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SPECTRAL SWITCH OF LIGHT INDUCED BY SCATTERING FROM A SYSTEM OF PARTICLES

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Abstract—The spectral switch phenomenon of light induced by scattering from a collection of particles is, to the best of our knowledge, reported for the first time. It is shown that when a spatially coherent light wave with a spectrum of Gaussian distribution is scattered from a collection of particles, the rapidly transition of the spectrum of the scattered field from red shift to blue shift (i.e., spectral switch) can be observed. It is also found that the spectrum of the scattered field will experience several spectral switches with the increase of the scattering angle.

1. INTRODUCTION

It has been known for some time that the spectrum of light generated by a partially coherent source may change on propagation, even in free space [1]. Subsequently, a similar phenomenon has also been found in light scattering [2]. After that, numerous papers have been published on the subject of scattering due to the potential application. For example, the spectral changes of light scattered from a spatially random medium have been discussed [3, 4], and the changes in the spectrum of light scattered from a collection of particles have been studied [5]. The inverse problem, i.e., the problem of finding the statistical properties of scattering medium from the spectral changes of the scattered field, has also been addressed, both for random medium [6, 7] and collection of particles [8, 9]. In addition, the scattering matrix theory for stochastic scalar fields has also been discussed [10–12].

On the other hand, in the discussing of spectral shifts on light propagation, it is shown that, by changing some parameters gradually,

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the spectral shift shows a rapid transition, i.e., the *spectral switch* [13], which has been investigated extensively [14–16]. An interesting problem can be addressed that is: does the spectral switch exist in the scattered spectrum? Although, as mentioned above, the spectral shifts of a light wave on scattering have been discussed extensively, as far as we know, there is no report on the spectral switch phenomenon of light wave on scattering.

In the present paper, we consider the spectral switch of a spatially coherent light wave scattered from a collection of particles with determinate distribution. It is found that the spectrum will experience several rapidly transitions from the red shift to the blue shift (i.e., spectral switch) with the increase of the scattering angle (i.e., the angle between the incident direction and the scattered direction).

2. THEORY

Let us assume that a spatially coherent, polychromatic plane wave with spectrum $S^{(i)}(\omega)$, with ω being the radiant frequency, incident on a scattering medium in a direction specified by a real unit vector $\mathbf{s}_0$ (see Fig. 1). The cross-spectral density function of the incident field at a pair of points specified by the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ may be expressed as ([17], Section 6.2)

$$W^{(i)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_0; \omega) = S^{(i)}(\omega) \exp(ik\mathbf{s}_0 \cdot (\mathbf{r}_2 - \mathbf{r}_1)), \tag{1}$$

where $S^{(i)}(\omega) = \langle a^*(\omega) a(\omega) \rangle$ is the spectral density of the incident light, which can be assumed as

$$S^{(i)}(\omega) = A \exp\left[-\frac{(\omega - \varpi)^2}{2\Gamma_0^2}\right], \tag{2}$$

where ϖ denotes the central frequency of the incident spectrum, Γ_0 stands for the line-width of spectrum, and A is a constant.

Let $F(\mathbf{r}, \omega)$ be the scattering potential of the medium, and assume that the scattering medium is so weak that the scattering can be analyzed within the accuracy of the first-order Born approximation ([18], Section 13.1.2). Let $C_F(\mathbf{r}_1, \mathbf{r}_2, \omega)$ be the correlation function of the scattering potential, which can be defined as ([17], Section 6.3.1)

$$C_F(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle F^*(\mathbf{r}_1, \omega) F(\mathbf{r}_2, \omega) \rangle, \tag{3}$$

where the angular brackets denote the average taken over the ensemble of the random medium.

The cross-spectral density function of the scattered field in the far zone, at two points specified by position vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$

($\mathbf{s}_1^2 = \mathbf{s}_2^2 = 1$), can be expressed as ([17], Section 6.3.2)

$$W^{(S)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0; \omega) = \frac{S^{(i)}(\omega)}{r^2} \tilde{C}_F(-\mathbf{K}_1, \mathbf{K}_2, \omega), \tag{4}$$

where

$$\tilde{C}_F(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint C_F(\mathbf{r}_1, \mathbf{r}_2, \omega) \exp[-i(\mathbf{K}_2 \cdot \mathbf{r}_2 + \mathbf{K}_1 \cdot \mathbf{r}_1)] d^3\mathbf{r}_1 d^3\mathbf{r}_2 \tag{5}$$

is the six-dimensional Fourier transform of the correlation function $C_F(\mathbf{r}_1, \mathbf{r}_2, \omega)$. and

$$\mathbf{K}_1 = k(\mathbf{s}_1 - \mathbf{s}_0), \quad \mathbf{K}_2 = k(\mathbf{s}_2 - \mathbf{s}_0). \tag{6}$$

The vectors $\mathbf{K}_1$ and $\mathbf{K}_2$ are analogous to the momentum transfer vector of quantum mechanical theory of potential scattering.

In this paper, we assume that the incident field is scattered by a collection of particles. For the sake of simplicity, we assume further that all the particles in the collection are identical, and located at point specified by position $\mathbf{r}_m$. The scattering potential of the whole system of the particles then can be expressed in the form [11],

$$F(\mathbf{r}, \omega) = \sum_{m=1}^{L} U(\mathbf{r} - \mathbf{r}_m, \omega), \tag{7}$$

where L is the total number of the particles, $U(\mathbf{r} - \mathbf{r}_m, \omega)$ is the scattering potential of the particle located at the point $\mathbf{r}_m$. In the discussing of the light scattering from a deterministic matter, the Gaussian centered scattering potential model is usually used [8, 9], i.e., the scattering potential of each particle has the form

$$U(\mathbf{r}, \omega) = B \exp\left(-\frac{\mathbf{r}^2}{2\sigma^2}\right). \tag{8}$$

where σ is a constant, which indicates the effective length of the scattering potential.

On substituting from Eq. (7) first into Eq. (3), then into Eq. (5), and calculating the six-dimensional Fourier transform, we can rewrite Eq. (5) as

$$\tilde{C}_F(\mathbf{K}_1, \mathbf{K}_2, \omega) = \tilde{F}^*(\mathbf{K}_1, \omega) \tilde{F}(\mathbf{K}_2, \omega), \tag{9}$$

where

$$\tilde{F}(\mathbf{K}, \omega) = \sum_{m=1}^{L} B(2\pi)^{(3/2)} \sigma^3 \exp\left[-\mathbf{K}^2 \sigma^2 / 2\right] \exp\left[-i\mathbf{K} \cdot \mathbf{r}_m\right]. \tag{10}$$

On substituting from Eq. (9) into Eq. (4), and letting $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}$, one can obtain the spectral density function of the scattered field

$$S^{(S)}(\mathbf{s}, \mathbf{s}_0; \omega) = \frac{S^{(i)}(\omega)}{r^2} B^2 (2\pi)^3 \sigma^6 \exp[-\mathbf{K}^2\sigma^2] \sum_{m=1}^{L} \exp[-i\mathbf{K} \cdot \mathbf{r}_m] \times \sum_{m=1}^{L} \exp[i\mathbf{K} \cdot \mathbf{r}_m], \quad (11)$$

where $\mathbf{K} = k(\mathbf{s} - \mathbf{s}_0)$, the angle between $\mathbf{s}$ and $\mathbf{s}_0$ is θ.

The scattered spectrum differs, in general, from the incident light wave, which is well-known as the spectral shift induced by scattering. In the following, we will present some graphs to show the properties of the spectrum of the scattered field.

As an example, we consider a two particles system, as shown in Fig. 1. Based on the formulas of Eq. (11), the normalized spectra of the incident light (the dotted line) and the scattered light (the solid line) with different scattering angle in such system are presented in Fig. 2. Generally, the spectrum of the scattered field is no longer Gaussian, but the spectrum is split into two peaks. According to some previous work (for example, Ref. [13]), the relative spectral shift of such a situation can be defined as

$$\frac{\delta\omega}{\varpi} = \frac{\omega_m - \varpi}{\varpi}, \quad (12)$$

where ω_m is the frequency at which $S^{(S)}(\mathbf{s}, \mathbf{s}_0; \omega)$ takes its maximum. If $\frac{\delta\omega}{\varpi} < 0$ (i.e., $\omega_m < \bar{\omega}$), the spectrum of the scattered field is red-shift. If $\frac{\delta\omega}{\varpi} > 0$ (i.e., $\omega_m > \bar{\omega}$), the spectrum of the scattered field is blue-shift.

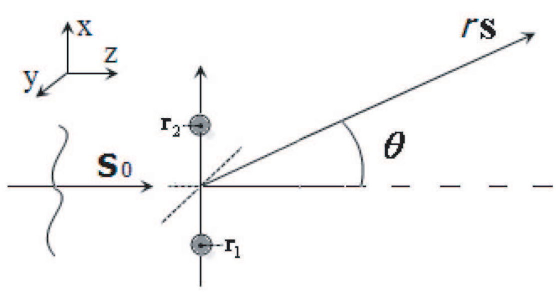

Figure 1. Illustration of the notation.

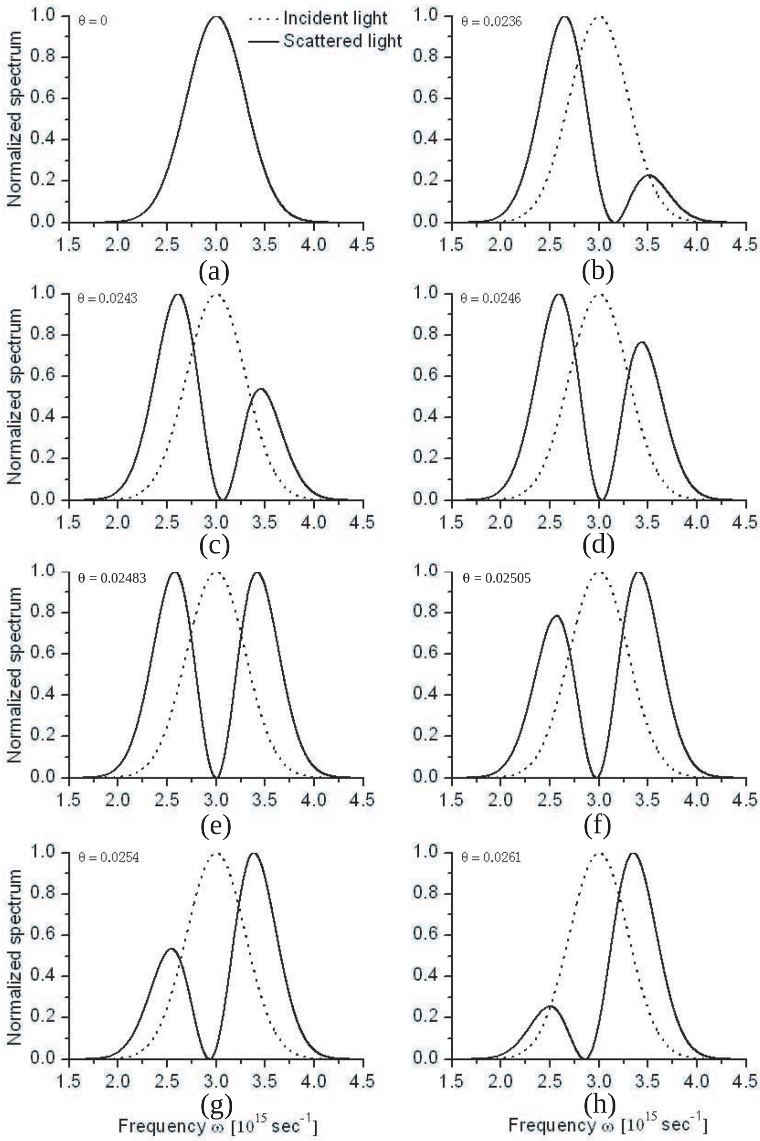

Figure 2. Changes in the normalized spectrum of the scattered field with scattering direction. The dotted curves are the normalized source spectrum. The parameters for calculations are $\varpi = 3 \times 10^{15}\, s^{-1}$, $\Gamma_0 = 0.1\varpi$, $\mathbf{r}_1 = (-6.328\,\mu\text{m}, 0, 0)$, $\mathbf{r}_2 = (6.328\,\mu\text{m}, 0, 0)$.

3. SPECTRAL SWITCH OF LIGHT SCATTERED FROM A TWO PARTICLES SYSTEM

As shown in Fig. 2(a), when the scattering direction is in accord with the incident direction (i.e., $\theta = 0$), the spectrum of the scattered field is the same as the spectrum of the incident light wave. However, with the increase of the scattering angle, the scattered spectrum is different from the spectrum of the incident light wave. For simplicity, we just present the spectrum of some typical scattering angles. When $\theta = 0.0236$, the relative spectral shift is $\delta\omega/\varpi = -0.116$, i.e., the spectrum is red shifted, and the intensity of the secondary peak is much smaller than the primary one (see Fig. 2(b)). If we continue to increase the scattering angle, the spectrum is also red shifted, but the intensity of the secondary peak of the spectrum increase (see Fig. 2(c) and Fig. 2(d)). It should be noted that there is a critical direction ($\theta = 0.02483$), at which the two peaks of the spectrum reach the same height (see Fig. 2(e)). This means that the spectral shift experience a sharply transition from the red shift to the blue shift, which has been defined as spectral switch [13].When the scattering angle continues to increase, the spectrum of the scattered field is blue shifted, and this shift decreases with the increase of the scattering angle (see Figs. 2(f)–2(h)). The detailed spectral parameters corresponding to Fig. 2 are listed in Table 1.

If we continue to increase the scattering angle, the spectrum of the scattered field will experience several spectral switches. Therefore, we present the related spectral shift of the scattered field with the changes of the scattering angle in Fig. 3 (the spectral switches marked by points in Fig. 3).

Table 1. Spectral parameters corresponding to Fig. 2.

	Scattering angle (θ)							
	0	0.0236	0.0243	0.0246	0.02483	0.02505	0.0254	0.0261
$\omega_m (10^{15} s^{-1})$	3	2.651	2.61	2.592	2.579;3.42	3.407	3.387	3.351
$\delta\omega/\varpi$	/	-0.116	-0.13	-0.136	-0.14;0.14	0.136	0.129	0.117
Spectrum	/	red shift	red shift	red shift	spectral switch	blue shift	blue shift	blue shift

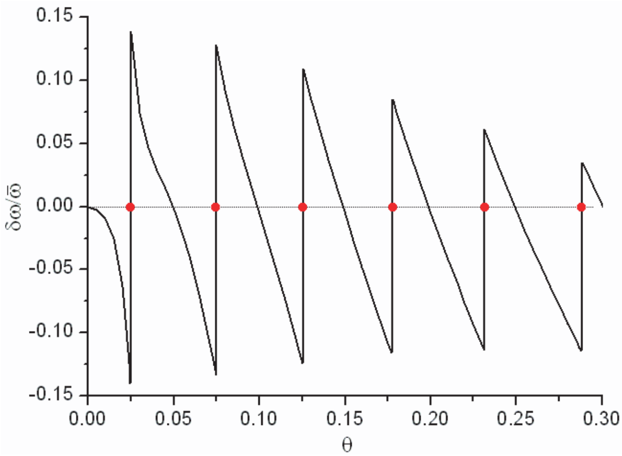

Figure 3. Relative spectral shifts of the scattered field as a function of scattering angle. The parameters for calculations are the same as in Fig. 2.

4. CONCLUSION

In this paper, we investigated the spectral switch induced by scattering of a spatially coherent light wave from a collection of particles. An example of a two-particle system has been discussed. It is found that the spectrum of the scattered field experiences several spectral switches with the increase of the scattering angle. A more complicated case of collection with more particles can be discussed in the same way. Due to the fact that the spectral switches can be used for information coding and propagation, and the properties of the spectral switch (for example, the locations or the numbers) is closely related with the properties of the particles system, it is hoped that the spectral switch can be used as a new way to determine the properties of the particles system, which is very important in areas such as medical diagnoses, remote sensing, and so on.

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