

## NONUNIFORM TRANSMISSION LINES AS COMPACT UNIFORM TRANSMISSION LINES

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**Abstract**—In this paper, we propose a new way to compact the transmission lines, which has a general application to miniaturization of RF and microwave circuits. In this way, we use Nonuniform Transmission Lines (NTLs) instead of Uniform Transmission Lines (UTLs). To synthesize the desired Compact Length Transmission Lines (CLTLs), the characteristic impedance function of the NTLs is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed structures is verified using some examples.

### 1. INTRODUCTION

We have to compact RF and microwave circuits in many applications. On the other hand, many RF and microwave circuits contain one or some single transmission lines of specified length to work at a desired frequency. Therefore, one possible way to compact the circuits is to compact (to reduce the length of) the transmission lines. Some efforts have been done to compact the transmission lines such as using DGS [1], EBG [2], high impedance and meandering lines [3], fractal lines [4] and stepped stubs [5]. In this paper, we propose a new way to compact the transmission lines. In this way, we use Nonuniform Transmission Lines (NTLs) instead of Uniform Transmission Lines (UTLs). To synthesize the desired Compact Length Transmission Lines (CLTLs), the characteristic impedance function of the NTLs is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed structures as is verified using some examples.

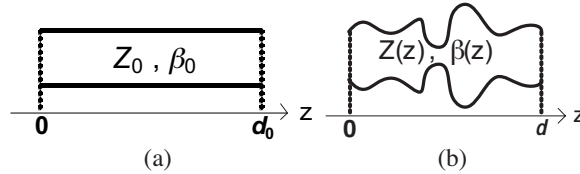
## 2. INTRODUCING CLTLS

In this section CLTLS are introduced. Figure 1(a) depicts a typical uniform transmission line with length  $d_0$ , whose characteristic impedance is  $Z_0$  and whose propagation constant at frequency  $f$  is  $\beta_0$ . Also, Figure 1(b) depicts a typical nonuniform transmission line with length  $d$ , whose characteristic impedance is  $Z(z)$  and whose propagation constant at frequency  $f$  is  $\beta(z)$ . We would like to design an NTL so that its  $ABCD$  parameters at frequency  $f_0$  be equal to those of a uniform transmission line. Furthermore, if we choose the length of the NTL,  $d$ , smaller than that of the uniform transmission line,  $d_0$ , the designed NTL will be a Compact Length Transmission Line (CLTL), in fact.

The  $ABCD$  parameters of the uniform transmission line is well known as follows

$$\begin{bmatrix} A_0(f) & B_0(f) \\ C_0(f) & D_0(f) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & jZ_0 \sin(\theta) \\ jZ_0^{-1} \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (1)$$

where  $\theta = \beta_0 d_0$  is the electrical length of the desired uniform transmission line. Moreover, we can use some methods to obtain the  $ABCD$  parameters of NTLs such as cascading many short sections [6, 7], finite difference [8], Taylor's series expansion [9], Fourier series expansion [10], the equivalent sources method [11], the method of Moments [12] and some approximate closed form solutions [13].



**Figure 1.** (a) A typical uniform transmission line (UTL). (b) A typical nonuniform transmission line (NTL).

## 3. SYNTHESIS OF CLTLS

In this section a general method is proposed to design optimally microstrip CLTLS. Firstly, we consider the following truncated Fourier series expansion for the normalized characteristic impedance  $\bar{Z}(z) = Z(z)/Z_0$ .

$$\ln(\bar{Z}(z)) = \sum_{n=0}^N C_n \cos(2\pi n z / d) \quad (2)$$

An optimum designed CLTL has to have the  $ABCD$  parameters as close as possible to the  $ABCD$  parameters of desired uniform transmission line. Therefore, the optimum values of the coefficients  $C_n$  in (2) can be obtained through minimizing the following error function.

$$\text{Error} = \sqrt{\frac{1}{4} \left( |A - A_0|^2 + Z_0^{-2} |B - B_0|^2 + Z_0^2 |C - C_0|^2 + |D - D_0|^2 \right)} \quad (3)$$

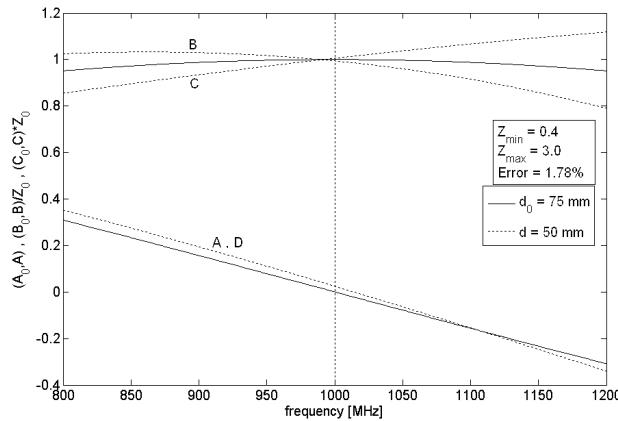
Moreover, the above defined error function should be restricted by some constraints such as easy fabrication and physical matching at two ends, like as the followings

$$\bar{Z}_{\min} \leq \bar{Z}(z) \leq \bar{Z}_{\max} \quad (4)$$

$$\bar{Z}(0) = \bar{Z}(d) = 1 \quad (5)$$

where  $\bar{Z}_{\min}$  and  $\bar{Z}_{\max}$  are the minimum and maximum available normalized characteristic impedance, respectively.

To solve the above constrained minimization problem, we can use the `fmincon.m` file in the MATLAB program. `fmincon` uses a Sequential Quadratic Programming (SQP) method, in which a Quadratic Programming (QP) subproblem is solved at each its iteration.

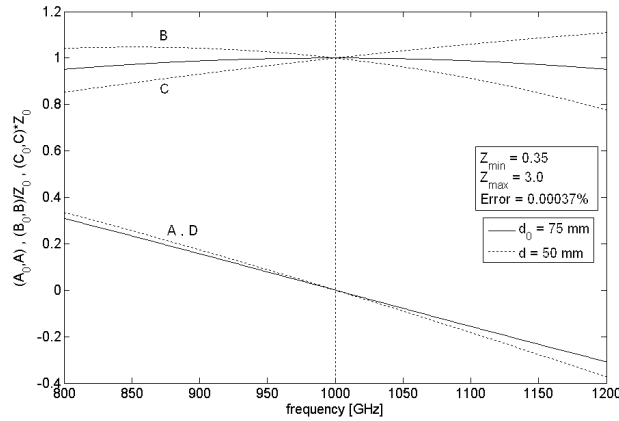


**Figure 2.** The  $ABCD$  parameters of the desired UTL and those of the designed CLTLs for min  $\bar{Z}_{\min} = 0.4$  and max  $\bar{Z}_{\max} = 3$ .

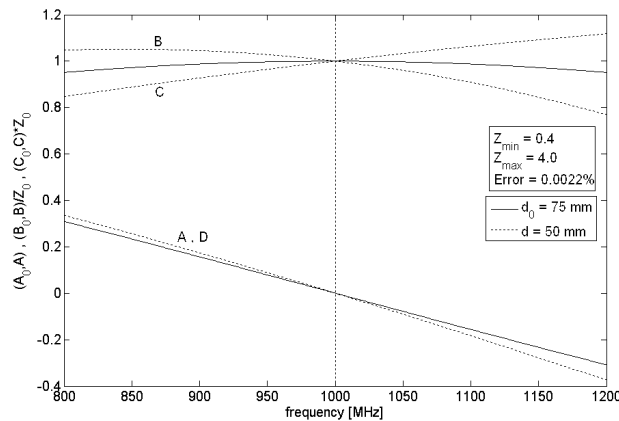
#### 4. EXAMPLES AND RESULTS

Consider an air filled UTL of length  $d_0 = 75$  mm and  $Z_0 = 50 \Omega$ , which acts as a quarter wavelength at frequency  $f_0 = 1000$  MHz. Three

CLTLs have been optimally designed at frequency  $f_0 = 1000$  MHz considering  $d = 50$  mm (33% compactness),  $N = 10$  for three different constraints. Figures 2–4 illustrate the  $ABCD$  parameters of the desired UTL and those of the designed CLTLs versus frequency along with the resulted errors for three cases of: 1. ( $\bar{Z}_{\min} = 0.4$  and  $\bar{Z}_{\max} = 3$ ), 2. ( $\bar{Z}_{\min} = 0.35$  and  $\bar{Z}_{\max} = 3$ ) and 3. ( $\bar{Z}_{\min} = 0.4$  and  $\bar{Z}_{\max} = 4$ ). Also, Table 1 and Figure 5 show the optimum values of the coefficients  $C_n$  and the resulted  $\bar{Z}(z)$ , respectively. It is observed that

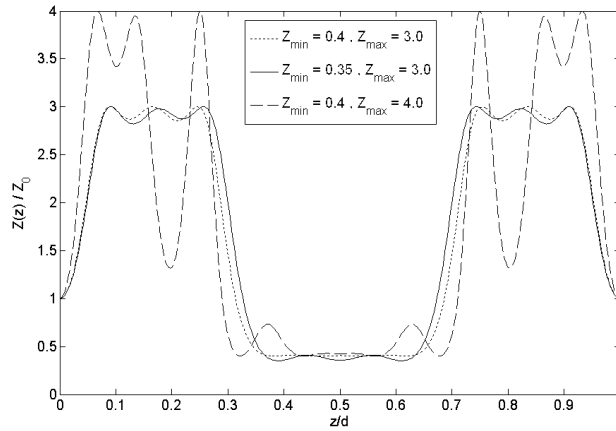


**Figure 3.** The  $ABCD$  parameters of the desired UTL and those of the designed CLTLs for min  $\bar{Z}_{\min} = 0.35$  and max  $\bar{Z}_{\max} = 3$ .

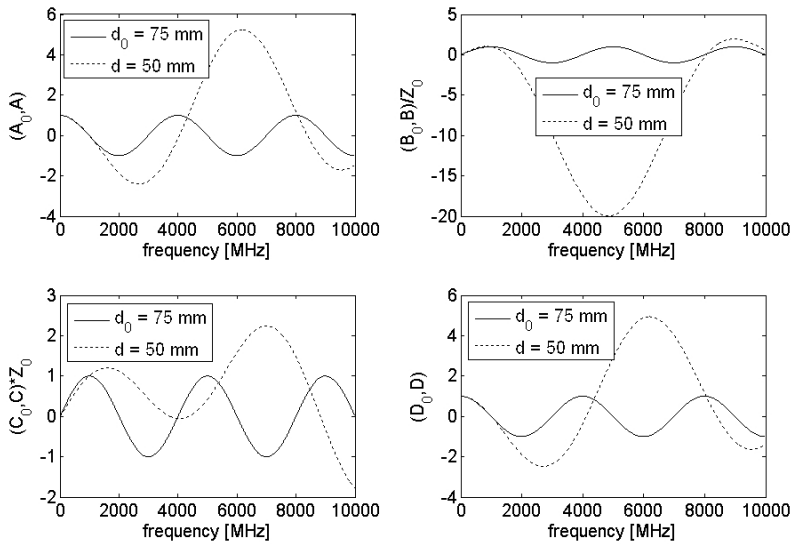


**Figure 4.** The  $ABCD$  parameters of the desired UTL and those of the designed CLTLs for min  $\bar{Z}_{\min} = 0.4$  and max  $\bar{Z}_{\max} = 4$ .

the  $ABCD$  parameters of the designed CLTLs are very close to those of the desired UTL. Of course, as the  $\bar{Z}_{\max}$  is increased or specially  $\bar{Z}_{\min}$  is reduced, the resulted error decreases. It is seen that the designed



**Figure 5.** The normalized characteristic impedance  $\bar{Z}(z)$  of the desired CLTLs.



**Figure 6.** The  $ABCD$  parameters of the desired UTL and those of the designed CLTLs for min  $\bar{Z}_{\min} = 0.35$  and max  $\bar{Z}_{\max} = 3$ .

CLTLs have no discontinuity. Also, one can see that the  $ABCD$  parameters of CLTLs have not a completely periodic behavior versus frequency and consequently the CLTLs have not spurious responses in contrary to UTLs. This is seen in Figure 6, which illustrates the  $ABCD$  parameters of the desired UTL and those of the designed CLTL in a broad frequency range for the case of ( $\bar{Z}_{\min} = 0.35$  and  $\bar{Z}_{\max} = 3$ ).

**Table 1.** The optimum values of the coefficients  $C_n$ .

	$C_0$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$	$C_8$	$C_9$	$C_{10}$
Case 1	0.2426	0.9882	-0.5762	-0.2974	0.1258	-0.1723	-0.1894	0.0139	-0.0461	-0.0807	-0.0084
Case 2	0.2684	0.9748	-0.6585	-0.2154	0.1333	-0.2456	-0.1399	0.0380	-0.1128	-0.0352	-0.0070
Case 3	0.2448	1.0423	-0.2859	-0.2399	-0.0170	-0.1327	-0.3757	-0.0615	0.2509	-0.1795	-0.2460

## 5. CONCLUSION

A new way to compact the transmission lines was proposed. In this way, we use Nonuniform Transmission Lines (NTLs) instead of Uniform Transmission Lines (UTLs). To synthesize the desired Compact Length Transmission Lines (CLTLs), the characteristic impedance function of the NTLs is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed structures is verified using some examples. It was seen that the  $ABCD$  parameters of the designed CLTLs could be close to those of the desired UTLs and as the maximum available impedance is increased or the minimum available impedance is reduced, the resulted error decreases. The designed CLTLs have no discontinuity and have not spurious responses in contrary to UTLs.

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