Progress In Electromagnetics Research Letters, Vol. 35, 19–28, 2012

APPLICATION OF THE PARTICLE SWARM OPTIMIZATION TO THE COMPUTATION OF THE GO/UTD REFLECTION POINTS OVER NURBS SURFACES

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Abstract—A new method to find the Geometrical Optics/Uniform Theory of Diffraction reflection points over Non Uniform Rational B-Splines surfaces is presented. The approach is based on the Particle Swarm Optimization (PSO) technique, and the cost function used to find the reflection points is based on Snell's law. The technique can be used as an alternative to classic minimization techniques in cases where convergence problems arise.

1. INTRODUCTION

Geometrical Optics (GO) combined with the Uniform Theory of Diffraction (UTD) is one of the most extensive deterministic techniques for high frequency electromagnetic analysis [1–3] in a number of applications, such as propagation in mobile communications, radiation of on board antennas, and others [4–6]. The main difficulty in GO/UTD is the ray-tracing calculation, especially when analyzing a complex environment [7, 8]. Ray tracing involves searching for the critical points (reflection and diffraction points). If the surfaces used to model the environment are planar, analytical expressions can be used to obtain these critical points [9, 10]. However, for arbitrarily curved surfaces, analytical expressions cannot be applied [11, 12]. Instead, minimization techniques such as the Conjugate Gradient Method (CGM) must be used [6, 13, 14]. If the surface is convex, no local minimal are present and minimization techniques generally provide

Received 25 June 2012, Accepted 11 September 2012, Scheduled 18 September 2012
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the point position with good accuracy. In contrast, in the analysis of concave surfaces, convergence problems can appear due to the presence of local minimal [15, 16]. This is an especially important problem in the analysis of systems with multiple interactions (combinations of reflections and diffractions) that consist of both convex and concave surfaces.

In this paper, we present an alternative method based on Particle Swarm Optimization (PSO) [17] that avoids convergence problems, even in complex systems. The PSO technique has been widely used for several applications that require the optimization of different problems [18–20] and can also be applied to minimization problems such as the ones presented in this work. A key advantage of PSO is that it avoids the problems seen in classic minimization techniques that arise due to local minimal [21, 22]. This approach, therefore, allows for the analysis of both convex and concave surfaces (and their combination) with the same accuracy. The approach has been applied only for the multiple reflection case, but could be easily generalized to the diffraction case by simply modifying the Fitness function. To prove the validity of the approach, cases are studied where the CGM cannot find a solution because of the presence of local minimal.

2. STATEMENT OF THE PROBLEM

The problem of obtaining reflection or diffraction points in the application of ray-tracing techniques over bodies that are modeled using parametric surfaces is typically addressed using minimization techniques [6, 8, 11]. Based on the Generalized Fermat's Principle [23], it can be stated that the path followed by a ray between two points (the source and the observer) is such that the geometrical path length is an extreme (maximum or minimum). This principle can be applied to complex bodies modeled by parametric surfaces. If one considers that the body is modeled by n surfaces, with each surface defined by two parametric coordinates, u_i and v_i, it follows that, in the case of an n-order reflection (Fig. 1) the reflection points can be obtained after the minimization of the following function:

$$d^n(u_1, v_1, u_2, v_2, \ldots, u_{n-1}, v_{n-1}, u_n, v_n) = d_0(u_1, v_1) + d_1(u_1, v_1, u_2, v_2) \\ +d_2(u_2, v_2, u_3, v_3) + \ldots + d_{n-1}(u_{n-1}, v_{n-1}, u_n, v_n) + d_n(u_n, v_n) \qquad (1)$$

where d^n is the total distance of the ray path followed by the nth-reflection, and d_i the different stretches in which d^n can be divided (Fig. 1).

This minimization cannot be solved analytically and, therefore, iterative minimization techniques such as the Conjugate Gradient

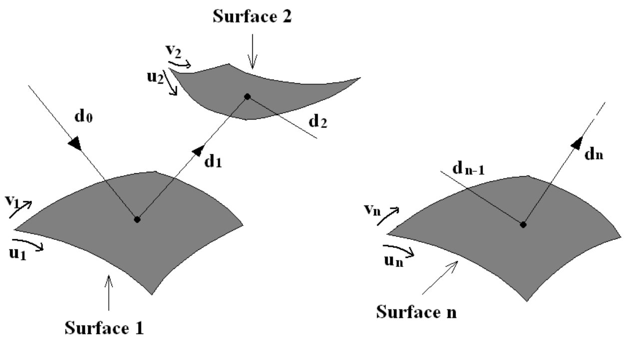

Figure 1. Ray-tracing for an n-order reflection.

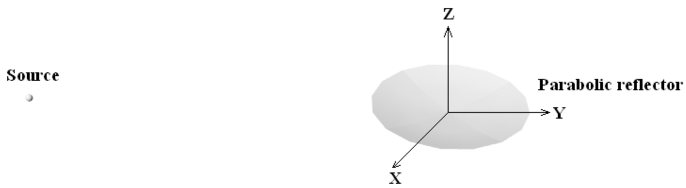

Figure 2. Reflection on a parabolic reflector.

Method (CGM) are often used [6, 13]. For most cases, these techniques provide a reliable solution. However, there are certain cases where minimization techniques do not find the correct solution. This situation occurs when the distance function presents local minimal, which occurs frequently in concave surfaces. Fig. 2 shows a case where the reflection on a parabolic surface must be calculated. The distance function is a two variable function depending on the two parametric coordinates of the surface. The distance as a function of both coordinates for a certain position of the source and the observer can be seen in Fig. 3, which clearly reveals the presence of a local minimum.

3. PARTICLE SWARM OPTIMIZATION

PSO is based on an evolutionary model used to solve engineering problems [17]: the behavior of a swarm of bees in search of the biggest concentration of flowers. The algorithm defines a number of particles,

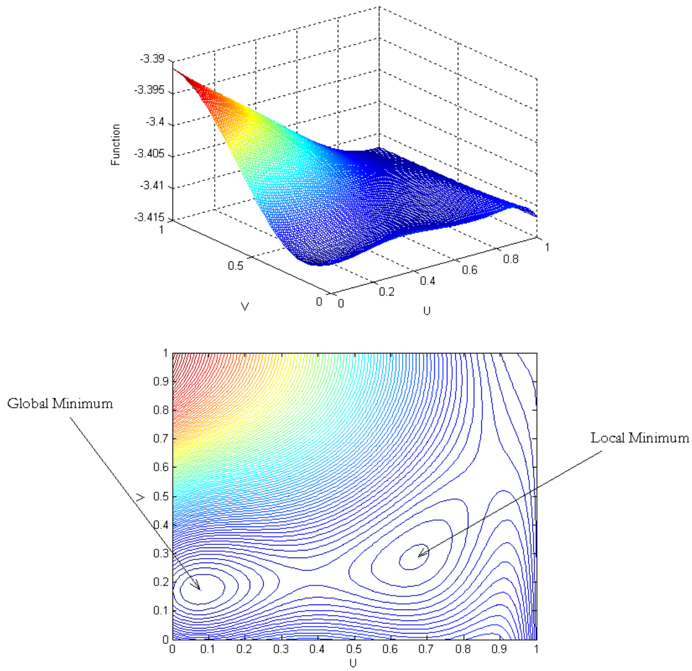

Figure 3. Isometric and plant view of the distance function.

or agents, inside the solution space as starting solutions. Each particle then moves randomly over the space, and its best position (pbest) for each iteration of the algorithm is recorded. The best position of all the particles (gbest) is stored as well. In the next iteration, this best position information is used by every particle when deciding where to move. The velocity is updated according to the following expression:

$$V_k = wV_{k-1} + c_1 r_1 \left(pbest_{k-1} - X_{k-1}\right) + c_2 r_2 \left(gbest_{k-1} - X_{k-1}\right) \quad (2)$$

where V_k is the particle velocity at the kth iteration, which is an n-dimensional vector with n being the dimension of the solution space; X_k is the particle position with the same dimension of the velocity; c_1 and c_2 are scalars that define the influence of the personal and the group experiences, respectively; w is the inertia value; and finally, r_1 and r_2 are random values.

According to this expression, the particle moves to the position given by:

$$X_k = X_{k-1} + \Delta_k V_k \quad (3)$$

The typical value for Δ_k is 1.

4. PSO APPLIED TO THE SEARCH OF REFLECTION POINTS

According to the GO theory, when a ray is reflected in a surface, the angle between the incident ray and the normal vector, and the angle between the reflected ray and the normal vector, must be the same (Fig. 4). In other words, the following expression must be satisfied:

$$\theta^i = \theta^r \tag{4}$$

with $\theta^i = -\cos^{-1}(\hat{n}\cdot\hat{s}^i)$ and $\theta^r = -\cos^{-1}(\hat{n}\cdot\hat{s}^r)$, where $\hat{n}$ is the normal vector to the surface at the reflection point, $\hat{s}^i$ the direction of the incident ray, and $\hat{s}^r$ the direction of the reflected ray. As another condition, the incident and reflected rays must be coplanar with the normal vector, thus defining the reflection plane. Taking into account these two conditions, the Fitness function used for the PSO algorithm is as follows:

$$Fitness = \left|\theta^i - \theta^r\right| + \left|\cos^{-1}(v_1\cdot v_2)\right| \tag{5}$$

where $v_1 = \hat{s}^i \times \hat{n}$ and $v_2 = \hat{s}^r \times \hat{n}$ are the normal vectors of the incident and reflected planes. If the direction of the incident and reflected rays are coplanar and the reflection point follows Snell's Law, then the Fitness function is zero.

At the beginning of the algorithm, each particle takes a random position over the surface. From this position, the Fitness function is determined from expression (5). For multiple reflections, each particle is described by the position of the points over each of the surfaces considered. In order to evaluate the Fitness function, the incident and reflected directions at each surface must be calculated. The generalized

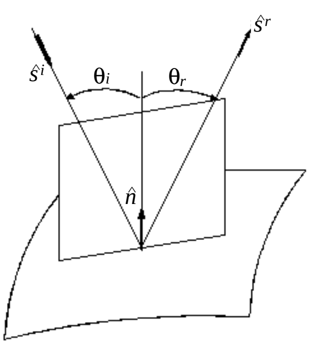

Figure 4. The reflection case: incident and reflection directions, angles and planes.

expression of the Fitness function for n reflections is:

$$Fitness = \sum_{j=1}^{n} \left|\theta_j^i - \theta_j^r\right| + \left|\cos^{-1}(v_{1j} \cdot v_{2j})\right| \qquad (6)$$

where θ_j^i and θ_j^r are the incident and reflected angles on the surface j, and v_{1j} and v_{2j} are the normal vectors to the incident and reflected planes, respectively.

5. RESULTS

Here, we analyze two different cases in order to demonstrate the feasibility of the method. It is important to notice that in the cases presented, the CGM was not able to find a proper solution for the reflection points due to the presence of local minimal in the distance function, as discussed in Section 2.

The first case analyzed was the parabolic surface in Fig. 2. In this case the function to minimize is shown in Fig. 3. Because of the random component of PSO, different simulations reach results with the same accuracy after a different number of iterations. By taking the average of the results, we can obtain the most information about the convergence for a particular case. We computed average values using different numbers of particles in order to study the relationship between particle number and convergence. As seen in Fig. 5, the convergence appears to be independent of the number of particles, with almost no difference in the result when particle number was changed from 10 to 500. Overall, for this case the method converges quickly, after only 100 iterations in the best solution.

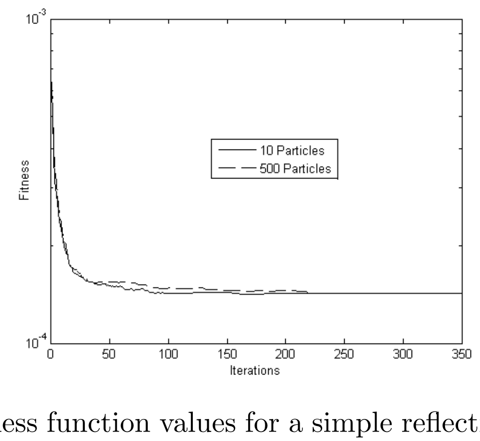

Figure 5. Fitness function values for a simple reflection case using 10 and 500 particles.

The next scenario analyzed consisted of a combination of three cylindrical surfaces: two concave and one convex. This geometry is shown in Fig. 6, with source and observation points indicated.

As in the previous case, the average Fitness function was obtained with different numbers of particles, with results given in Fig. 7. We can see that for this case, the convergence strongly depends on the number of particles. The number of particles becomes more important in the triple reflection case because the number of parameters to minimize has tripled, compared to the simple reflection case. When only ten particles are used, the Fitness function is approximately 0.2 after 350 iterations. When the particle number is increased to 500, the Fitness function is reduced to below 1.0e-4 after the same number of iterations.

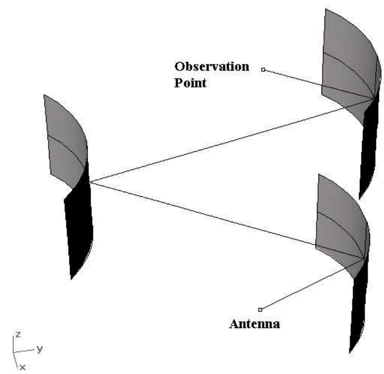

Figure 6. The triple reflection case.

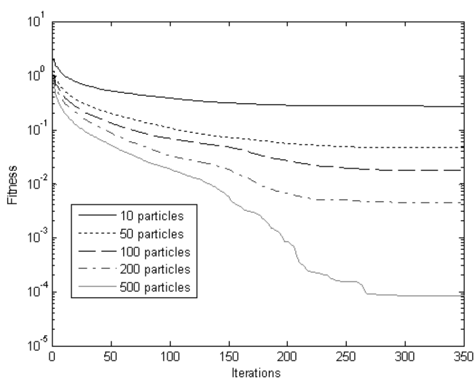

Figure 7. Fitness function values for a triple reflection case using different numbers of particles.

Table 1. Computational time (in seconds) for different numbers of particles and iterations.

		Fitness function values			
		0.1	0.01	0.001	0.0001
Particles	10	0.46			
	50	0.66	1.95		
	100	0.69	2.32	3.46	
	200	0.62	2.79	3.78	4.31
	500	0.77	3.99	5.45	5.96

It is worth noting that with the increase in the number of particles comes an increase in the computational time.

Table 1 shows the average computational time used to reach different values of the Fitness function using a Pentium IV at 3 GHz with 2 GBytes of RAM. In this table, a blank cell means that the corresponding fitness was not reached after 500 iterations. Depending on the desired precision, the number of particles can be increased or decreased. In this triple reflection case, to obtain a Fitness value of 1e-2, 50 particles are sufficient, whereas almost 200 particles must be used to obtain a Fitness value of 1e-4. The user must take into account the balance between computational time and precision when choosing the number of particles.

6. CONCLUSIONS

In this work, a new method to calculate the GO/UTD reflection points based on the Particle Swarm Optimization is presented. The method is an alternative to other minimization methods such as CGM and is especially useful when the CGM does not converge. We analyze a case that combines convex and concave surfaces and show that PSO finds the correct solution while CGM does not, thus proving the validity of the method for these problematic situations.

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