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TRANSMISSION PROPERTIES OF STACKED SRR METASURFACES IN FREE SPACE

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Abstract—In this paper, transmission properties of stacked split ring resonators metasurfaces in free space and under normal incidence are investigated experimentally and numerically. Emphasis is put on studying the interaction between adjacent SRRs metasurfaces. The thorough analysis of the electromagnetic fields shows that both magnetic and electric coupling can occur between adjacents metasurfaces for vertical and horizontal polarization. In addition, we found that all propagating bands within our spectral window (up to 20 GHz) support right-handed behaviour. Both simulation and experiment results in the microwave regime are in good agreement.

1. INTRODUCTION

The split ring resonator (SRR) was proposed by Sir John Pendry et al. in 1999 [1]. This resonant unit exhibits a strong magnetic response leading to a negative permeability within a certain frequency range and can be utilized for the realization of left-handed metamaterials [2]. The SRR is usually excited by an external time-varying magnetic field applied normally to the particle plane. This, in turn, produces a quasi-static resonance at wavelengths that are much larger than its own size. Alternatively, because of its bi-anisotropy it can also be excited by an external time-varying electric field perpendicular to the air gaps [3]. Owing to those landmark properties, i.e., negative permeability and subwavelength dimension, the SRR has been

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recently the subject of intensive research activities [2–4] either for designing negative refractive index medium notably with the addition of metallic wires [2], compact microwave planar circuits [3] or perfect lens [4]. Moreover, the SRR is found to support the propagation of the so-called magnetoinductive waves (MIWs) [5–8] at the quasi-static resonance when nearest-neighbor coupling is included in the analysis of arrays of SRRs. It is found that MIWs in the axial configuration are forward waves with the phase and group velocities in the same direction, that is, right-handed propagation. However, in the planar configuration MIWs are backward waves with phase and group velocities in opposite direction, that is, left-handed propagation. These MIWs have attracted much attention recently and have been exploited in many potential applications such as propagation [9], nonlinear effect [10], delay lines [11] and focusing [12, 13].

Most studies on the electromagnetic properties of arrangements of SRRs reported up to date have been focused on the fundamental mode (quasi-static resonance). To our knowledge study of the high order resonances interactions is very limited in the literature [14–16]. With the present work, we aim to bridge this gap with a detailed study of the transmission properties across stacked SRRs (axial configuration) metasurfaces putting particular emphasis on the interaction between adjacent SRRs metasurfaces. The analysis shown below comprises not only numerical results, but also experiments at the microwave regime for stacked SRRs metasurfaces under normal incidence illumination with respect to the two orthogonal orientations of the electric field. When the electric field is parallel to the air gaps, we term the illumination as horizontal polarization; in contrast, the vertical polarization is defined when the electric field is perpendicular to the air gaps; see Figure 1.

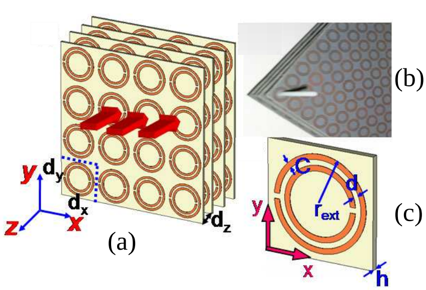

Figure 1. Schematic of stacked SRR (a), fabricated prototype (b) and unit cell (c).

2. MODEL

The design used here is connected to our previous work based on a single SRR metasurface [17]. Indeed, it is the natural extension of that work. Figure 1(a) shows a schematic drawing of four stacked SRRs metasurfaces with longitudinal periodicity $d_z = 2.145\,\text{mm}$. Note that this periodicity leads to an electromagnetic band-gap with a center rejected frequency outside the spectral window of our measurements. Thus, we are working far below the stop-band defined by the stack periodicity. The dimensions of the unit cell of SRR (see Figure 1(c)) are the same as those employed in [17, 18]. Namely, the outer radius of the SRR is $r_{ext} = 3.5\,\text{mm}$, and widths of the gaps, metal as well as the air ring between inner and outer rings are 0.4 mm. The arrangement of SRRs rests on a commercial low loss microwave substrate (ARLON CuClad 250) with relative permittivity $\epsilon_r = 2.43$ and dielectric height $h = 0.49\,\text{mm}$, coated with a conductive layer of copper of thickness 35 µm where the SRRs are etched. With these parameters, the quasi-static resonance is expected within the C band of the microwave spectrum according to the theory developed in [19]. The in-plane lattice constants along x- and y-direction are equal to $a_x = a_y = 8\,\text{mm}$, which is approximately 1/8 of the free space wavelength at the quasi-static resonance and 8/15 at the higher frequency of the experiment. Notice that the onset of the first grating lobe falls beyond our experimental highest frequency. The SRRs metasurfaces displayed in Figure 1(a) were fabricated with a total size of 200 mm × 200 mm (25 × 25 elements), see Figure 1(b). The free space transmission measurements were performed using an Agilent 8722ES vector network analyzer and different microwave standard horn antennas to cover the frequency bandwidth (3–20 GHz). The distance between horn antennas is maintained at 500 mm. Since $2D^2/\lambda = 1333\,\text{mm}$ (for the quasi-static resonance), where D is the lateral size of the metasurface, these measurements have been done in the near field (Fresnel zone) [20]. Therefore, rippled response may be expected due to the diffraction and multiple reflection effects.

The measured transmission characteristic of the stacked SRR metasurfaces under normal incidence is displayed in Figure 2 where three significant resonances in the range from 3 GHz to 20 GHz manifest themselves when the electric field is along the y-axis (Figure 2(c)). The first resonance around 5 GHz is the quasi-static resonance that arises from the circulating current along the SRR induced by the external electric field. This first resonance vanishes when the electric field is along x-axis, leaving only the dynamic resonances (Figure 2(d)). Moreover, from Figure 2 it follows that the resonances for vertical

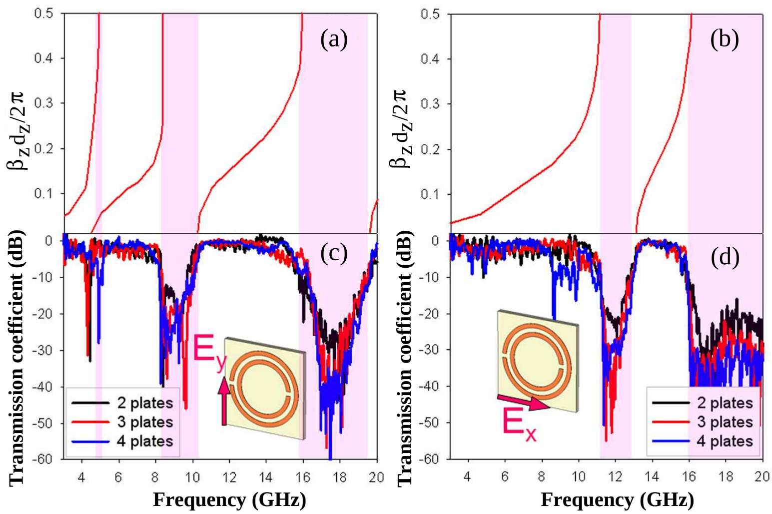

Figure 2. Simulated dispersion diagram for SRRs in axial configuration, vertical (a) and horizontal (b) polarization. Measurement of the transmission coefficient as a function of different number of stacked metasurfaces for vertical (c) and horizontal (d) polarizations. Insets: unit cell along with incident electric field.

polarization do not match in frequency with those observed for horizontal polarization. This can be explained by the different nature of current circulation at each resonance [21]. It anticipates that the first and second resonance for vertical polarization corresponds essentially to the excitation of both outer and inner ring, but with different direction: the induced current density circulates at each ring in the same [17] and opposite (see Figure 3(a)) direction for the first and second resonance, respectively. For this reason they are classified as anti-symmetric and symmetric, correspondingly. To finish with this polarization, the third resonance stems from the excitation of the outer ring (see Figure 3(b)). According to the node distribution, the total length of the arc corresponds to 2λ, and thus, the resonance may fall theoretically around 17.8 GHz, which is in agreement with the experiments. Note that the quasi-static resonance of four stacked metasurfaces shows a frequency up shift in comparison to that of two and three stacked metasurfaces. This can be due to misalignments or buckling of the stacked metasurfaces in the experiments.

With regard to the horizontal polarization, the first and the second resonances are essentially created from current circulation of the outer and inner ring, respectively. In addition, owing to the magnetic

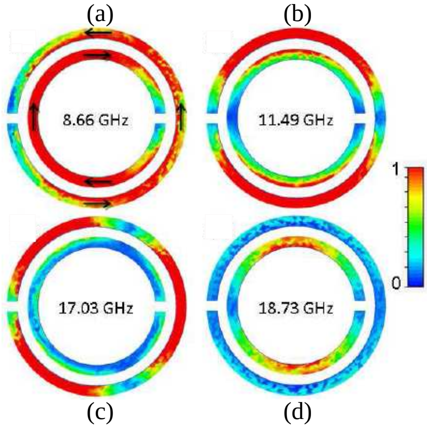

Figure 3. The current density distribution for vertical (a), (b) and horizontal (c), (d) polarisation, respectively, at each correspondence frequency.

symmetry along x, the current vanishes at the middle plane of the SRR, see Figures 3(c), (d), and thus, each ring can be seen as two disconnected wires oriented mainly horizontally.

In the following, we investigate the coupling mechanism between stacked SRRs metasurfaces. As it has been intensively studied for the quasi-static resonance [8], the electric field component of the incident wave can excite circulating currents along SRRs which in turn give rise to a magnetic dipole moment perpendicular to the SRR plane, that is, along z according to our coordinates definition [1, 3]. When several SRRs metasurfaces are stacked the dipole moment of the first SRR excites the next second-layer SRR, which in turn induces another dipole moment along z in the second SRR. This process is repeated until the end of the stack. The presence of such coupling supports a set of slow waves that propagate at a velocity less than that of the light. In addition, the electric field is always perpendicular to the gaps of SRRs. The electric coupling between adjacent SRRs metasurfaces can thus not be ignored. Therefore, at the first resonance we need to consider the magnetic and electric coupling. This is indeed the mechanism governing the MI waves. To gain insight into the coupling mechanism that can emerge for the rest of higher passbands, we plot the z-component of the electric and magnetic field at a certain frequency within each passband, see Figure 4. One can safely state that both electric and magnetic coupling is noticeable for both polarizations, since the z-component of the magnetic and electric field extends from one layer of SRRs to the adjacent one. Due to the electric symmetry along y (under horizontal polarization), the z-component of the electric field vanish at the central yz-cutting plane, which likely hides coupling features if just that cutting plane is displayed. For this reason we also plot the electric field along the yz-cutting plane at a certain x position different from the centre, see Figure 4 (right column). This lets us show that the coupling mechanism is complex for higher order bands and is not restricted to occur only along the central yz-cutting plane as it happens for the magneto-inductive band. This study proves that the subwavelength stack of SRRs layers cannot be modelled simply by an LC tank connected to a transmission line. It is required to add elements accounting for the coupling shown in Figure 4.

We have discussed so far the interaction between adjacent SRRs metasurfaces. Now we investigate the propagation bands emerging within the spectral window of our measurements. To identify the nature of the electromagnetic propagation inside a structure, four methods have been commonly used. The first one is to retrieve the effective material parameters from simulated or measured scattering parameters [22]. The second one is founded on a refraction/wedge experiment [23]. The third one is to use the dispersion diagram

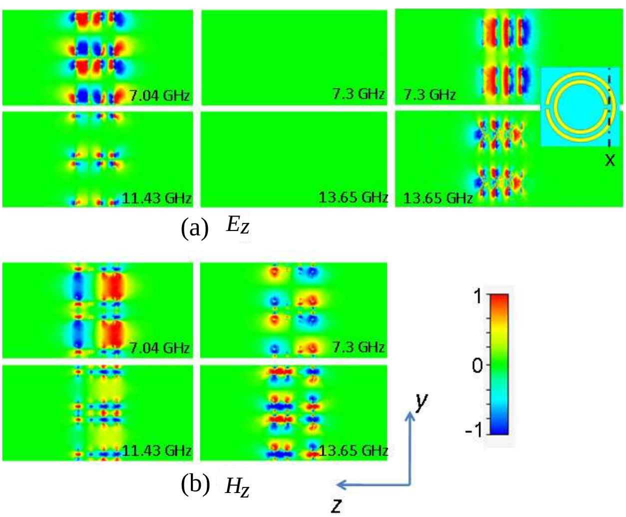

Figure 4. The simulated electric (a) and magnetic (b) fields distributions for the four stacked SRRs metasurfaces at the corresponding resonances, along the yz-middle-cutting plane, for vertical (left column) and horizontal (central column) polarizations, and along the x position of the yz-cutting plane for horizontal polarization (right column).

to characterize the refractive index [24], and the last method is to observe the phase response as a function of the number of stacked metasurfaces [24]. In this paper, we use the third (shown in Figure 2) and fourth methods to characterize the nature of the wave propagation for all propagating bands. It is worthwhile to mention that for the upper frequency band the dimension of the unit cell is on the order of the wavelength meaning that our structure is no longer in the effective medium limit. Therefore, the use of the first method (retrieval method) would be misleading.

By using the eigenmode solver of CST Microwave Studio^{TM}, the band structure (along z) of an infinite three dimensional stack of SRR metasurfaces is computed for a fixed longitudinal lattice $d_z = 2.145\,\mathrm{mm}$ with respect to two orthogonal polarizations, see Figure 1(c). By applying periodic boundary condition in all directions, the infinite stack is replaced with only a unit cell. A specific phase shift across the cell is defined and the eigenmodes frequencies are obtained at several values of this phase shift, whereas in the cross-sectional dimensions the phase shift is zero.

The band structure for each polarization is presented in Figure 2 (top panels). The calculated dispersion diagram shows a good agreement with the transmission characteristics shown at the bottom of Figure 2. The wave propagation properties of those modes can be deduced from the dispersion curves. As it is seen from Figure 2 (top panels), there are passband and rejections bands. The origin of the latter comes from the excitation of the rings, whereas the passbands emerge when the rings are not in resonance, i.e., the incoming wave does not see the metasurfaces. Particular interest is the case related to the magneto-inductive phenomenon, because in this case the passband is generated when the SRRs are in resonance [8]. The positive slope exhibited by the passbands accounts for right handed behaviour, i.e., effectively the stack behaves as a positive refractive index medium. For the magneto-inductive case, the nature of the propagation depends on the sign of the coupling between adjacent elements. Indeed for axial configuration the magnetic field of each of the elements has the same direction [8]. Therefore, the mutual inductance is positive between the elements, i.e., the phase velocity (ω/β) is parallel to the group velocity ($d\omega/d\beta$). Note that there have been studies on the left handed behavior of the metamaterials composed of SRR pairs [25, 26]. It is found that the coupling along the transverse and propagation direction plays a key role in generating left handed behavior in SRR pairs. Thus, it will be of great importance to study the same concept in double SRR pairs as a future work.

To confirm this conjecture experimentally, we make use of the fourth method, and observe the phase difference as the number of stacked layers increases. The obtained experimental results are found to be in qualitative agreement with the band structure (Figure 2). Indeed, for all propagation bands, Figure 5 shows that the phase increases as the number of stacked SRRs metasurfaces increases, which accounts for parallel behavior between phase velocity and group velocity.

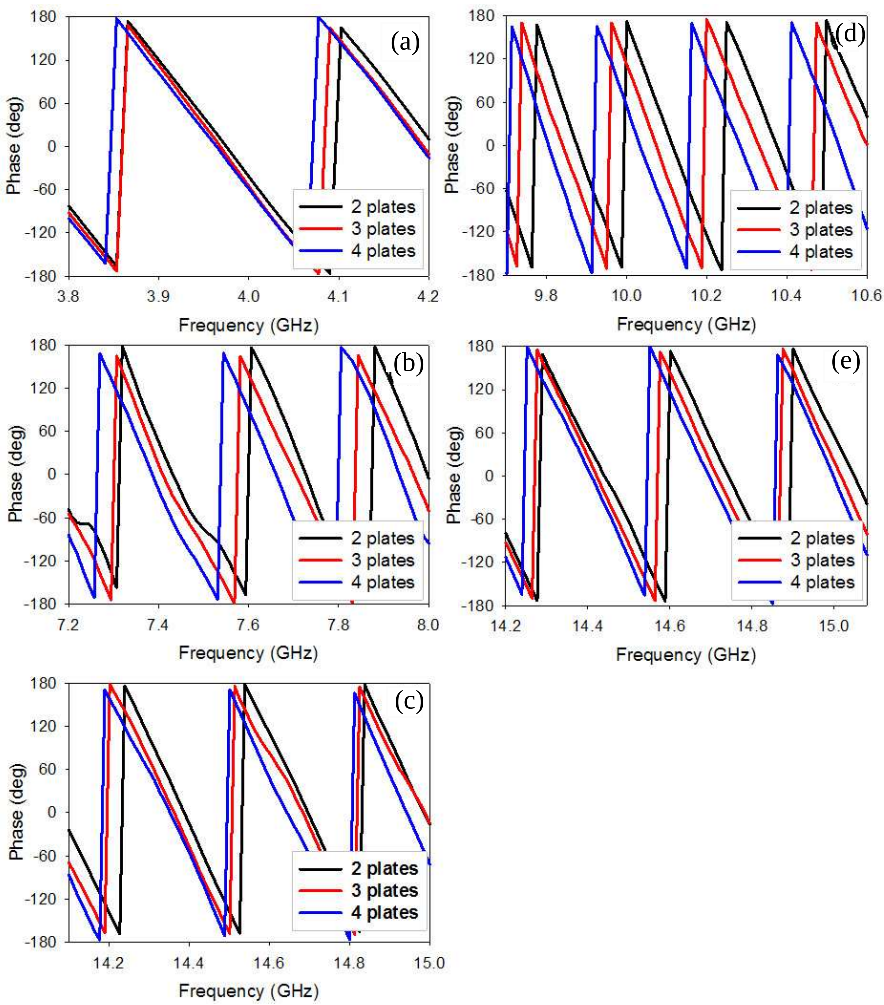

Figure 5. Phase response corresponding to the measured transmission coefficient for vertical (a) and horizontal (b) polarization, respectively.

3. CONCLUSION

In summary, we have studied experimentally and numerically the transmission properties through stacked SRRs metasurfaces when an electromagnetic wave impinges perpendicularly to the particle plane. The analysis has been done for both orthogonal polarizations: vertical and horizontal. We have found that both magnetic and electric coupling can occur between adjacent metasurfaces for vertical and horizontal polarization. In addition, we demonstrated that all propagation bands support right-handed behavior and it cannot simply be modelled by an LC tank together with transmission line because of the couplings. We verified also numerically that the current flow at the higher order resonances is not circular and the classification as symmetric or antisymmetric is no longer valid. Indeed, each ring of the SRR particle behaves as two disconnected wires for horizontal polarization. This work may help in the design of spatial filters based on metasurfaces, where spurious/higher bands are of primary concern for the performance of the device. Additionally, the interaction between SRR metasurfaces at microwave frequencies may provide further insight on some characteristics at higher frequencies (Infrared or optical).

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