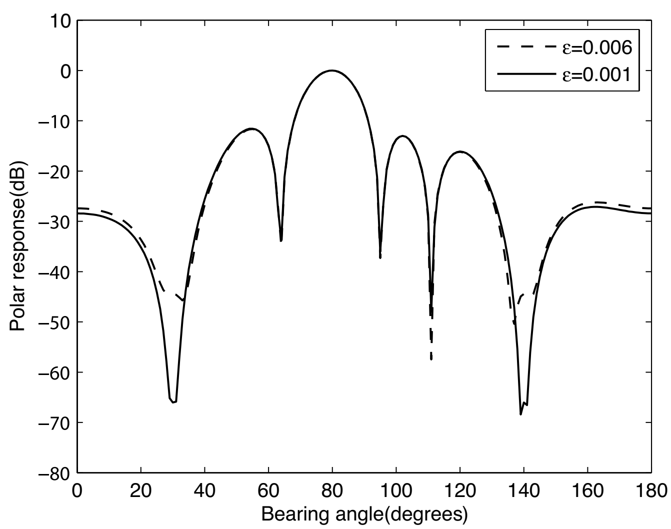

Figure 3. The beam pattern with different ϵ.

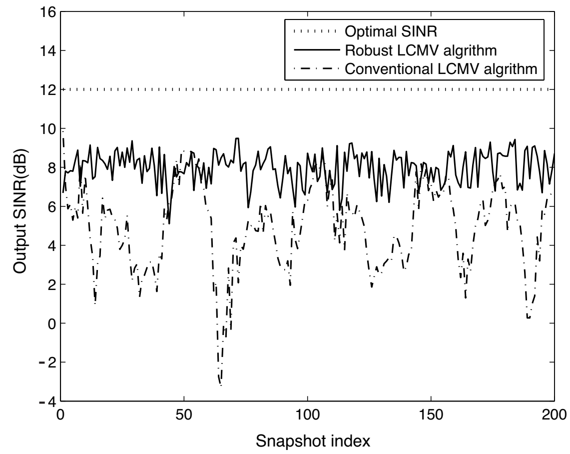

Figure 4. The output SINR of a 16-element linear array with two moving jammer sources.

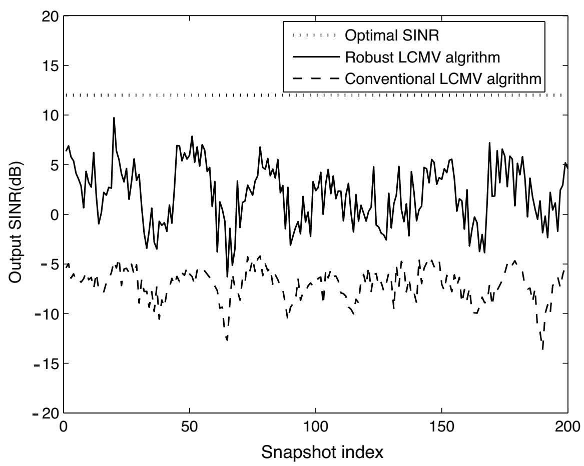

Figure 5. The output SINR of a 16-element linear array with broadband jammer source.

4.3. Broadband Jammers

The single broadband jammer source is considered in the next example. The jammer imping on the 16-element array from the direction 30° and has the bandwidth 10% as compared with the central frequency. The value of the ϵ is the same as that in the last example. Fig. 5 shows the output SINR of the conventional LCMV algorithm and constrained

LCMV algorithm. It clear from the figure that the conventional LCMV algorithm fails when the jammer is broadband signal while the robust LCMV algorithm retains low losses.

5. CONCLUSIONS

This paper has imposed additional constraints known as the derivative constraints on LCMV algorithm to maintain a broader null of the spatial power response width in the vicinity of interference directions. The optimal weight vector is solved by minimizing the weight vector subject to linear and derivative constraints on the weight vector. Results of computer simulations have demonstrated the effects of introducing derivative constraints to the optimum processor. As a consequence of derivative constraints, the null widths are broadening to alleviate the broadband and moving jammer problem.

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