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COMPRESSIVE ESTIMATION OF CLUSTER-SPARSE CHANNELS

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Abstract—Cluster-sparse multipath channels, i.e., non-zero taps occurring in clusters, exist frequently in many communication systems, e.g., underwater acoustic (UWA), ultra-wide band (UWB), and multiple-antenna communication systems. Conventional sparse channel estimation methods often ignore the additional structure in the problem formulation. In this paper, we propose an improved compressive channel estimation (CCE) method using block orthogonal matching pursuit algorithm (BOMP) based on the cluster-sparse channel model. Making explicit use of the concept of cluster-sparsity can yield better estimation performance than the conventional sparse channel estimation methods. Compressive sensing utilizes cluster-sparse information to improve the estimation performance by further mitigating the coherence in training signal matrix. Finally, we present the simulation results to confirm the performance of the proposed method based on cluster-sparse.

1. INTRODUCTION

In wireless broadband communication systems, frequency-selective fading is generally induced by the reflection, diffraction and scattering of the transmitted signals due to the buildings, large moving vehicles,

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mountains, etc.. Such fading phenomenon distorts received signals and poses critical challenges in the design of communication systems for high-rate and high-mobility wireless communication applications. High-rate data symbols, after transmitting through multipath channel, often spread into neighboring symbol periods, and cause serious inter-symbol interference (ISI) at the receiver side. The frequency-selective fading significantly affects communication system performance. High-rate data transmission over multipath channel often gives rise to a large number of propagation parameters in different signal space. However, exact knowledge of these parameters is not critical for reliable communication of data over the multipath channel. Rather, we are only interested in characterizing the interaction between the physical propagation environment and the transmitter/receiver signal space. Hence, channel estimation techniques try to capture this relationship. The conventional sparse channel model assumes that nonzero taps are distributed randomly in a channel vector $\mathbf{h}$ [1, 2]. However, in many propagation environments, there exist several big obstacles, e.g., buildings and hilly-terrains environment, which give rise to cluster-structure in multipath channel [3, 4]. Hence, accurate channel estimation becomes a fundamental problem of such communication systems.

Hence, accurate channel estimation becomes a fundamental problem of such communication systems. In last several years, various linear estimation methods have been proposed based on the assumption of rich multipath channel model which is shown in Fig. 1. However, recently, a lot of physical channel measurements verified the channel taps exhibit sparse distribution which is shown in Fig. 2. Hence,

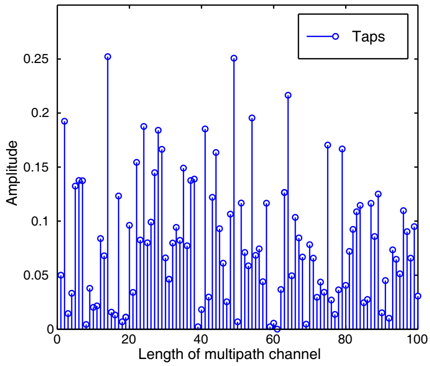

Figure 1. Rich multipath fading channel model.

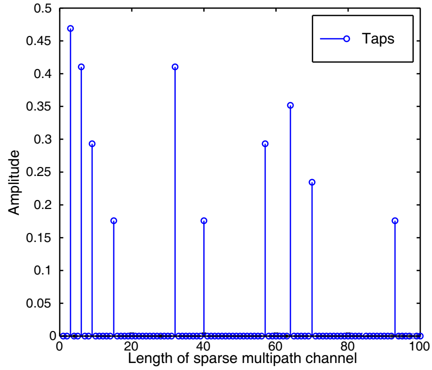

Figure 2. Sparse-based multi-path fading channel model.

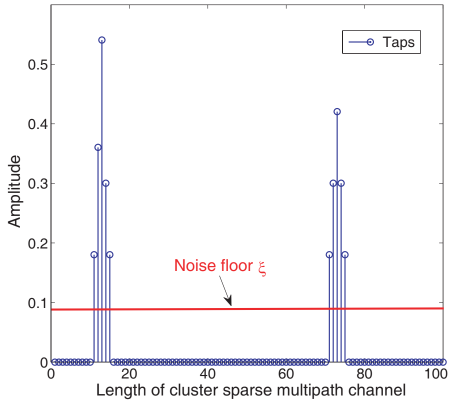

Figure 3. Cluster-sparse multipath fading channel model.

CCE methods have been proposed by exploiting the channel sparsity. Compared to traditional linear methods, CCE has the following advantages: First, by utilizing shorter training sequence, we still attain the same estimation performance. Hence, CCE can improve the spectral efficiency. Second, by exploiting channel sparsity, less channel freedom of degree is acquired and hence the lower bound of estimator is obtained [1]. Recent years, sparse-based channel estimation methods have been proposed [1, 5–7]. Many real-world channels of practical interest, such as in underwater acoustic communication [8], terrestrial signals transmission of high definition television (HDTV) [9] and residential ultra wideband (UWB) systems [10–12], tend to have sparse or approximately sparse impulse responses which is shown in Fig. 3, and conventional linear channel estimation methods such as the least-squares method fail to capitalize on the anticipated sparsity.

In contrast to the existing works in the sparse approximation and compressive sensing literature on sparse channel estimation and based on our previous works [13–15], in this work, we proposed a compressive cluster-sparse channel estimation method using block orthogonal matching pursuit algorithm (BOMP). Accurately, the proposed algorithm serves only for cluster sparse signal recovers in noiseless case [16, 17]. And the BOMP algorithm based on sensing dictionary has been proposed [14]. All of the proposed methods are based on the noiseless case. And also our proposed method differs from the pioneering work on the noise sparse signal recovery using orthogonal matching pursuit algorithm (OMP) [18–21]. The main difference is that OMP-based recovery method neglects cluster-structure in the signal while BOMP-based method can exploit the cluster-structure information well. Hereby, more accurate estimation performance can be obtained. We compare the OMP-based sparse channel estimation performance by computer simulations in Section 4 in this paper. Due to on real channel estimation, noise interference is unavoidable and robust channel estimator is required. In this paper, we propose a cluster-sparse channel estimation method using the modified BOMP for noise-interference and fading channels.

Throughout the paper, we denote vectors by boldface lowercase letters, e.g., x, and matrices by boldface uppercase letters, e.g., $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]$, the $N \times N$ identity matrix is written as $\mathbf{I}_N$. For a given matrix $\mathbf{X}$, $\mathbf{X}^T$, $\mathbf{X}^H$, and $\mathbf{X}^\dagger$ denote its transpose, conjugate transpose, trace, and pseudo inverse, respectively. $\langle \mathbf{x}_i, \mathbf{x}_j \rangle$ denotes vector inner product operation between $\mathbf{x}_i$ and $\mathbf{x}_j$. For a given vector $\mathbf{h}$, its Euclidean norm is $\|\mathbf{h}\|_2 = \sqrt{\mathbf{h}^H\mathbf{h}}$, ℓ_1-norm is denoted by $\|x\|_1 = \sum_i |h_i|$, $\|x\|_\infty = \max_i |h_i|$ is the ℓ_∞-norm which finds the maximum entry in the vector, $\|x\|_0$ counts the number of nonzero taps and $\|\mathbf{h}\|_{C,0}$ denotes its cluster sparse measurement.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Assuming that the multipath channel comprises L paths, the channel impulse response, $\mathbf{h}$, can be expressed as [22, 23]

$$\mathbf{h} = \sum\nolimits_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \tag{1}$$

where h_l and τ_l are the complex-valued path gain with $E[\sum_{l=0}^{L-1} |h_l|^2] = 1$ ($E[\cdot]$ denotes the expectation operation) and the symbol-spaced time delay of the l-th path, respectively. In Eq. (1), we assumed that the channel satisfies sparse distribution, i.e., most of channel taps h_l, $l = 0, 1, \ldots, L-1$, are zero or approximately zero. The channel vector $\mathbf{h}$ is K-sparsity if

$$\|\mathbf{h}\|_0 = K \ll L, \tag{2}$$

where $\|\cdot\|_0$ denotes ℓ_0 operation which counts the number of nonzero coefficient in a vector. The training signal $\boldsymbol{x}$ is denoted by $\boldsymbol{x}$ with normalized $\mathbf{E}[|\mathbf{x}|^2] = 1$. The corresponding complex baseband transmitted signal and channel output are related as

$$y(t) = \sum\nolimits_{l=0}^{L-1} h_l \mathbf{x}(t - \tau_l) + v(t). \tag{3}$$

Eq. (3) can be written using the matrix form as

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{n}, \tag{4}$$

where $\mathbf{y}$ is the $N + L - 1$ dimensional complex measurement signal vector, $\mathbf{X}$ is the $(N + L - 1) \times L$ complex circultant training signal matrix with first row $[\mathbf{x}^T, \mathbf{0}_{1\times(L-1)}]^T$ and $\mathbf{h}$ is given in Eq. (1). As discussed in cluster-sparse model [14, 16, 17], the nonzero taps of $\mathbf{h}$ appear in clusters rather than being arbitrarily spread over the vector. We assume that the vector $\mathbf{h} \in \mathbb{C}^{L\times 1}$ is a concatenation of C blocks and each block has d channel taps. Hence, the cluster-sparse multipath channel $\mathbf{h}$ can be rewritten as

$$\mathbf{h} = [\underbrace{h_1, \ldots, h_d}_{\mathbf{h}^T[1]}, \underbrace{h_{d+1}, \ldots, h_{2d}}_{\mathbf{h}^T[2]}, \ldots, \underbrace{h_{L-d+1}, \ldots, h_L}_{\mathbf{h}^T[c]}]^T \tag{5}$$

where $L = Cd$. Due to the unavoidable noise and channel multipath fading, based on the proposed method of block-sparse measurement in noiseless case, we extend the previous work to the case of presence of noise in this paper. As a result the sparsity measure of vector $\mathbf{h}$ is defined as

$$\|\mathbf{h}\|_{cluster,0} = \sum_{c=1}^{C} I(\|\mathbf{h}[c]\|_2 > \xi), \tag{6}$$

where

$$I(\|\mathbf{h}[c]\|_2 \geq \xi) = \begin{cases} 1, & \|\mathbf{h}[c]\|_2 \geq \xi \\ 0, & \|\mathbf{h}[c]\|_2 < \xi \end{cases}, \quad c = 1, 2, \ldots C \tag{7}$$

denotes the indicator function. For a cluster channel $\mathbf{h}$, that is to say, the number K of cluster is very small, i.e., $K \leq C$. If a cluster is over the given noise floor ξ and then we count the cluster. In other words, each cluster has several taps over the noise floor ξ. Hence, we can easily show that a K-cluster sparse channel $\mathbf{h}$ is defined as a channel vector that satisfies $\|\mathbf{h}\|_{cluster,0} \leq K$. It is worth noting that K-cluster sparse and T-sparse of a channel $\mathbf{h}$ satisfies $K \leq T$. In the next part, our aim is that how to exploit the K-cluster structure information and to accurate estimate the channel taps in each cluster.

3. COMPRESSIVE ESTIMATION OF CLUSTER-SPARSE CHANNEL

Before discussing the compressive estimation for cluster-sparse channel, we review several fundamental theories in compressive sensing [24, 25]. At first, we introduce a mutual incoherent property (MIP) [26, 27] for training signal matrix $\mathbf{X}$. Due to the cluster-sparse property of the unknown recovered signal, we analyze the block MIP (BMIP) on training signal matrix $\mathbf{X}$ rather than MIP. From the matrix theory perspective, we can verify the BMIP less than MIP in a training signal matrix $\mathbf{X}$ and hence a more accurate channel estimator is obtained. Because the looser MIP is acquired on compressive channel estimation, hence, we can obtain a more accurate sparse channel estimator by using the same training signal matrix (TSM).

3.1. MIP and BMIP of TSM

According to Eq. (6), we rewrite the (complex) training signal matrix $\mathbf{X}$ in a block-structure style as

$$\mathbf{X} = [\underbrace{\mathbf{x}_1, \ldots, \mathbf{x}_d}_{\mathbf{X}[1]}, \underbrace{\mathbf{x}_{q+1}, \ldots, \mathbf{x}_{2d}}_{\mathbf{X}[2]}, \ldots, \underbrace{\mathbf{x}_{L-q+1}, \ldots, \mathbf{x}_N}_{\mathbf{X}[c]}], \tag{8}$$

where each block $\mathbf{x}[c] = [\ x_{c,1} \ \ \ldots \ \ x_{c,d}\ ]$ is an $(N+L-1) \times d$ submatrix. Conventionally, the MIP [26, 27] of the training signal matrix $\mathbf{X}$ is defined by

$$\mu := \max_{(i,j)\neq(k,l)} \left|\left\langle \mathbf{x}_{(i,j)} \mathbf{x}_{(k,l)} \right\rangle\right|, \tag{9}$$

and each of its columns has unit norm. We define the BMIP of training signal matrix $\mathbf{X}$ as

$$\mu_B = \max_{\ell, r\neq\ell} \frac{1}{d} \rho\Big(\mathbf{X}^H[\ell]\mathbf{X}[r]\Big). \tag{10}$$

where $\rho(\mathbf{X}^H[\ell]\mathbf{X}[r])$ denotes the spectral norm of training matrix $\mathbf{X}^H[\ell]\mathbf{X}[r]$. Note that $\mathbf{X}^H[\ell]\mathbf{X}[r]$ is the (ℓ, r)th $d \times d$ block of the $N \times N$ matrix $\mathbf{X}^H\mathbf{X}$. It is easy to see that definition in Eq. (11) is invariant to the choice of orthogonal basis $\mathbf{X}[\ell]$ for $R(\mathbf{X}[\ell])$. This is because

$$\rho\Big(\mathbf{X}^H[\ell]\mathbf{X}[r]\Big) = \rho\Big(\mathbf{X}_\ell^{\mathbf{H}}\mathbf{X}^{\mathbf{H}}[\ell]\mathbf{X}[\mathbf{r}]\mathbf{X}_{\mathbf{r}}\Big). \tag{11}$$

Comparing Eq. (10) with Eq. (11), we can easily show that $0 \leq \mu_B \leq \mu \leq 1$. The detailed proof can be found in [16]. When $d = 1$ in $\mathbf{X}$, μ_{B} reduces to the conventional MIP according to

$$\mu = \max_{\ell, r\neq\ell} \left|\mathbf{x}_\ell^H \mathbf{x}_r\right|. \tag{12}$$

Our goal is to provide accurate recovery conditions on the training matrix $\mathbf{X}$ ensuring that the cluster-sparse channel vector $\mathbf{h}$ can be reconstructed from measurements of the form (1) through computationally efficient algorithms. We extend the conventional mutual incoherence measurement to cluster by defining block-coherence. A sufficient condition for BOMP to robust reconstruct $\mathbf{h}$ is

$$\rho_c\left(\mathbf{X}_0^\dagger \mathbf{X}_0^c\right) < 1, \tag{13}$$

where

$$\rho_c(\mathbf{X}) = \max_j \sum_i \rho(\mathbf{X}[i,j]) \tag{14}$$

where $\rho(\mathbf{X})$ denotes the spectral norm of training matrix $\mathbf{X}$ and returns the largest singular value of $\mathbf{X}$, $\mathbf{X}_0$ denotes the matrix whose blocks correspond to the clusters of $\mathbf{h}$, $\boldsymbol{X}_0^c$ denotes the complementary of $\mathbf{X}_0$ in $\mathbf{X}$, and $\mathbf{X}[i,j]$ is the (i,j)th block of $\mathbf{X}$ [16]. Eldar et al. proved that any block of K-sparse vector $\mathbf{h}$ can be recovered from Eq. (1) using BOMP if

$$Kc < \frac{1}{2}\left(\frac{1}{\mu_B} + c - (c-1)\frac{\eta}{\mu_B}\right), \tag{15}$$

where

$$\eta = \max_l \max_{i,j\neq i} \left|\mathbf{x}_i^H \mathbf{x}_j\right|, \quad \mathbf{x}_i,\ \mathbf{x}_j \in \mathbf{X}[l], \tag{16}$$

is termed as sub-coherence in a block sub-matrix [16]. If $c = 1$, the cluster sparse structure of channel will reduce to sparse. Hence, the requirement of cluster-sparse signal recovery in Eq. (15) will coincide with OMP-based sparse signal recovery condition [18, 28, 29] given as

$$Kc < \frac{1}{2}\left(\frac{1}{\mu_B} + c\right). \tag{17}$$

We can find that accurate recovery condition of BOMP is looser than OMP. Hence, by using the same training signal matrix on channel estimation, BOMP-based channel estimation can obtain more accurate channel estimator.

3.2. Compressive Channel Estimation

Compressive estimation of cluster-sparse channel using BOMP algorithm is similar to conventional OMP, and can serve as a low computationally attractive alternative to convex optimization method. The algorithm begins by initializing the residual as $\mathbf{r}_0 = \mathbf{y}$. At the k-th step, BOMP selects the block that is the best coherent to the current residual according to

$$i_k = \arg\max_i \left\|\mathbf{X}^H[i]\mathbf{r}_{k-1}\right\|_2 \tag{18}$$

and $\boldsymbol{r}_{k-1}$ is the residual.

$$i_\ell = \arg\min_i \left\|\mathbf{X}^H[i]\mathbf{r}_{\ell-1}\right\|_2. \tag{19}$$

Once the block index i_ℓ is chosen, we find the optimal coefficients by computing $\mathbf{h}_\ell[i]$ as the channel estimator to

$$\mathbf{h}_\ell[i] = \arg\min_{i_\ell} \left\| \mathbf{y} - \sum_{i\in\Omega} \mathbf{X}[i]\mathbf{h}_\ell[i] \right\|_2^2. \tag{20}$$

Here Ω is the set of chosen indices i_j, $1 \le j \le \ell$. The residual is then updated as

$$\mathbf{r}_\ell = \mathbf{y} - \sum_{i\in\Omega} \mathbf{X}[i]\mathbf{h}_\ell[i]. \tag{21}$$

BOMP-based compressive channel estimation can be summarized as:
(1) Initialization: Let the estimation residual error $\mathbf{r}_0 = \mathbf{y}$, the iteration counter $k = 1$ and the taps index set Ω_0 be the empty set.
(2) Cluster-position sensing step: Find the block index i_k by solving the optimization (3). Then, $\Omega_k = \Omega_{k-1} \cup \{i_k\}$.
(3) Updating the residual: $\mathbf{r}_k = (\mathbf{I}_{kp} - \mathbf{X}_{\Omega_k}\mathbf{X}^{\dagger}_{\Omega_k})\mathbf{y}$, where $\mathbf{I}_{kp}$ is the identity matrix of size $kp\times kp$, is a set of blocks $\mathbf{X}_{\Omega_k} = [\mathbf{X}[i_1], \ldots, \mathbf{X}[i_k]]$ and $(\cdot)^\dagger$ denotes the pseudo-inverse.
(4) Algorithm iteration: Set $k = k+1$, and return to step (2) if $k \le K$.

4. SIMULATION RESULTS

In this section, the average mean square error (average MSE) performance of the proposed estimator will be evaluated by computer simulations. For the purpose of comparison, the MSE performance of other existing channel estimation methods such as LS, OMP, and CoSaMP algorithms will also be evaluated. In addition, the lower

Table 1. Simulation parameters.

Estimation methods	Linear method: LS Compressive methods: OMP [28] CoSaMP [5] Proposed method in this paper
channel fading	Frequency-selective block fading
Channel length	100
Cluster-sparse channel	Number of cluster-sparse: 2 Total number of nonzero taps: 10
SNR	0–30 dB
Length of training signal	$N = 20$–80

bound (known position of dominant channel taps) of channel vector is also calculated as a reference. The parameters used in the computer simulation are listed in Table 1. To illustrate the performance of the proposed algorithm, Fig. 4 shows the average estimation error, employing different methods. The average MSE function is defined by

$$average\ MSE(\Delta\mathbf{h}) = \frac{1}{M}\sum_{m=1}^{M}\left\|\mathbf{h}-\hat{\mathbf{h}}_{\mathrm{m}}\right\|_2^2, \tag{22}$$

where M is the number of Monte Carlo runs, $\hat{\mathbf{h}}$ denotes the channel estimator. We adopt $M = 10000$ Monte Carlo runs for averaging. In this computer simulation, we generate the cluster-sparse randomly, i.e., the position of cluster is random; channel taps satisfy random Gaussian distribution with $\mathcal{CN}(0,1)$ in a cluster, and the channel vector $\mathbf{h}$ is normalized $\|\mathbf{h}\|_2^2 = 1$. Fig. 4 shows that the compressive channel estimators (OMP, CoSaMP and BOMP) are better than the linear channel estimator (LS). In addition to the compressive channel estimation by BOMP, the proposed method further exploits cluster-sparse structure information and hence the proposed estimator performs better than conventional sparse estimators which are based on the OMP and CoSaMP algorithms. The main reason is that proposed method considers the BMIP rather than MIP in the training signal matrix $\mathbf{X}$. From CS [24, 25] perspective, μ_B is smaller than μ in $\mathbf{X}$. As a result, it will acquire more accurate channel estimator due to mitigating more coherent interference in $\mathbf{X}$. At the second place, we consider compressive channel estimation according to signal-to-noise

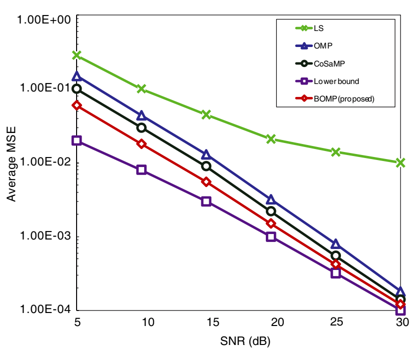

Figure 4. Estimation performance (average MSE) versus SNR.

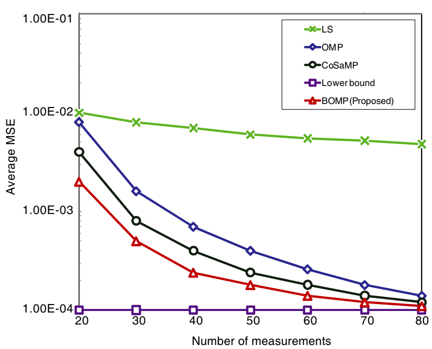

Figure 5. Estimation performance (average MSE) versus number of measurements.

ratio (SNR) which is defined as

$$SNR = 10 \log \left(P/\sigma_n^2 \right). \tag{23}$$

It is shown in Fig. 5 that compressive methods (OMP, CoSaMP and proposed) are better than least square (LS)-based linear method. As the length of training signal increases, more accurate channel estimators are obtained. However, LS-based linear channel estimator does not have distinct improvement which is caused by undetermined system. If we use a sufficiently long training signal at the cost of reducing the bandwidth efficiency, lower bound of LS-based linear channel estimator is still obtained. From Fig. 5, we can also find that the proposed method (cluster-sparse based) obtain more accurate channel estimate than sparse-based conventional compressive methods (OMP and CoSaMP).

5. CONCLUSION

In this paper, we proposed a compressive channel estimation method for cluster-sparse multipath broadband communication systems. By utilizing the proposed method, we captured potential cluster-structure of multipath channel comparing with conventional sparse channel estimators which are based on a simple sparse structure assumption. We derived the theoretic performance of the proposed estimator higher than previous channel estimator without exploiting the cluster structure information. Computer simulation results also confirmed the proposed method efficiency. Hence, we predict that the proposed method will have an application in wireless communication, especially in practical UWA and UWB communication systems.

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