Progress In Electromagnetics Research M, Vol. 24, 113–126, 2012

RESONANT TRANSMISSION THROUGH A PAIR OF RIDGE-LOADED CIRCULAR SUB-WAVELENGTH APERTURES

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Abstract—This paper deals with resonant transmission through a pair of ridge-loaded circular sub-wavelength apertures in an infinite perfect electric conductor (PEC) plane. The effect of the distance between the two resonant circular sub-wavelength apertures allocated along the ridge direction ("parallel" case) and perpendicular to the ridge direction ("collinear" case) on the transmission cross section (TCS) is analyzed numerically by using a method of moments (MoM). It is found that the TCS for the parallel case varies more sensitively to the distance than that for the collinearly located case, and the maximum TCS for the parallel case is tripled compared to the TCS value of a single resonant aperture. For the case of maximum TCS in the parallel configuration, the directivity in the broadside direction is about 8.76 times (= 9.43 dB) compared to that for the single resonant aperture. For the purpose of validation, the single resonant aperture and a pair of resonant apertures in the parallel configuration with a distance for maximum TCS are fabricated on a stainless steel plate with 0.3 mm thickness, and their transmission characteristics are measured. Experimental results show that the transmittance, which is a transmitted power density measured at 50 cm away from the aperture plane, for the parallel resonant apertures is about 7 times (= 8.43 dB) higher than that for the single aperture, which agrees well with the simulation.

Received 19 February 2012, Accepted 2 April 2012, Scheduled 9 April 2012

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1. INTRODUCTION

Since *Bethe* reported the transmission phenomenon through a small circular aperture on an infinite thin conducting plane, there have been numerous studies to improve the transmission through electrically small sub-wavelength apertures in physics and optics areas. It was found that the transmittance normalized to the area for small circular aperture scales as $(a/\lambda)^4$ where a is the aperture radius and λ is the wavelength and, therefore, the transmittance decreases abruptly as the aperture radius reduces [1]. To solve this problem, various approaches have been introduced [2, 3], and numerous research results in the microwave area have also been reported recently [4–10].

Representative methods to enhance the transmittance through a small aperture are to modify the shape of the aperture into some self-resonant shapes such as C-shaped or H-shaped apertures [2–6], to place a dielectric or conducting object close to a sub-wavelength aperture [7, 8], and to surround the aperture with grooves [9–11]. Recently, metamaterials have been employed to make the small aperture resonant [12]. It was found that the transmission cross-section (TCS) when the maximum power transmission phenomenon occurs in a small resonant aperture is $3\lambda^2/(4\pi)$, regardless of the shape or the size of the aperture [7]. This was also verified in studies on the transmission characteristics of a ridge-loaded small circular aperture [4–6]. Most recent studies, however, have been restricted to the single aperture problem. Hence we need to examine whether the transmission characteristics (such as total power transmission, radiation patterns of the transmitted wave, received power density in an observation point, etc.) of two adjacent identical apertures can be considerably enhanced compared to the single aperture case.

In this paper, we consider the problem of electromagnetic transmission through two identical resonant apertures in a thin conducting plane. The resonant aperture is a sub-wavelength circular slot loaded with ridges [4–6]. Two different configurations have been investigated: the two apertures are placed along the ridge direction in the "parallel" case while, in the "collinear" case they are aligned perpendicular to the ridge direction. The effects of the distance between the resonant apertures on the transmission characteristics are numerically studied using the method of moments (MoM) and the Finite Difference Time Domain (FDTD) method. The numerical results for the variations of the TCS and the transmission power density in the two different cases are compared to those in the single aperture case. In addition, the effect of the thickness of the conducting plane on the transmission is examined, and the numerical results are validated by experimental results.

2. NUMERICAL ANALYSIS

Figure 1(a) shows the geometry of two identical resonant apertures allocated in an infinite perfect electric conductor (PEC) plane ($0 < z < t$) with distances of X_d in the x-direction and Y_d in the y-direction. According to earlier studies [4–6] on electromagnetic transmission through a single resonant aperture of the same shape as that in Fig. 1, the non-resonant sub-wavelength circular aperture can be resonated by loading double ridges across the aperture. In Fig. 1(a), D is the diameter of the aperture, W is the width of the ridge, and G is the gap between the ridge edges. The thickness of the PEC plane is t. All media except for the PEC plane are assumed to be free space with constitutive parameters (ε_0, μ_0). A plane wave polarized parallel to the ridge axis (y-axis direction) and propagating in the z-direction is assumed to be normally incident upon the plane ($z = 0$).

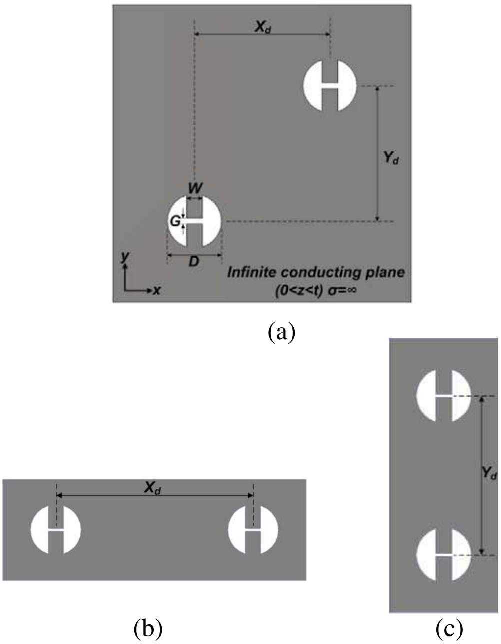


Figure 1. Geometry of a pair of ridge-loaded circular apertures in an infinite conducting plane: (a) Aperture pair in an arbitrary position in the plane. (b) Collinear case ($Y_d = 0$). (c) Parallel case ($X_d = 0$).

In recent studies [4–6], the problem for the case of a single resonant aperture in an infinitesimally thin conducting plane ($t = 0$) was solved numerically using both the MoM and the FDTD. In the analysis, the MoM employed the Rao-Wilton-Glisson (RWG) basis function, which uses a triangular basis function, and is efficient for the scattering problem from either closed conducting bodies or open surfaces [13]. In order to solve the problem of electromagnetic coupling through two identical apertures as shown in Fig. 1, the MoM used in the recent studies on the single aperture is applied. The procedure for solving the problem using the MoM is summarized as follows.

1) First, the equivalence principle is employed to divide the original problem into two equivalent situations in each region, $z > 0$ and $z < 0$, respectively

2) The fields in each region can be expressed in terms of the fields due to the incident plane wave and the equivalent magnetic current patches, which are equal to the tangential electric field distributions, over the two shorted slots.

3) The continuity of tangential fields across the apertures leads to the coupled integral equations for the magnetic current distributions.

4) By employing the RWG basis function, the equations are solved numerically.

5) From knowledge of the magnetic current distributions, one can obtain all the field components in each region and then the quantities of interests can be computed.

For the FDTD, the conformal finite-difference time-domain (CFDTD) code is used with the perfectly matched layer (PML) type of absorbing boundaries [14]. The analysis methods used for the single aperture problem are employed to study the dual aperture geometry in Fig. 1. The analysis procedures for the problem are described in detail in [4–6], and so are not given in this paper.

To investigate the effects of the direction and the distance between the two resonant apertures, two different configurations have been considered, as shown in Figs. 1(b) and 1(c). The two apertures are placed along the ridge direction for the "parallel" case ($X_d = 0$), while they are allocated perpendicular to the ridge direction for the "collinear" case ($Y_d = 0$).

According to Poynting's theorem, the transmitted power P_t to the region $z > t$ through the two resonant apertures can be obtained via the surface integral over the aperture area ($z = t$). The transmission cross section (TCS) is then calculated by normalizing P_t with the power density $\tilde{p}_0$ of the incident plane wave as

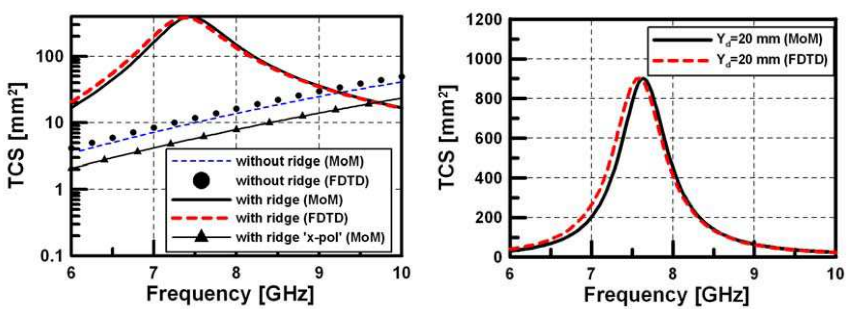
$$\text{TCS} = \frac{P_t}{\tilde{p}_0} \left[\text{m}^2\right] \tag{1}$$

in which $\tilde{p}_0 = E_0^2/(2\eta_0)[W/\mathrm{m}^2]$, E_0 is the magnitude of the incident electric field, and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$.

Figure 2 shows the variations of the TCS of a single resonant aperture and dual apertures with geometrical parameters $t = 0\,\mathrm{mm}$, $D = 10\,\mathrm{mm}$, $W = 3\,\mathrm{mm}$, and $G = 0.5\,\mathrm{mm}$. As shown in Fig. 2(a), the TCS obtained by the MoM is maximized to $396\,\mathrm{mm}^2$ at the resonant frequency $f = 7.44\,\mathrm{GHz}$, which is about a 40-fold (16 dB) increase from the value of $9.7\,\mathrm{mm}^2$ for the aperture without the ridge [4–6]. The maximum value of $396\,\mathrm{mm}^2$ agrees well with the TCS value ($= 3\lambda^2/4\pi = 388\,\mathrm{mm}^2$) of a small resonated aperture at the corresponding resonant frequency [4–6]. We note that the TCS values are plotted in log scale to show the differences clearly.

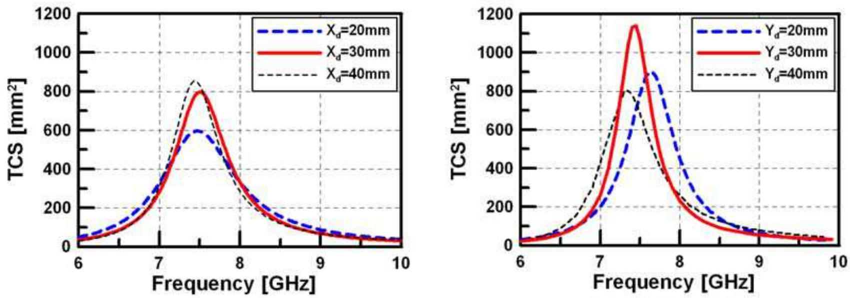
In Fig. 2(a), the "*x-pol*" case denotes that the electric field vector of the incident plane wave is changed to the x-direction, which is perpendicular to the ridge axis. The TCS for the "*x-pol*" case is decreased from that for the circular aperture without the ridges, since the loaded ridges block the aperture, making the effective aperture area (TCS) even less.

Figure 2(b) shows the TCS of the dual resonant apertures for the *parallel* case ($X_d = 0$) with $Y_d = 20\,\mathrm{mm}$. In Fig. 2(b), when it is compared to the single aperture case, the resonant frequency, obtained by the MoM, is shifted to $f = 7.64\,\mathrm{GHz}$, and the maximum TCS amounts to $901\,\mathrm{mm}^2$, which differs from twice the TCS of $390\,\mathrm{mm}^2$ of the single resonant slot. Due to the mutual coupling between two adjacent apertures, the transmission characteristics, such as TCS, resonant frequency, and transmitted field pattern, might be changed in either this parallel case ($X_d = 0$) or in the collinear case ($Y_d = 0$).



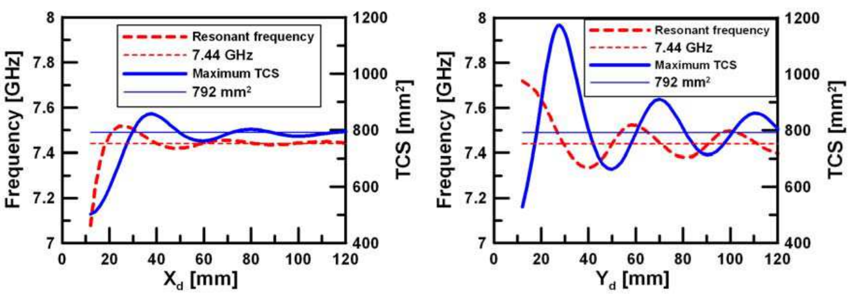
(a) (b)

Figure 2. TCS variations: (a) Single aperture cases. (b) Dual apertures ($X_d = 0\,\mathrm{mm}$, $Y_d = 20\,\mathrm{mm}$). $t = 0\,\mathrm{mm}$, $D = 10\,\mathrm{mm}$, $W = 3\,\mathrm{mm}$, and $G = 0.5\,\mathrm{mm}$.



(a) (b)

Figure 3. TCS characteristics [MoM]: (a) Collinear case. (b) Parallel case.



(a) (b)

Figure 4. Resonant frequency and maximum TCS [MoM]: (a) Collinear case. (b) Parallel case.

In Figs. 2(a) and 2(b), the results obtained by the FDTD are also plotted to show the validity of the data calculated by the MoM. The agreements between the data obtained by two different numerical methods verify well the results for the TCS.

In order to examine the effects of the distances X_d and Y_d on the transmission characteristics, we obtained the TCS as a function of frequency, and some plots for sampled distances are shown in Fig. 3. From Fig. 3, it is observed that the resonant frequency and the maximum TCS are changed, along with the variation of distance between two identical resonant apertures. They change more sensitively to the distance Y_d in the parallel case, rather than X_d in the collinear case.

The data for the resonant frequency and the maximum TCS are searched in terms of varying the distance between the apertures for the collinear and parallel cases, respectively, and the results are given in Fig. 4. In both cases of Fig. 4(a) and of Fig. 4(b), as the distances X_d and Y_d are increased, the resonant frequency converges to $f = 7.44\,\text{GHz}$ in the single aperture case, and the maximum TCS at resonant frequency approaches $792\,\text{mm}^2$, which is double the maximum TCS ($396\,\text{mm}^2$) of the single aperture. From Fig. 4, we observe that the transmission characteristics are more sensitive to the distance Y_d in the parallel case, rather than the X_d in the collinear case. Note that the transmission characteristics are not significantly affected by the distance X_d when it is larger than 70 mm. These phenomena are thought to be reasonable, considering that the mutual impedance between two identical resonant dipoles aligned side by side varies more sensitively to the distance than the collinearly arranged case [15, 16], because the mutual coupling between two resonant dipoles is the dual of this coupling between the two resonant apertures.

In a study on the gain enhancement of two parallel dipoles fed by uniform current sources [15], it was observed that the gain of an array of two identical resonant dipoles, arranged in parallel, is maximized to about 3 times the gain of single dipole when the spacing between the dipoles approaches to about 0.7λ at the resonant frequency (see Fig. 14 in [15]). From the duality theorem, if two resonant slots, which are the dual structure of the two resonant dipoles arranged in parallel, are fed by uniform voltage sources, the gain of the array of two slots would be maximized to about 3 times the gain of single resonant slot radiator when the spacing between the slots is near to 0.7λ. Then, from the reciprocity theorem, the TCS of the array of two resonant slots in parallel would be maximized to about 3 times the TCS of single resonant aperture when the plane wave, having an electric field component polarized perpendicular to the slot axis, is incident on the slots from the broadside direction which is the maximum radiation direction when the slots are fed by uniform voltage sources. Similarly, in this study on the TCS of two identical small resonant slots, the maximum TCS ($1{,}174\,\text{mm}^2$) in parallel case at the resonant frequency of 7.44 GHz corresponds to about three times compared to the single aperture case. In addition, the spacing $Y_d = 29\,\text{mm}$ for the maximum TCS corresponding to 0.72λ at $f = 7.44\,\text{GHz}$ agrees well with the spacing of 0.7λ between two dipoles for the maximum gain. In the same way, it is found that the variation in the gain of two dipole array, arranged in parallel, against the separation between the dipoles has nearly the same shaped curve as the TCS of two resonant apertures in parallel case [15].

Figure 5 compares the radiation patterns at the resonant frequency $f = 7.44\,\text{GHz}$ for the case of maximum TCS in Fig. 4(b). As shown in Fig. 5, the directivity in the E-plane for the parallel dual apertures having maximum TCS is improved much more compared to that for the single resonated aperture. Owing to the improvement in TCS (3 times = 4.77 dB) and the increase in the directivity in the radiation patterns, we have a maximum enhancement of 8.76 times (about 9.43 dB) in the power density in the broadside direction. In addition, the results obtained by the MoM agree very well with those obtained by the FDTD.

In the foregoing discussions, the results for the transmission characteristics of the resonant apertures are limited to the case of an infinitesimally thin conducting plane ($t = 0$), which is impractical since the thickness of the conducting sheet cannot be ignored in the simulation frequency range (5–10 GHz). Moreover, the simulation data for the TCS and radiation patterns are hard to verify by experiments (though the validity from the agreements between the results obtained

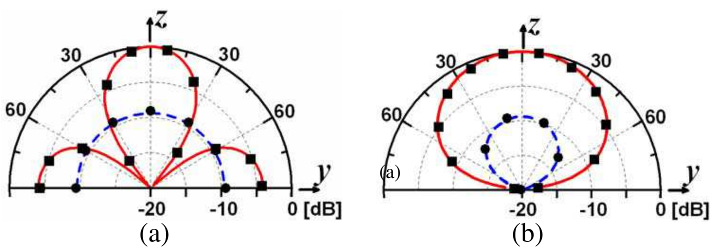


Figure 5. Radiation patterns at $f = 7.44$ GHz for the two parallel apertures having maximum TCS ($Y_d = 29$ mm): (a) E-plane (yz-plane). (b) H-plane (xz-plane). $t = 0$ mm, $D = 10$ mm, $W = 3$, and $G = 0.5$ mm. Dual aperture: Solid line (MoM), ■ (FDTD). Single aperture: Dashed line (MoM), • (FDTD).

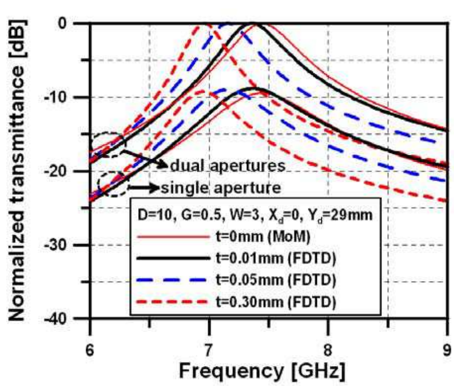


Figure 6. Normalized transmittance of a single aperture and parallel aperture pair ($Y_d = 29$ mm) for $t = 0$ mm, 0.01 mm, 0.05 mm, and 0.30 mm [MoM and FDTD].

by using two different methods, MoM and FDTD, has already been shown in Fig. 2). Therefore, what is needed is to check the effect of thickness t of the conducting plane by observing some measurable quantities that are closely related to the transmission characteristics of the apertures. The power density at an observation point that is sufficiently distant in the broadside direction from the conducting plane might be included in suitable quantities.

It is worthwhile to mention that the electromagnetic coupling problem through apertures in a conducting plane with finite thickness ($t > 0$) can be solved by MoM only for some special cases of aperture shapes, so that the fields inside the conducting plane can be expressed as the sum of eigen modes [17]. Otherwise, the MoM cannot be directly employed, but should be combined with another numerical method, such as the finite element method (FEM), to solve the problem. Since the FDTD is not limited by the conductor thickness, we used the FDTD in order to obtain the data for the transmission characteristics of the apertures in a conducting plane of finite thickness.

The results in Fig. 6 are the normalized power density at a distance of 50 cm from the apertures in the broadside direction for $t = 0$ mm, 0.01 mm, 0.05 mm, and 0.30 mm. Here, the transmittance is a ratio of the power density at an observation point to the incident power density of the plane wave arriving at the apertures. It is clearly seen that the resonant frequency is decreased, and the quality factor of the frequency response is increased as the thickness of the conducting plate is increased, which is due to the increased effective capacitance formed between ridges loaded inside the circular aperture. In addition, for $t = 0.30$ mm, the difference in the transmittance between the single aperture and the dual apertures at $f = 6.96$ GHz is about 9.27 dB (8.45 times), that is, much higher than the difference in the TCS, which is predictable from the directivity enhancement in the far-zone radiation patterns in Fig. 5.

3. EXPERIMENTAL RESULTS

To validate the numerical results, an experimental set-up consisting of a vector network analyzer (VNA) and two broadband horns, as shown in Fig. 7, is used to measure the transmission characteristics of the two fabricated resonant apertures. The transmitting horn (1) is positioned at a perpendicular distance of 50 cm from the sample aperture and oriented such that the incident electric field vector is in the y-direction. A broadband amplifier (SHF Communication Technologies, SHF100APP) with Gain $>$ 18 dB is employed between the VNA (Anritsu 37397C) and the transmitting antenna to amplify the input power density to the sample. A 40 mm × 100 mm rectangular hole surrounded by a pyramidal microwave absorber is positioned between the broadband horn antennas. The horn antennas (ETS-Lindgren, Model # 3115) are double-ridged waveguide horns with a gain of 9.5–11.5 dBi over 5–10 GHz and the dimension is 244 mm(width) × 279 mm(depth) × 159 mm (height).

Two sample apertures are fabricated on a 110 mm × 160 mm stainless steel plate with 0.3 mm thickness, as shown in Fig. 8. One is a single ridge-loaded aperture and the other is a pair of apertures in parallel with the distance for the maximum TCS ($Y_d = 29$ mm). The receiving horn (2) is positioned at a perpendicular distance of 50 cm from the exit side of the sample. The samples are mounted in the center of the hole in the pyramidal absorber. The transmission characteristics of the sample can be obtained from the received power density of the receiving horn, which can be calculated from the measured S_{21} of the VNA.

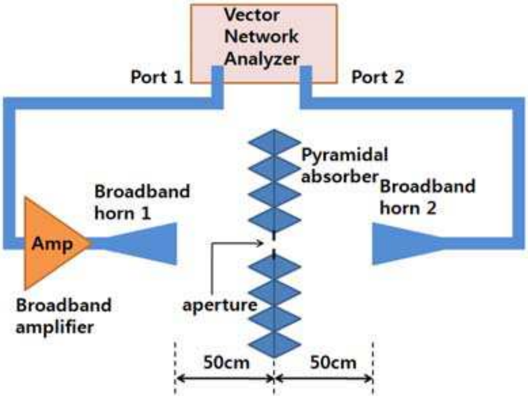


Figure 7. Experimental set-up for measurement.

(a) (b)

Figure 8. Photographs of apertures fabricated on a stainless steel plate: (a) Single aperture. (b) Dual apertures ($Y_d = 29$ mm).

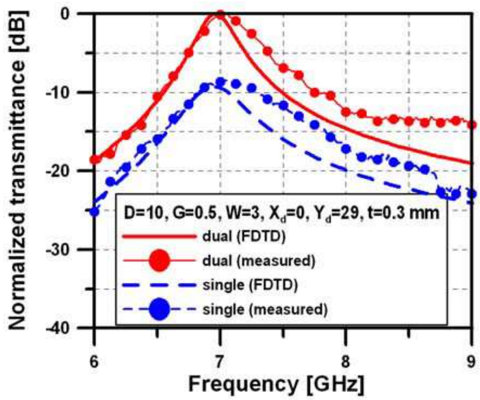


Figure 9. Comparison of the simulated and the measured transmittance characteristics of a single aperture and a parallel aperture pair ($Y_d = 29\,\text{mm}$) for $t = 0.3\,\text{mm}$.

Figure 9 presents the comparison of the simulated and the measured transmittance characteristics of a single aperture and a parallel aperture pair ($Y_d = 29\,\text{mm}$) for $t = 0.3\,\text{mm}$. The measured TCS for the apertures is maximized at $f = 7.05\,\text{GHz}$, which is close to the simulated resonant frequency $f = 6.96\,\text{GHz}$. The measured maximum transmittance of the dual apertures is 8.43 dB higher than that of the single aperture, while there is a gap of 9.27 dB between the simulated maximum transmittances for the single and dual apertures. In Fig. 9, the transmittances of the apertures, obtained by the FDTD, are normalized to the maximum level of $-17.62\,\text{dB}$ at resonance and the transmittances measured by a VNA are normalized to the maximum of $-23.75\,\text{dB}$ which is 6.13 dB smaller than the simulated maximum. The difference between the simulated and measured results are thought to be mainly due to the finite effect of the conducting plane, surrounded by the pyramidal microwave absorber, on which the electromagnetic wave is incident. Although the measured data levels are somewhat different from the simulated data, the resonant frequency and the enhancement in the transmittance are close to the simulation, as shown in Fig. 9, which again verifies the numerical results in this study. The measured transmittance is obtained from the difference between the $|S_{21}|_{\text{dB}}$ measured by a VNA under two situations. One is the experimental set-up as shown Fig. 7, while, in the other situation, the absorbing wall containing the conducting plate is eliminated and two horns are faced each other at a distance of 50 cm.

The results and the methods of the ridge-loaded aperture pair presented in this paper are expected to be useful for the research on achieving very high transmission efficiency through a sub-wavelength aperture, multiple apertures, or aperture arrays in the microwave and optics areas [2, 3].

4. CONCLUSIONS

We have presented an example of resonant transmission through a pair of ridge-loaded circular sub-wavelength apertures. The effect of the distance between the two resonant circular sub-wavelength apertures in an infinite perfect electric conductor (PEC) plane on electromagnetic wave transmission is analyzed. The transmission characteristics of the aperture pair are investigated for two different alignment configurations: in the "parallel" case, the apertures are allocated along the ridge direction, while, in the "collinear" case, they are placed perpendicular to the ridge direction. From the numerical results, it is found that the TCS for the parallel case varies more sensitively to the distance between the apertures than that for the collinearly located case. The maximum TCS for the parallel case is tripled compared to the TCS value of a single resonant aperture, and the directivity for the maximum TCS case is 9.43 dB increased from that for the single resonated aperture.

For the purpose of validation, a single resonant aperture and a pair of resonant apertures in the parallel case with a distance for maximum TCS ($Y_d = 29\,\mathrm{mm}$) are fabricated on a stainless steel plate with 0.3 mm thickness, and their transmission characteristics are measured. Experimental results show that the transmittance, which is a transmitted power density measured at 50 cm away from the aperture plane, for the parallel resonant apertures is about 7 times (8.43 dB) higher than that for a single aperture, which agrees fairly well with the numerical results. We expect that the results and the methods for the ridge-loaded aperture pair presented in this paper will be useful for further research about resonant transmission through multiple apertures or aperture arrays in the microwave, millimeter wave, and optics regions.

ACKNOWLEDGMENT

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF), funded by the Ministry of Education, Science, and Technology (2011-0027022), and also funded by the fifth stage of Brain Korea 21 (BK21).

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